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Markov Memory in Multifractal Natural Processes

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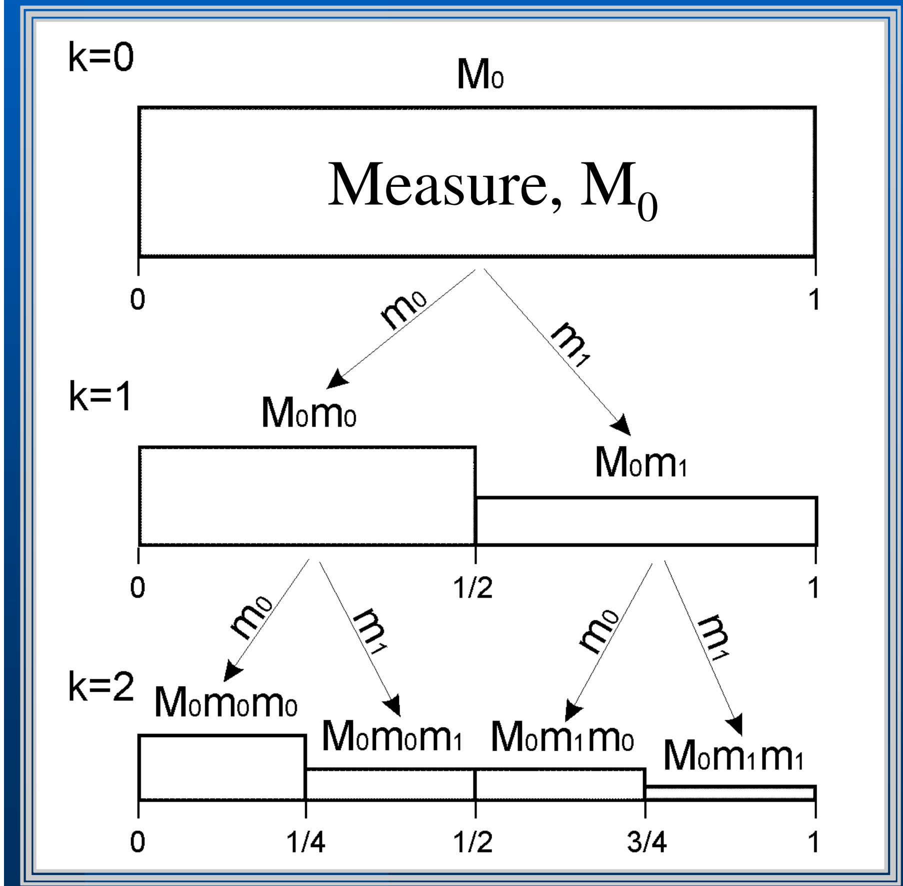
Presentation outline

- Markovian memory in multifractal spectrum
- Multiplicative cascades & Markovian processes
- Application: Turbulence

