What do I do?

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November 09, 2015

- PhD in 2012 on neutron star (NS) spacetimes
- 6 month DAAD scholarship to work with K. Kokkotas
- Postdoc with T. Sotiriou that started at SISSA (2013)...
- ... and continued at the U. Nottingham (2014-15)
- This summer moved to Oxford, MS

- Neutron star spacetimes. Analytic spacetimes with appropriate matching parameters. Comparison between numerical spacetimes and their analytic counterparts.

- Neutron star multipole moments. Calculation from numerical models, range for realistic neutron stars, and properties and relations between them (universal "3-hair" relations).

- Properties of Neutron star spacetimes and astrophysical applications (neutron star spacetimes are *not* Kerr). QPOs and relativistic precession model. Properties of accretion discs in neutron star spacetimes (spectra, selftrapping c-modes).

- Neutron stars in alternative theories of gravity. Post-TOV formalism, a theory independent approach to neutron star properties and structure.

- Characterising spacetimes in alternative theories of gravity. Multipole moments and geodesics of spacetimes in scalar-tensor theories of gravity.

Rotating neutron Stars

The line element for a stationary and axially symmetric spacetime (the spacetime admits a timelike, ξ^a, and a spacelike, η^a, killing field, i.e., it has rotational symmetry and symmetry in translations in time) is [1],

$$ds^2 = -e^{2\nu}dt^2 + r^2 \sin^2\theta B^2 e^{-2\nu}(d\phi - \omega dt)^2 + e^{2(\zeta-\nu)}(dr^2 + r^2 d\theta^2).$$

Field equations in the frame of the ZAMOs:

$$\mathbf{D}\cdot(B\mathbf{D}\nu) = \frac{1}{2}r^2\sin^2\theta B^3 e^{-4\nu}\mathbf{D}\omega\cdot\mathbf{D}\omega + 4\pi B e^{2\zeta-2\nu}\left[\frac{(\epsilon+p)(1+u^2)}{1-u^2} + 2p\right],$$

$$\mathbf{D}\cdot(r^2\sin^2\theta B^3 e^{-4\nu}\mathbf{D}\omega) = -16\pi r\sin\theta B^2 e^{2\zeta-4\nu}\frac{(\epsilon+p)u}{1-u^2},$$

$$\mathbf{D}\cdot(r\sin\theta\mathbf{D}B) = 16\pi r\sin\theta B e^{2\zeta-2\nu}p,$$

Komatsu, Eriguchi, and Hechisu[2] proposed a scheme for integrating the field equations using Green's functions. This scheme is implemented by the RNS numerical code to calculate rotating neutron stars [3].

Asymptotic expansion of the metric functions:

$$\nu = \sum_{l=0}^{\infty}\nu_{2l}(r)P_{2l}(\mu), \quad \omega = \sum_{l=0}^{\infty}\omega_{2l}(r)P_{2l+1,\mu}(\mu), \quad B = \sum_{l=0}^{\infty}B_{2l}(r)T_{2l}^{1/2}(\mu),$$

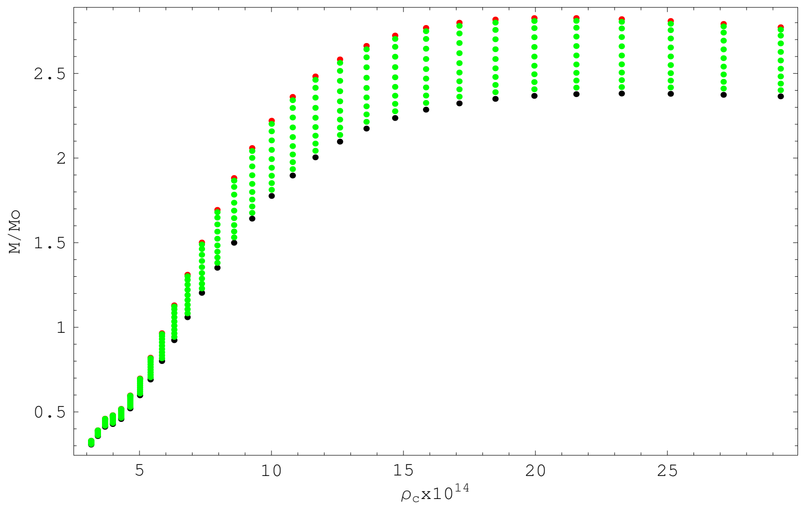
where P_l are the Legendre polynomials, $\mu = \cos\theta$, and $T_l^{1/2}$ are the Gegenbauer polynomials.

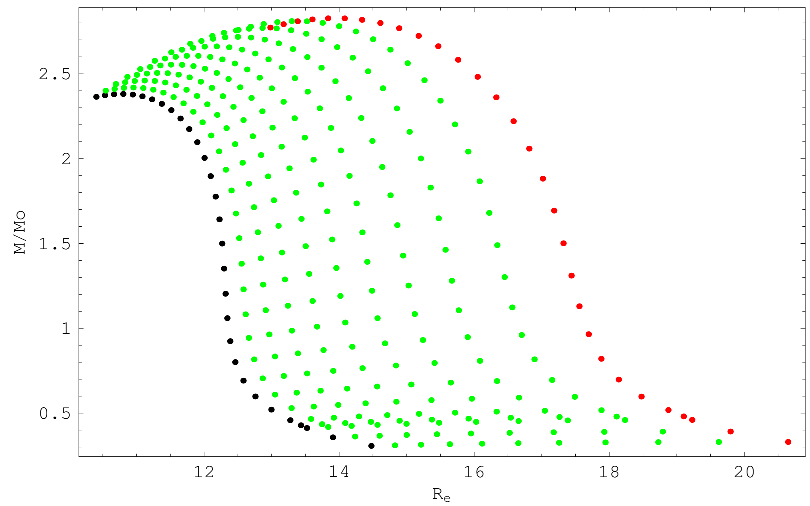
[1] E. M. Butterworth and J. R. Ipser, ApJ **204**, 200 (1976).

[2] H. Komatsu, Y. Eriguchi, and I. Hechisu, MNRAS **237**, 355 (1989).

[3] N. Stergioulas, J.L. Friedman, ApJ, **444**, 306 (1995).

One can use RNS to calculate models of rotating neutron stars for a given equation of state. For example we show here some models for the APR EOS:





The models with the fastest rotation have a spin parameter, $j = J/M^2$, around 0.7 and a ratio of the polar radius over the equatorial radius, r_p/r_e, around 0.56.

The code, except from the various physical characteristics of the neutron stars, provides the metric functions in a grid on the coordinates x and μ in the whole space (for values from 0 to 1 for both variables), where $\mu = cos\theta$, $r = \frac{xr_e}{1-x}$ and r_e is a length scale.

