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Complex Dynamics in Quantum Systems
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HGSFP
Heidelberg Graduate School of Fundamental Physics

Tutorial on Quantum Chaos

nHS, Phil.weg 12, Wednesday 14:15 - 16:00 (SS 2012)

Problem Sheet 10

Problem 21 – Quantum Kicked Rotor (QKR)

The QKR is described by the following kick-to-kick time evolution operator (Floquet operator):

$$\hat{U} = \hat{K}\hat{F}, \quad \text{with} \quad \hat{K} = e^{-ik\cos(\hat{\theta})} \quad \text{and} \quad \hat{F} = e^{-iT\hat{n}^2/2}. \qquad (1)$$

k and T are two independent parameters characterizing the kick strength and period, respectively. For a rotor, periodic boundary conditions are assumed, i.e. in position/angle representation we have for all times $\psi(\theta = 0) = \psi(\theta = 2\pi)$.

(a) Compute the matrix elements of $\hat{K}$ in the position (or angle θ) representation for a finite basis of length N and a grid $\theta_i = \frac{2\pi}{N} i$, for $i = 1, 2, ..., N$.

(b) Compute the matrix elements of $\hat{K}$ in the angular momentum representation for a finite basis of length N and a grid $n = -N/2, -N/2 + 1, ..., N/2$.

(c) For $k = 5$ and $\tau = 1$ and an initial state in momentum representation of the form $\psi(n) = \delta_{0,n}$, compute numerically the temporal evolution induced by $\hat{U}$, i.e. compute

$$\psi(t) = \hat{U}^t \psi(0)\,, \tag{2}$$

for $t = 1, 2, ..., 200$.

(d) Plot the angular momentum distribution $|\psi(n,t)|^2$ for t =1, 5, 10, 50, 100, 200 kicks, making sure that your wavefunction is properly normalized, i.e. that $\sum_n |\psi(n,t)|^2 = 1$ for all times t.

(e) Compute the second moment of the angular momentum distribution, i.e. the expectation value of the energy $E(t) = \langle \psi(n,t)|\frac{n^2}{2}|\psi(n,t)\rangle$ for all $t = 1, 2, ..., 200$, and plot it versus t.

(f) For a better interpretation of your results, plot also the time-averaged quantities corresponding to (d) and (e), i.e. the quantities $X(t) = \sum_{t'=1}^{t} Y(t')/t$, for $Y(t) = |\psi(n,t)|^2$ or $Y(t) = E(t)$, again as a function of t.

Hints for the time evolution in (c):

1) First compute $\hat{F}\psi(0)$ in momentum respresentation, then use a Fast Fourier Transform (FFT) into position representation before computing $\hat{K}\hat{F}\psi(0)$. Then use the inverse FFT to get back to position representation to compute $\hat{F}\hat{K}\hat{F}\psi(0)$, and continue now this procedure iteratively. Choose a grid which is adequate for applying FFT, i.e. $N = 2^x$ for some integer x.

2) Make sure that the matrix dimension N is large enough to guarentee a numerically exact evolution. You can directly check this by looking at $|\psi(n,t)|^2$ versus n for various values of $N = 2^x$ and $x = 7, 8, 9, 10, 11, ...$.