The binomial multiplicative cascade



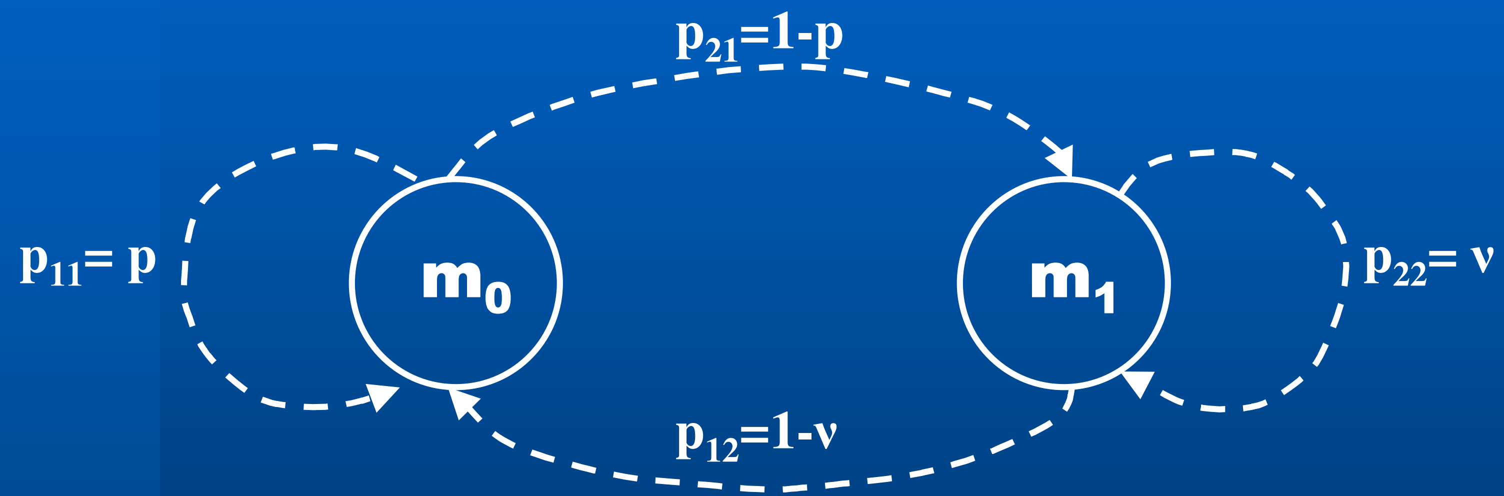
Fixed multipliers m_o, m_1

Fixed probability of multipliers

Probabilities of multipliers determined by Markovian memory

The First-Order, Two-State Markovian Process

The probability of the new state depends only on the previous one



There are 4 transition probabilities

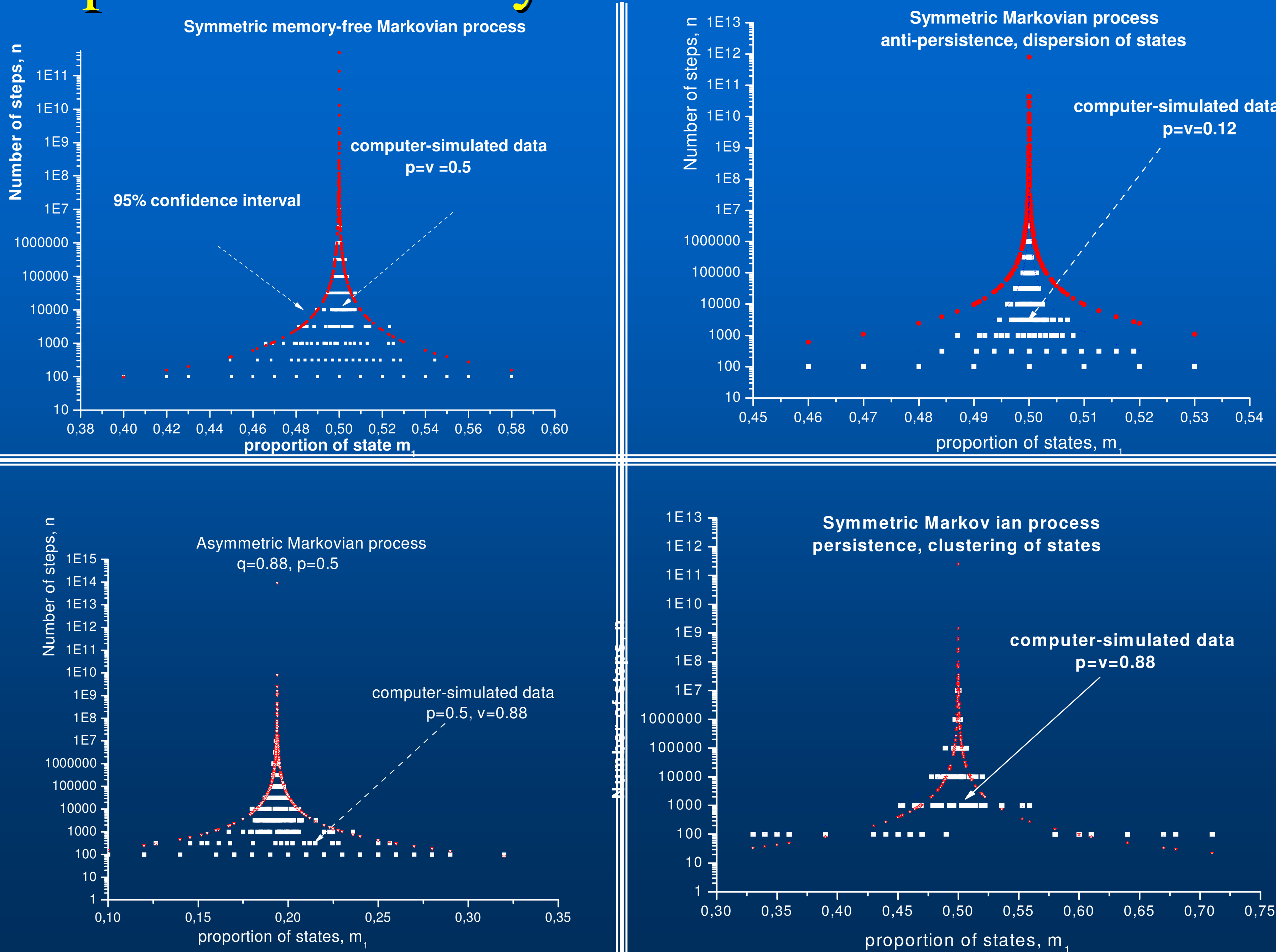
$$\overline{m}_0 \overset{n>>}{\underset{p=\nu}{=}} 50\%$$