But, numerical spacetimes in tabulated form are not very practical. An analytic spacetime that captures the neutron star spacetime properties would be very useful.

How should we relate the neutron star spacetime to an analytic spacetime?
Relativistic multipole moments characterise stationary axisymmetric spacetimes.

Multipole moments of numerical spacetimes

The RNS code can calculates the first non-zero multipole moments, i.e., M, $S_1 \equiv J$, M_2, S_3 and M_4[4].

$$\begin{aligned}
M_0 &= M, \\
S_1 &= jM^2, \\
M_2 &= -\frac{1+4b_0+3q_2}{3}M^3, \\
S_3 &= -\frac{3(2j+8jb_0-5w_2)}{10}M^4, \\
M_4 &= \frac{19-18j^2+160b_0+120q_2+336b_0^2+360b_0q_2-105q_4-192b_2}{105}M^5
\end{aligned}$$

where $j \equiv J/M^2$, and the various parameters are given by the integrals,

$$\begin{aligned}
Q_{2l} = M^{2l+1}q_{2l} &= -\frac{r_e^{2l+1}}{2}\int_0^1 \frac{ds' s'^{2l}}{(1-s')^{2l+2}}\int_0^1 d\mu' P_{2l}(\mu')\tilde{S}_\rho(s',\mu'), \\
W_{2l-2} = M^{2l}w_{2l-2} &= -\frac{r_e^{2l}}{4l}\int_0^1 \frac{ds' s'^{2l}}{(1-s')^{2l+2}}\int_0^1 d\mu' \sin\theta' P^1_{2l-1}(\mu')\tilde{S}_{\hat{\omega}}(s',\mu'), \\
\tilde{B}_{2l} = M^{2l+2}b_{2l} &= -\frac{16\sqrt{2}\pi r_e^{2l+4}}{2l+1}\int_0^{1/2} \frac{ds' s'^{2l+3}}{(1-s')^{2l+5}}\int_0^1 d\mu' \sin\theta' p(s',\mu')Be^{2(\zeta-\nu)}T^{1/2}_{2l}(\mu'),
\end{aligned}$$

where in the second integral $l \geq 1$, in contrast to the other two integrals where $l \geq 0$.

[4] G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012),
K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Neutron star multipole moments properties in GR

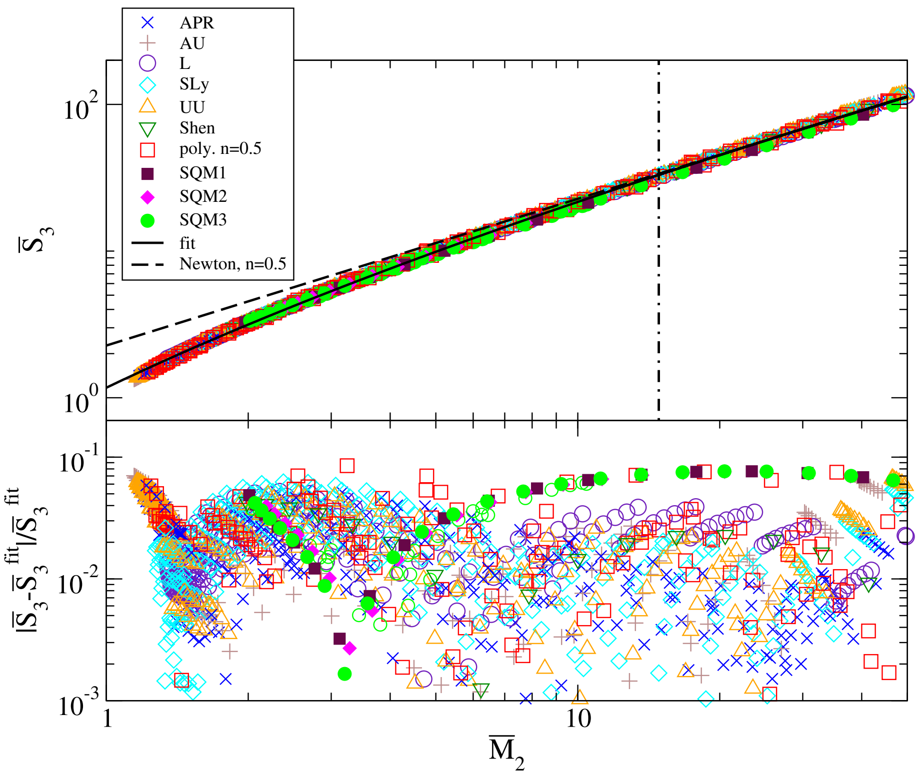
Black Hole-like behaviour of the moments[5]:

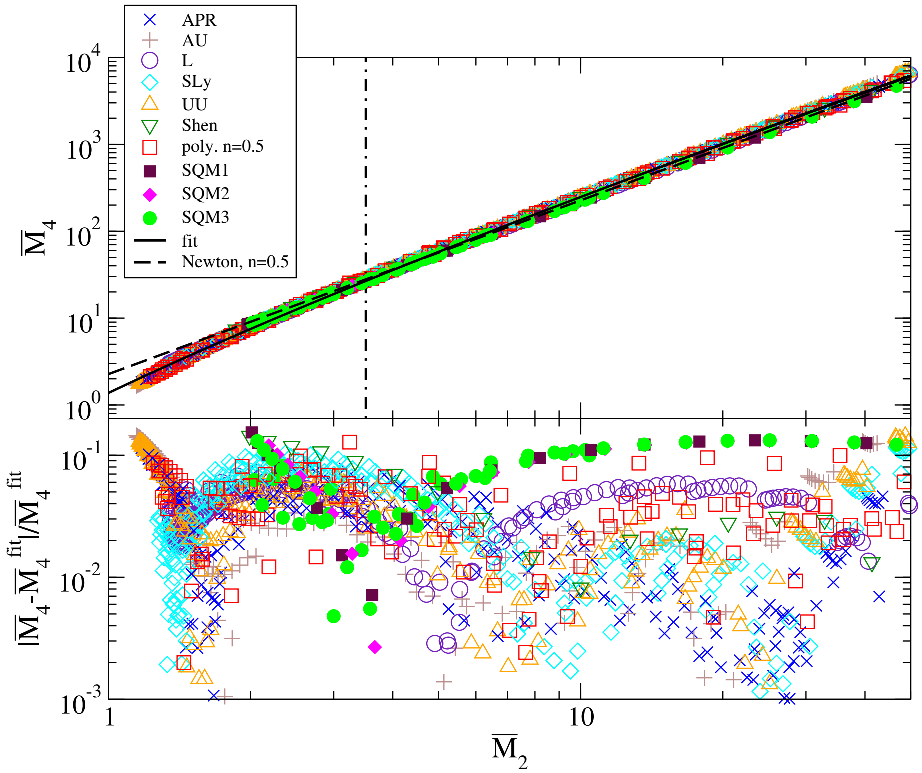
Kerr moments	Neutron star moments
$M_0 = M,$	$M_0 = M,$
$J_1 = J = jM^2,$	$J_1 = jM^2,$
$M_2 = -j^2M^3,$	$M_2 = -a(EoS, M)j^2M^3,$
$J_3 = -j^3M^4,$	$J_3 = -\beta(EoS, M)j^3M^4,$
$M_4 = j^4M^5,$	$M_4 = \gamma(EoS, M)j^4M^5,$
$\vdots$	$\vdots$
$M_{2n} = (-1)^n j^{2n} M^{2n+1},$	$M_{2n} = ?,$
$J_{2n+1} = (-1)^n j^{2n+1} M^{2n+2}$	$J_{2n+1} = ?$

