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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 11

Problem 22 – Poisson distribution

Given n real random numbers $x_1, ..., x_n$, uniformly distributed in the interval $x_i \in [0, n]$. Compute numerically for $n = 10^5$ the probability density $P(s)$ such that the distance on the x-axis of a given x_i to its nearest (larger) neighbour x_j equals s.

Hints:

1. Compute 10^5 real random numbers in $[0, 10^5]$ using a standard random number generator (cf., e.g., *Numerical Recipes*, available online).
2. Sort the x_j in such a way that $x_i < x_j \; \forall i < j$ with $i, j \in \{1, 2, \ldots, 10^5\}$.
3. Count how many distances $s = x_{j+1} - x_j$ are in the intervals $[0, 0.1], [0.1, 0.2], \ldots, [9.9, 10]$, and plot the normalized numbers (relative frequencies) in a histogram.

4. What is the functional form of the obtained distribution $P(s)$?

Problem 23 – Rectangular billard

a) Show that for

$$V(x,y) \;=\; \begin{cases} 0 \;, & 0 < x < a \quad 0 < y < b \\ \infty \;, & \text{otherwise} \end{cases} \tag{1}$$

the exact quantum mechanical spectrum coincides with the WKB solution from the lecture:

$$E_{n,k} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n^2}{a} + \frac{k^2}{b}\right) .$$

b) Fix $a = 1, b = \frac{1}{2}\left(\sqrt{5} - 1\right), \frac{\hbar^2\pi^2}{2m} = 1$, and compute the probability distribution $P(s)$.

Hint:
Use the 10^5 lowest eigenvalues and count how many distances are in the intervals $[0, 0.1], [0.1, 0.2], \ldots, [9.9, 10]$, respectively.