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Complex Dynamics in Quantum Systems
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Heidelberg Graduate School
of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 3

Problem 7 – Separable Motion

Find the solution of the Hamilton-Jacobi equation for the motion of a particle of unit mass in 3D governed by the Stark-Hamiltonian:

$$H(|\vec{r}| = r, \vec{p}) = \frac{p^2}{2} - \frac{1}{r} + Fz\,.$$

You can solve this problem step by step.

a) Using parabolic coordinates (ξ, η, ϕ) obtained from the cylindrical coordinates (ρ, ϕ, z) via $\xi = r + z$ and $\eta = r - z$, with $r = \sqrt{\rho^2 + z^2}$ and $\rho = \sqrt{\xi\eta} = \sqrt{x^2 + y^2}$, show first that

$$p_\phi = \xi\eta\dot{\phi}\,,\; p_\eta = \frac{1}{4\eta}(\xi + \eta)\dot{\eta}\,,\; p_\xi = \frac{1}{4\xi}(\xi + \eta)\dot{\xi}\,.$$

b) Now show that

$$H(\eta, \xi, \phi; p_\eta, p_\xi, p_\phi) = 2\frac{\xi p_\xi^2 + \eta p_\eta^2}{\xi + \eta} + \frac{p_\phi^2}{2\eta\xi} - \frac{2}{\xi + \eta} + \frac{F}{2}(\xi - \eta) \ .$$

c) Finally, write down the time-independent Hamilton-Jacobi equation

$$\tilde{H}\left(\frac{\partial S}{\partial \xi}, \frac{\partial S}{\partial \eta}, \frac{\partial S}{\partial \phi}; \xi, \eta, \phi\right) = E \ ,$$

and solve it with the separation ansatz for $S = S_1(\phi) + S_2(\xi) + S_3(\eta)$.

Problem 8 – Canonical Perturbation Theory

Find the generating function and the new Hamiltonian in second order perturbation theory for the Hamiltonian of Problem 5 with $a \equiv -3\epsilon$, i.e., for

$$H = H_0(x, p) + \epsilon H_1(x) = \frac{p^2 + \omega^2 x^2}{2} + \epsilon x^3 \ .$$