[5] W.G. Laarakkers and E. Poisson, Astrophys. J. **512** 282 (1999).
G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012).
K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Neutron star multipole moments properties in GR

EoS independent behaviour of the moments[6] :





$$\bar{M}_{2n} = |M_{2n}/(j^{2n}M^{2n+1})|, \ \bar{J}_{2n+1} = |J_{2n+1}/(j^{2n+1}M^{2n+2})|$$

All these are properties that characterize the spacetime around neutron stars as well as the gravitational aspects of the stars themselves.

[6] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014).
K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

An analytic neutron star spacetime would be very useful to do astrophysics.

The vacuum region of a stationary and axially symmetric space-time can be described by the Papapetrou line element[7],

$$ds^2 = -f\left(dt - w d\phi\right)^2 + f^{-1}\left[e^{2\gamma}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right],$$

where f, w, and γ are functions of the Weyl-Papapetrou coordinates (ρ, z).

By introducing the complex potential $\mathcal{E}(\rho, z) = f(\rho, z) + i\psi(\rho, z)$ [8], the Einstein field equations take the form,

$$(Re(\mathcal{E}))\nabla^2\mathcal{E} = \nabla\mathcal{E}\cdot\nabla\mathcal{E},$$

where, $f = \xi^a\xi_a$ and ψ is defined by, $\nabla_a\psi = \varepsilon_{abcd}\,\xi^b\nabla^c\xi^d$.

An algorithm for generating solutions of the Ernst equation was developed by Sibgatullin and Manko [9]. A solution is constructed from a choice of the Ernst potential along the axis of symmetry in the form of a rational function

$$\mathcal{E}(\rho = 0, z) = e(z) = \frac{P(z)}{R(z)},$$

where $P(z), R(z)$ are polynomials of z of order n with complex coefficients in general.

[7] A. Papapetrou, Ann. Phys., **12**, 309 (1953).

[8] F.J. Ernst, Phys. Rev., **167**, 1175 (1968); Phys. Rev., **168**, 1415 (1968).

[9] V.S. Manko, N.R. Sibgatullin, Class. Quantum Grav., **10**, 1383 (1993).

Two-Soliton spacetime: This is a 4-parameter analytic spacetime which can be produced if one chooses the Ernst potential on the axis to have the form:

$$e(z) = \frac{(z - M - ia)(z + ib) - k}{(z + M - ia)(z + ib) - k}$$

The parameters a, b, k of the spacetime can be related to the first non-zero multipole moments through the equations,

$$J = aM, \quad M_2 = -(a^2 - k)M, \quad J_3 = -\left[a^3 - (2a - b)k\right] M,$$

where M is the mass.

One can use the multipole moments M, J, M_2, and J_3 of a numerically calculated neutron star and produce an analytic two-soliton spacetime that reproduces very accurately the numerically calculated spacetime [10]. Instead of using a specific set of values for the moments, one could reproduce any neutron star spacetime using the universal relations

$$\sqrt[3]{\bar{J}_3} = A + B_1 \left(\sqrt{\bar{M}_2}\right)^{\nu_1} + B_2 \left(\sqrt{\bar{M}_2}\right)^{\nu_2},$$

Therefore the first higher moments of a general neutron star spacetime can be expressed in terms of only three parameters, the mass M, the angular momentum J, and the quadrupole M_2,[11] having thus a universal analytic spacetime.

[10] G. P., and T. A. Apostolatos, MNRAS, **429**, 3007 (2013); Other analytic spacetimes have been proposed in the past, see for eg. E. Berti, and N. Stergioulas, MNRAS, **350**, 1416 (2004)

[11] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014): $\sqrt[3]{\bar{J}_3} = -0.36 + 1.48 \left(\sqrt{\bar{M}_2}\right)^{0.65}$

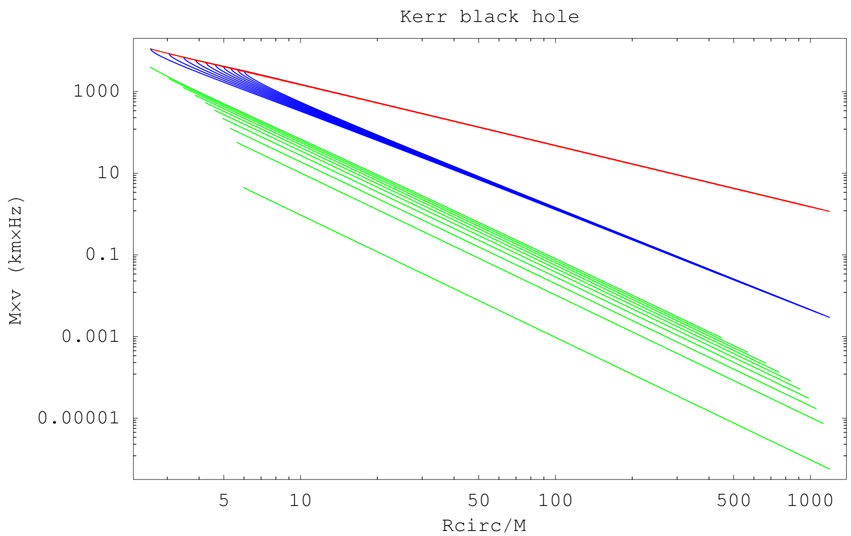
"Scaled" Frequencies for the Kerr spacetime:

Scaled Orbital frequency: $M\Omega = \frac{299790\sqrt{1/r^3}}{1+j(1/r)^{3/2}}$

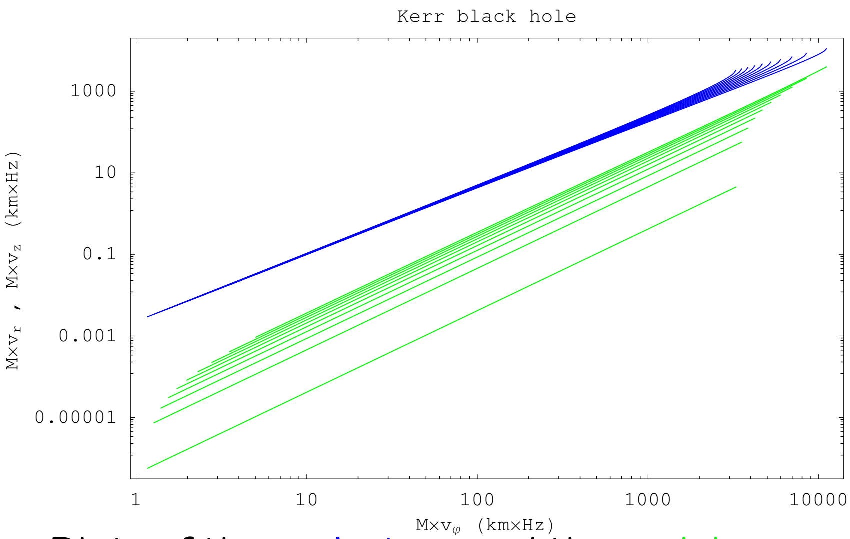
Scaled Radial frequency: $M\kappa_\rho = M\Omega\left(1 - 6(1/r) + 8j(1/r)^{3/2} - 3j^2(1/r)^2\right)^{1/2}$

Scaled Vertical frequency: $M\kappa_z = M\Omega\left(1 - 4j(1/r)^{3/2} + 3j^2(1/r)^2\right)^{1/2}$

Orbital, $M\nu_\phi$, and precession, $M\nu_a$, "scaled frequencies" for Kerr black holes for various j (0.01-0.91).



Plots of the orbital, periastron and nodal precession "scaled frequencies".

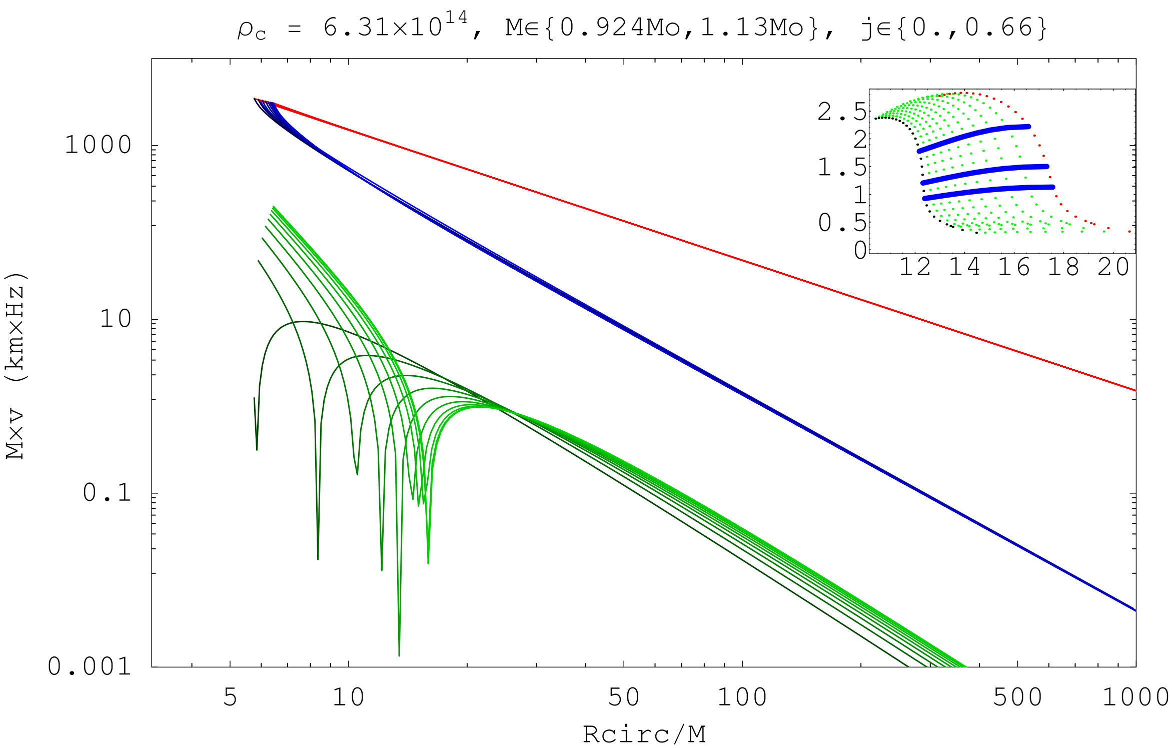


Plots of the periastron and the nodal precession "scaled frequencies" against the orbital "scaled frequency".

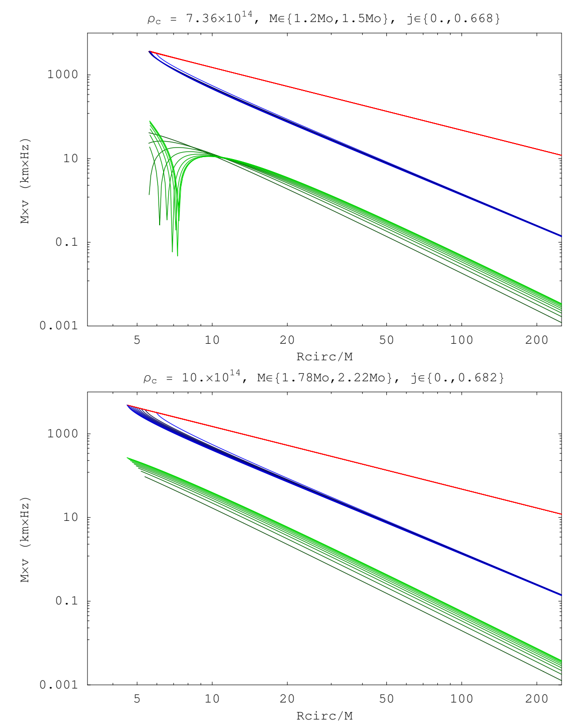
The general effect of rotation is to increase the observed frequencies (and reduce the ISCO radius; for $j \sim 1$ the horizon and ISCO radii go to $1M$).

Frequencies for the spacetime around neutron stars: The effect of rotation

Orbital frequency, periastron precession frequency and nodal precession frequency:
Orbital, $M\nu_\phi$, and precession, $M\nu_a$, ‘‘scaled frequencies’’ for neutron star models constructed with the APR EOS for various j up to the Kepler limit(0-0.7) and the same central density.

