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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 5

Problem 10 – Kicked Rotor

Given the time-dependent Hamiltonian

$$H(p,q,t) = H_0(p) + V(q)T \sum_{n=-\infty}^{\infty} \delta(t - nT) \; ,$$

how must one choose $H_0(p), V(q)$ and T such that the following mapping corresponds to the Standard Map (see next problem (1))?

$$\begin{pmatrix} p_n \\ q_n \end{pmatrix} \mapsto \begin{pmatrix} p_{n+1} \\ q_{n+1} \end{pmatrix} \text{with} \begin{pmatrix} p_n \\ q_n \end{pmatrix} = \lim_{\delta \to 0^+} \begin{pmatrix} p(t = nT - \delta) \\ q(t = nT - \delta) \end{pmatrix}$$

Problem 11 – Chaos and Stochasticity

The Standard Map is defined as

$$\begin{aligned} p \mapsto p' &= p + K \sin(q) \\ q \mapsto q' &= q + p' \end{aligned} \tag{1}$$

In this problem we do not take the modulus operation in the evolution of momenta p (in the evolution of q it might as well be taken, why?).

a) For $K = 10$, compute numerically how the energy $p^2/2$ increases with the number of iterations of the map. To do so define ca. 1000 initial points at $p = 0$, with equidistant values $q \in [1.0001, 1.0002, ..., 1.1]$. What kind of stochastic-like motion do you observe?

b) Calculate analytically how, for large $K \gg 1$, the energy $p^2/2$ increases on average with the number of iterations of the map. You should express p_n as a function of p_0 and $q_0, q_1, ..., q_{n-1}$. Use the fact that the iterated values of $\sin q$ are (in good approximation) uncorrelated for large $|p|$.

Problem 12 – Resonance Islands and Self-Similarity

The usual form of the Standard Map is the following:

$$p \mapsto p' = p + K \sin(q) \pmod{2\pi}$$
$$q \mapsto q' = q + p' \pmod{2\pi}\,. \tag{2}$$

Plot the phase space for $K = -1.5$ in the regions

a) $q \in [-3, -2], p \in [1, 2]$

b) $q \in [-2.7, -2.6], p \in [1.2, 1.4]$

c) $q \in [-2.65, -2.64], p \in [1.25, 1.26]$

d) $q \in [-2.6475, -2.6465], p \in [1.253, 1.2535]$

for 100 initial points equidistantly distributed in the given regions (plotting only iterated points within the region). For a good resolution you need ca. 5000 iterations in a), 20000 in b), 10^5 in c), and 5×10^5 in d).

What do you observe in this case of mixed regular-chaotic dynamics?