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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 9

Problem 19 – Density Operator

Given the density operator of a quantum system

$$\hat{\rho} = \sum_n c_n |\psi_n\rangle\langle\psi_n| \quad \text{with} \quad c_n \in [0,1]\ ,\ \sum_n c_n = 1\,, \tag{1}$$

answer the following question:

have the states $|\psi_n\rangle$ to be orthogonal in general?

and prove the following statements:

(a) the eigenvalues λ_i of $\hat{\rho}$ are real; (b) $\lambda_i \in [0,1]$; (c) $\sum_i \lambda_i = 1$; (d) $\sum_i \lambda_i^2 \leq 1$.

Problem 20 – Wigner function

(a) Prove for a 1D system that both of the following definitions give the same Wigner function $W_{|\psi\rangle}(x,p)$:

$$\frac{1}{2\pi\hbar}\int ds\ \hat{\psi}^*(p+\frac{s}{2})\hat{\psi}(p-\frac{s}{2})e^{ixs/\hbar} \tag{2}$$

$$\frac{1}{2\pi\hbar}\int ds\ \psi^*(x+\frac{s}{2})\psi(x-\frac{s}{2})e^{-ips/\hbar} \tag{3}$$

(b) Prove the trace product rule for two density matrices $\hat{\rho}_1$ and $\hat{\rho}_2$:

$$tr\,(\hat{\rho}_1\hat{\rho}_2) = 2\pi\hbar \int dx \int dp\ W_{\hat{\rho}_1}(x,p)W_{\hat{\rho}_2}(x,p)\,. \tag{4}$$

(c) A quantum state $|\psi\rangle$ may be projected onto phase space by the following definition:

$$Q_{|\psi\rangle}(x,p) \equiv \frac{1}{\pi}\left|\langle\Phi_{p,q}|\psi\rangle\right|^2\ , \tag{5}$$

with

$$\Phi_{p,q}(x) \equiv N \exp\left(-\frac{(x-q)^2}{2\sigma_q^2} - \frac{i}{\hbar}px\right)\ , \tag{6}$$

with appropriate normalization constant $N = (\pi^{1/4}\sigma_q^{1/2})^{-1}$. Prove that $Q_{|\psi\rangle}(x,p)$ coincides with the definition of the Husimi function $H_{|\psi\rangle}(x,p)$ defined in the lectures.