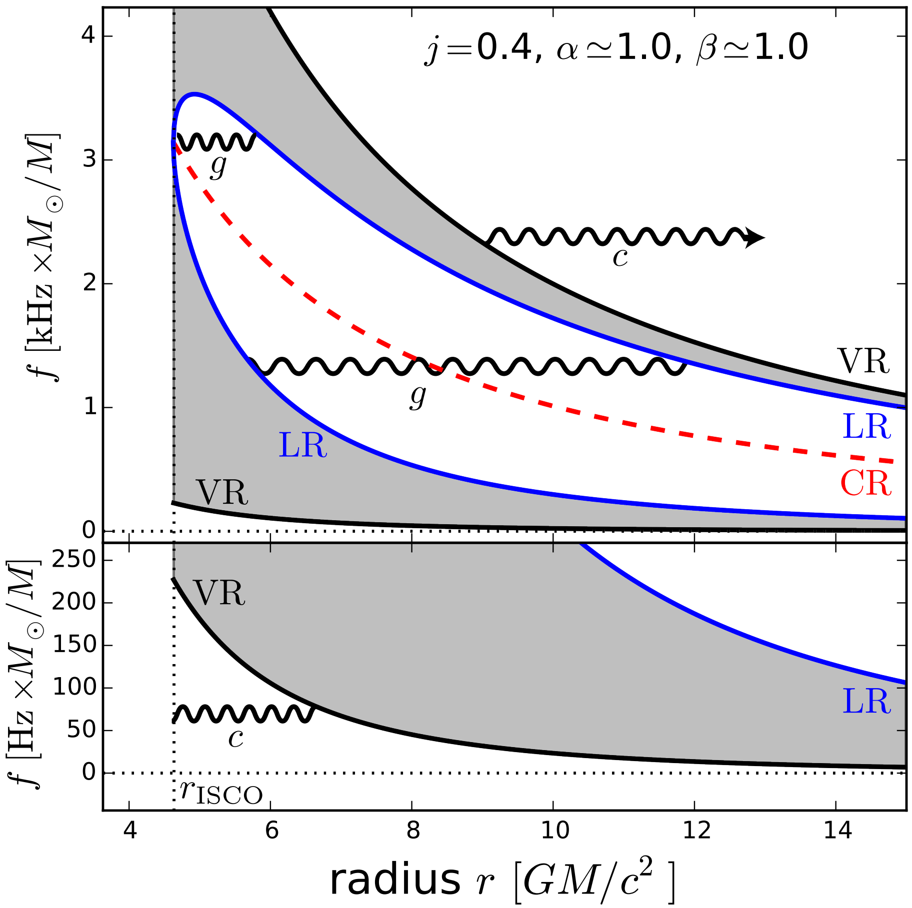


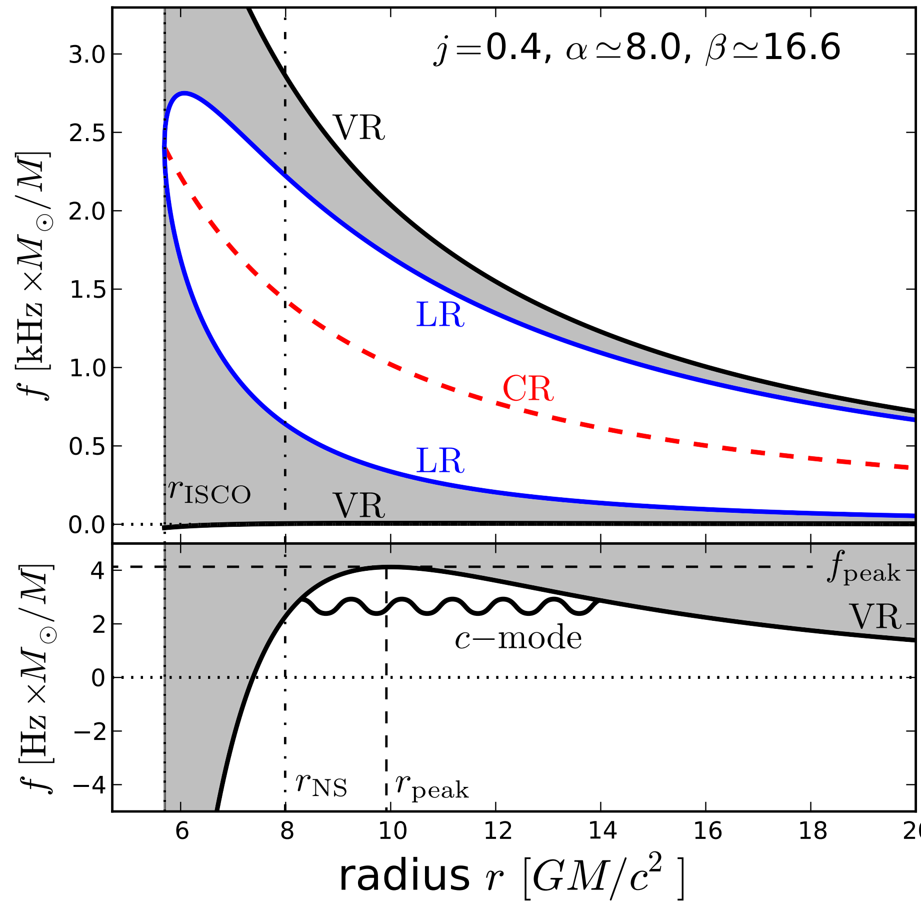
Plots of the orbital, periastron and nodal precession frequencies for different rotations for models with $\rho_c = 6.3 \times 10^{14} g/cm^3$. Rotation in the range, $f \sim 0.3 - 0.9$kHz.



Same plots for $\rho_c = 7.3 \times 10^{14} g/cm^3$ (upper) and $\rho_c = 10^{15} g/cm^3$ (lower).

Application: Corrugation modes in accretion disks around neutron stars.

