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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 12

Problem 24 – Avoided Crossing

A spin-1/2 system may be described by the following quantum Hamiltonian in matrix form

$$H = \begin{pmatrix} E_0 + \Delta & V \\ V^* & E_0 - \Delta \end{pmatrix}, \tag{1}$$

with $E_0, \Delta \in \mathbb{R}$ and $V \in \mathbb{C}$.

a) Compute the eigenvalues of H.

b) Imagine a dependence $E_0(\lambda), \Delta(\lambda), V(\lambda)$ on a real parameter λ. Argue that in general there is no such value λ to obtain a degenerate spectrum.

Hint:
How many real equations must λ satisfy to realise a degeneracy of the eigenenergies?

How does the situation change for purely real V?

c) Sketch the eigenlevels of H as a function of λ for $\Delta(\lambda) = \lambda\Delta_0$ and fixed values of E_0 and V (both assumed to be independent on λ). What happens to the eigen*state* of the lower level (and of the higher level, respectively) when λ is adiabatically varied between $\lambda = -100\frac{V}{\Delta_0}$ and $\lambda = 100\frac{V}{\Delta_0}$?