Sandro Wimberger & Georgios Kordas
Complex Dynamics in Quantum Systems
Institut für Theoretische Physik,
Universität Heidelberg
Philosophenweg 19, D-69120 Heidelberg
Email: s.wimberger@thphys.uni-heidelberg.de
g.kordas@thphys.uni-heidelberg.de
Phone: (++49-(0)6221) 54 - 9449
(++49-(0)6221) 54 - 9319

HGSFP
Heidelberg Graduate School of Fundamental Physics

# Tutorial on Quantum Chaos

## nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

## Problem Sheet 4

### Problem 9 – Henon-Heiles Problem – Poincaré Surface of Section

The Henon-Heiles Hamiltonian is defined as follows

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\left(x^2 + y^2\right) + x^2 y - \frac{1}{3}y^3 \,.$$

a) Draw the contour lines of the potential $V(x, y)$ in the $(x, y)$ plane.

b) For $E > 0$, the Poincaré surface of section is defined by the condition $x = 0$. Plot $(p_y, y)$ in this plane intersecting the full dynamical phase space for the following set of energies: $E = 0.01$, $E = 0.1$, $E = 0.12$, $E = 0.14$, $E = 0.16$, and $E = 1/6$.

Hints:

1. For fixed $E$ define a grid of possible initial conditions in phase space, e.g. by $x(t = 0) = 0, y(0) = -0.5, -0.4, ..., 1$ and $p_y(0) = 0, \pm 0.1, ..., \pm 0.5$ (for very small $E \leq 0.01$ you may eventually have to use a finer grid). Choose initial points for which $V(x, y) + p_y^2/2 \leq E$ and, particularly, $p_x$ by requiring that the total energy equals $E$.

2. Compute numerically the trajectories starting with the initial conditions defined in (1.) up to a time for which ca. 500 crossings with the surface of section have occurred.

3. The values for $(p_y, y)$ at the crossing point are best obtained by a linear interpolation at the two points at which $x$ is changing sign.

4. The equations of motion can be integrated, e.g., by a Runge-Kutta method, as described, for instance, in *Numerical Recipes* (available online for free!).