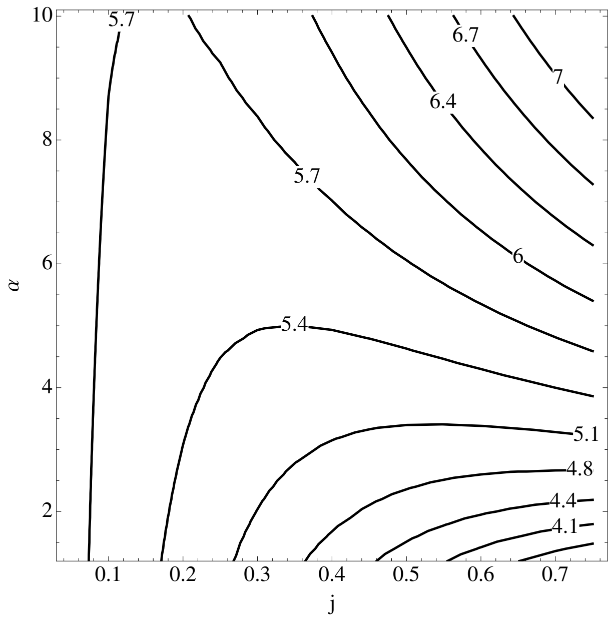




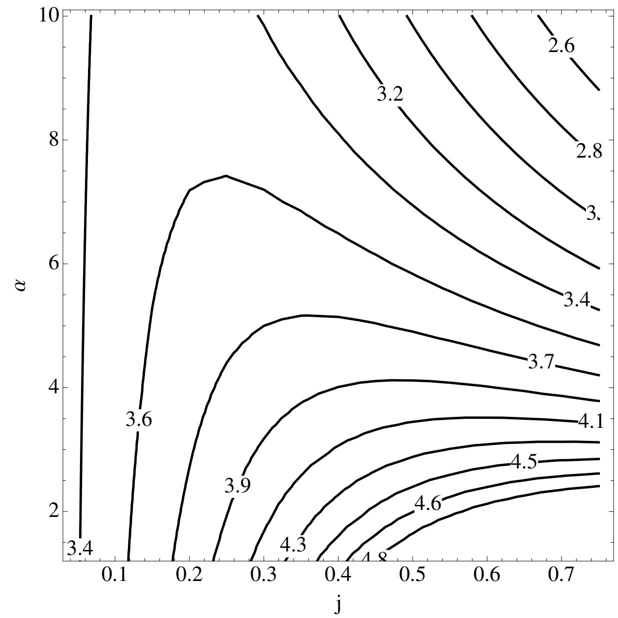
Upper: Diskoseismic propagation diagram for a Kerr black hole with spin parameter $j = 0.4$ for one-armed waves with $m = 1, n = 1$. Waves can propagate in the white regions exterior to $r_{\rm ISCO}$, and are evanescent in the shaded regions between the vertical resonances (VR) and Lindblad resonances (LR). Inertial modes (g-modes), with $m = 1, n = 1$, can become self-trapping due to the turnover of the outer Lindblad resonance, while lower frequency g-modes are quickly damped by corotation (CR). Corrugation waves can propagate at high frequencies exterior to the outer VR, and at low-frequencies interior to the inner VR. **Lower**: Enlargement of the propagation diagram at low frequencies.

Upper: Same plot but for a neutron star spacetime with spin parameter $j = 0.4$, quadrupole rotational deformability $\alpha = 8$, and spin-octupole deformability $\beta \simeq 16.6$. Waves with frequency $f = \omega/2\pi$ can propagate in the white regions exterior to the NS radius $r_{\rm NS}$ (or wherever the disk is truncated). Wave regions are qualitatively similar to the Kerr black hole, except for the low-frequency c-mode region, where $\omega < \Omega - \Omega_\perp$. **Lower**: At low frequencies c-modes can be self-trapped due to the turnover of the Lense-Thirring frequency, $\Omega - \Omega_\perp$, at radius $r_{\rm peak}$, and frequency $f_{\rm peak}$, as a result of the spacetime quadrupole contribution.

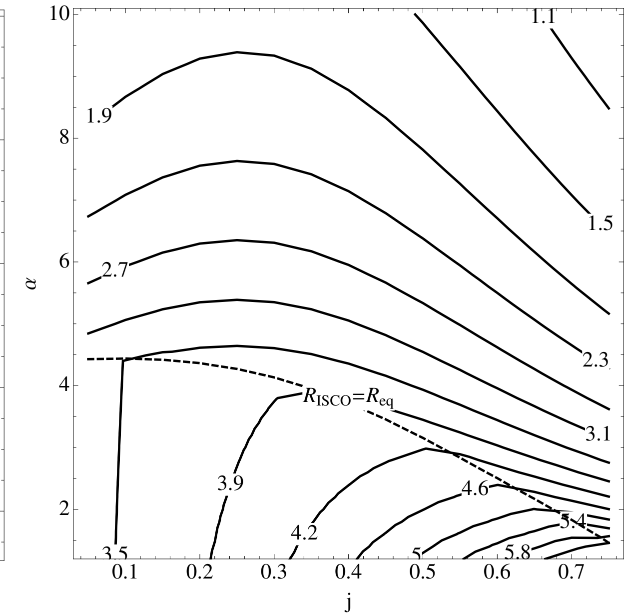
Orbital and precession frequencies: G.P. MNRAS 454, 4066 (2015)



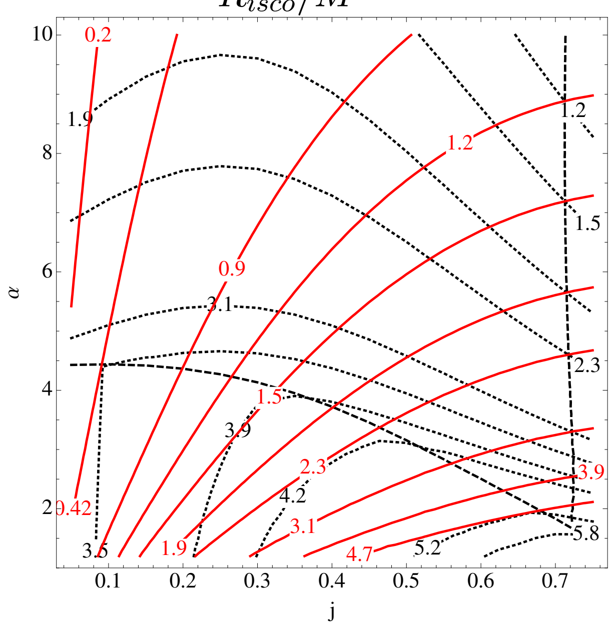
R_{isco}/M



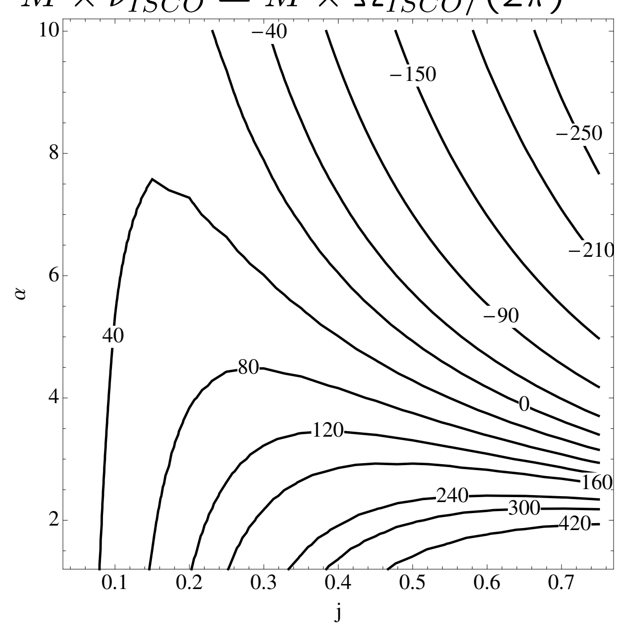
$M \times \nu_{ISCO} = M \times \Omega_{ISCO}/(2\pi)$



