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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 8

Problem 17 – Mean Density of States

A semi-classical expression for the mean density of states in a system with n degrees of freedom is given by the Thomas-Fermi formula

$$\bar{\rho} = \frac{1}{(2\pi\hbar)^n} \int \int \delta(E - H(\mathbf{p}, \mathbf{r})) d^n p d^n r,$$

where $\mathbf{r} = (r_1, ..., r_n)$ are the space coordinates, $\mathbf{p} = (p_1, ..., p_n)$ are the corresponding momenta and $H(\mathbf{p}, \mathbf{r})$ is the classical hamiltonian of the system. The formula is based on the consideration that there is, on average, exactly one quantum state per Planck-cell with the phase space volume $(2\pi\hbar)^n$.

1. Show for $n = 3$ and $H = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r})$:

$$\bar{\rho} = \frac{m}{2\pi^2\hbar^3} \int d^3 r \sqrt{2m(E - V(\mathbf{r}))}\Theta(E - V(\mathbf{r})),$$

where $\Theta(x) = \begin{cases} 1, x > 0 \\ 0, x < 0 \end{cases}$ is the heavyside step function.

2. Calculate the mean density of states for:

   (a) the three dimensional infinite box potential with edge lengths a, b and c,

   (b) the three dimensional isotrop harmonic oscillator with the frequency ω,

   (c) the hydrogen atom.

   Compare each case with the *exact* density of states of the quantum mechanical system.

Hint: $\int_0^1 x^2\sqrt{1-x^2}dx = \int_0^1 x^2\sqrt{\frac{1}{x}-1}dx = \frac{\pi}{16}$

Problem 18 – Poisson Summation Formula

1. Show for $x \in \mathbb{R}$:

$$\sum_{m=-\infty}^{\infty} e^{2\pi imx} = \sum_{n=-\infty}^{\infty} \delta(x-n).$$

2. The function $f(x)$ and its Fourier transformation

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{ikx}dx$$

   are given. Show for $\alpha \in \mathbb{R}$:

$$\sum_{n=-\infty}^{\infty} f(\alpha n) = \frac{\sqrt{2\pi}}{\alpha} \sum_{m=-\infty}^{\infty} F(\frac{2\pi m}{\alpha}).$$