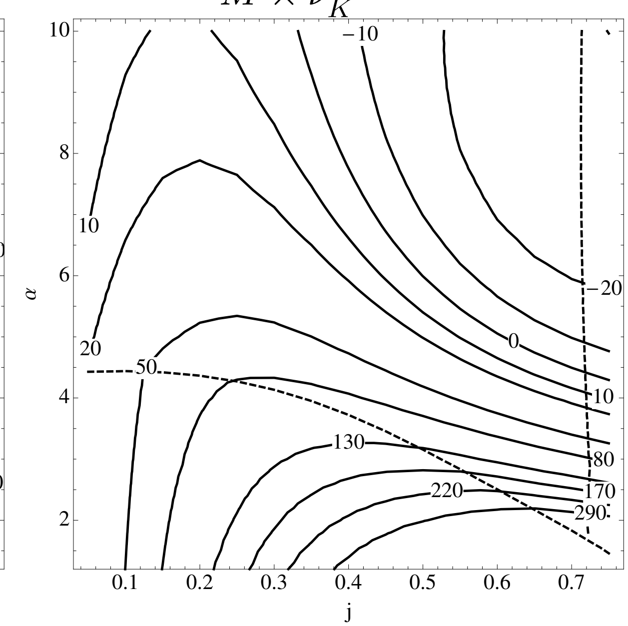
$M \times \nu_K^{max}$



$M \times \nu_K^{max}$ & f_{NS}

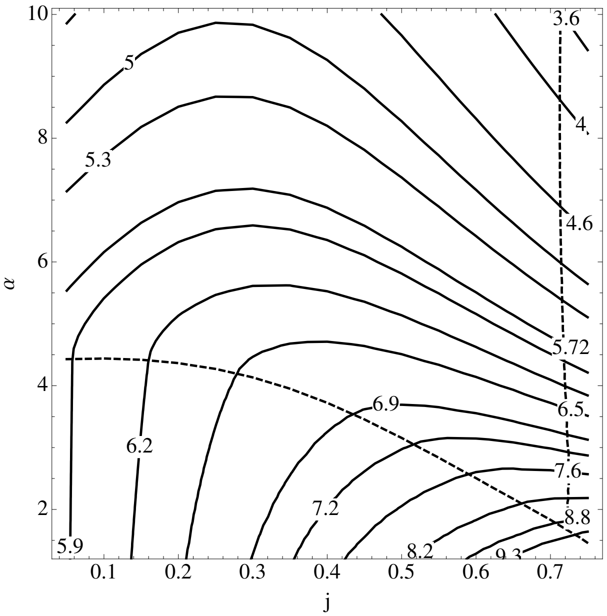


$M \times (\Omega_z/2\pi)_{ISCO}$

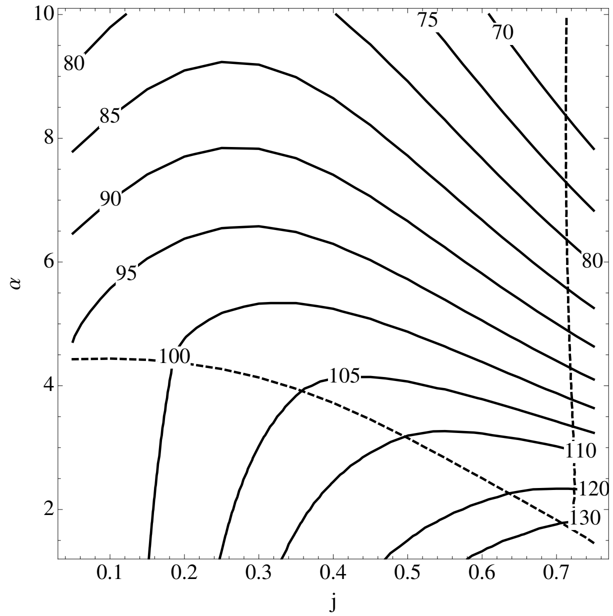


$M \times (\Omega_z/2\pi)_{inn}$

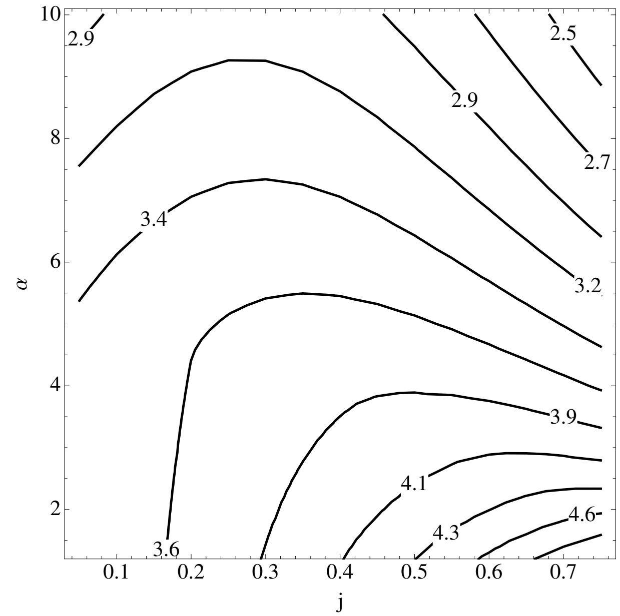
Accretion disc properties:



$\eta = 1 - \tilde{E}\big|_{\text{in}}$



$\bar{T}^{\text{max}}_{\text{eff}}$ ($\dot{M}_0$=10$^{-12}M_\odot$year^{-1})



$\nu L_\nu / \dot{M}_0$

Some basic scalings:

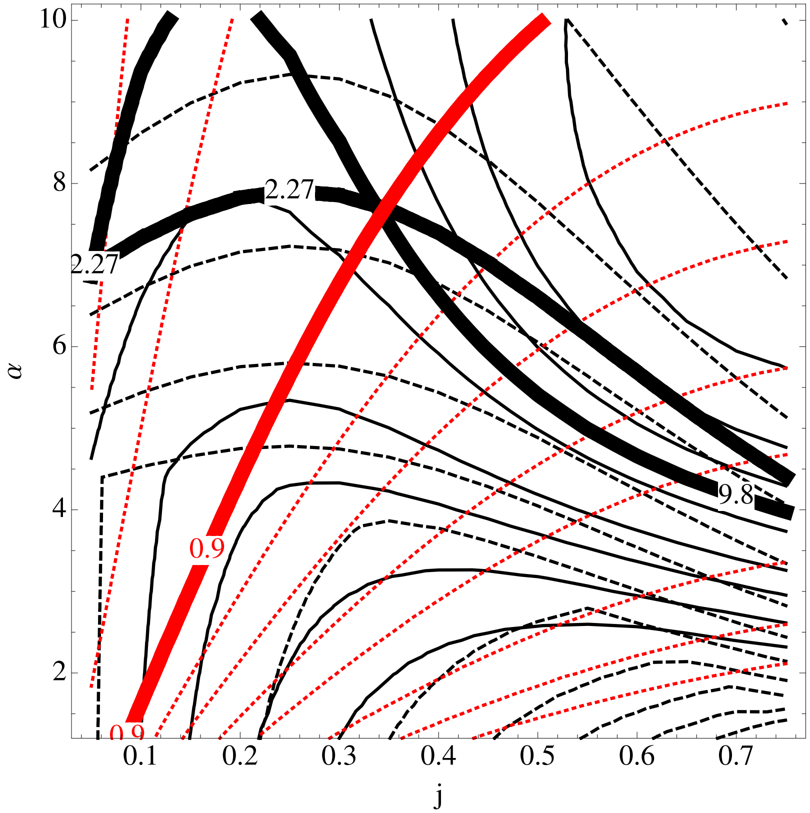
$\rho = M\bar{\rho}$, $\Omega = M^{-1}\bar{\Omega}$, $\tilde{E} = M^0\bar{\tilde{E}}$, and $\tilde{L} = M\bar{\tilde{L}}$.

Some more involved scalings:

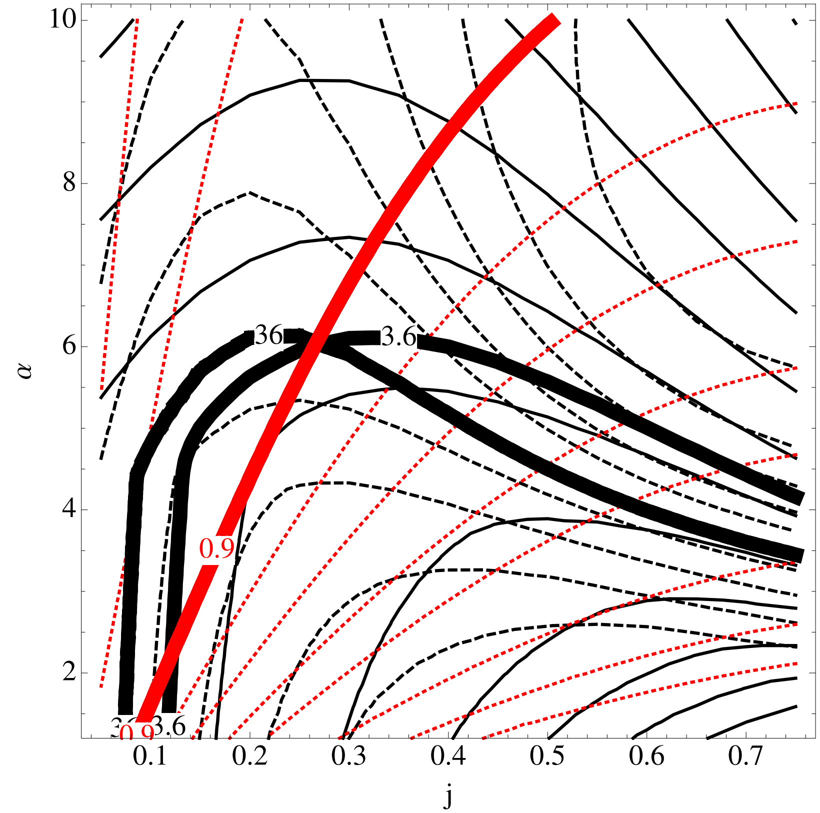
$F(\rho)/\dot{M}_0 = M^{-2}\overline{\left(F(\bar{\rho})/\dot{M}_0\right)}$, $T = M^{-1/2}(\dot{M}_0)^{1/4}\bar{T}$, $\varepsilon_{h\nu} = M^{-1/2}(\dot{M}_0)^{1/4}\bar{\varepsilon}_{h\nu}$,

and finally, $L_\nu = M^{1/2}(\dot{M}_0)^{3/4}\bar{L}_\nu$, and $\nu L_\nu = (\dot{M}_0)\overline{\nu L_\nu}$.

Combining the different properties:



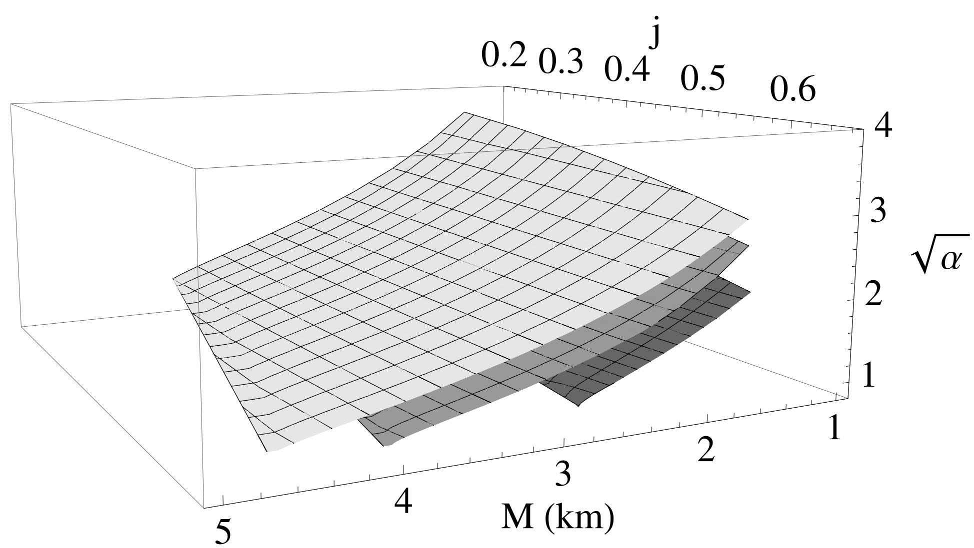
Contour plots of the orbital frequency at the orbit closest to the stellar surface (dashed black lines), the nodal precession frequency at the same orbit (solid black lines), and the rotation frequency of the star itself (dotted red lines)..



Contour plots of the maximum integrated luminosity (solid black lines), the nodal precession frequency at the orbit closest to the stellar surface (dashed black lines), and the rotation frequency of the star itself (dotted red lines).

Identifying the EoS: Determining the parameters α and j and the independent knowledge of the mass of the neutron star (assuming for example that it is known from the binary system observations), one can evaluate the first three multipole moments.

Such a “measurement”[12] of the first 3 moments (M, J, M_2) could select an EOS[13] out of the realistic EOS candidates.



[12] G.P. MNRAS **454** 4066 (2015),
and for an alternative proposal see, G.P., 2012 MNRAS, 422, 2581-2589.

[13] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014)

Thank You.