Frequency of state m_0

St. deviation of $\overline{m}_0$

$$\sigma \underset{p=\nu}{=} \frac{0.5}{\sqrt{n}}\sqrt{\frac{p}{1-p}}$$

Proportion of binary states and variance effects

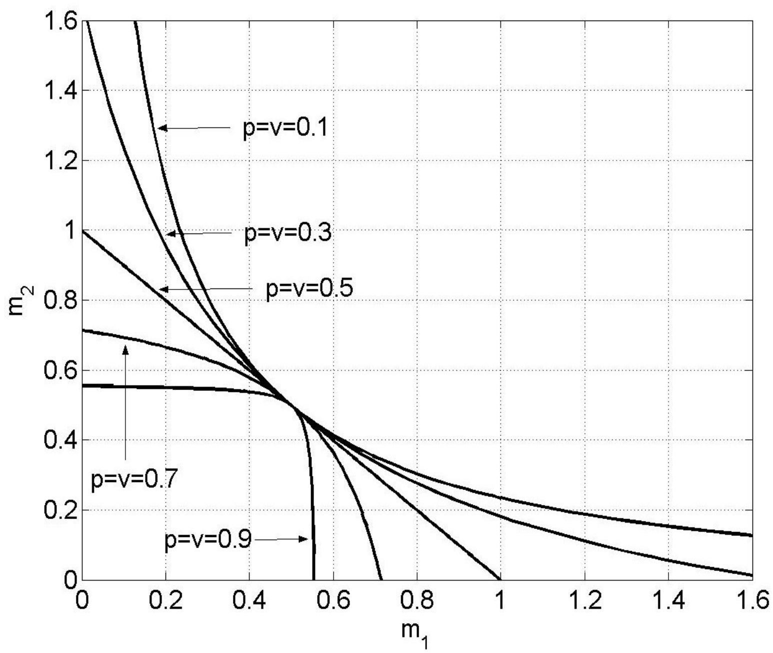


Binomial Cascades & Multipliers

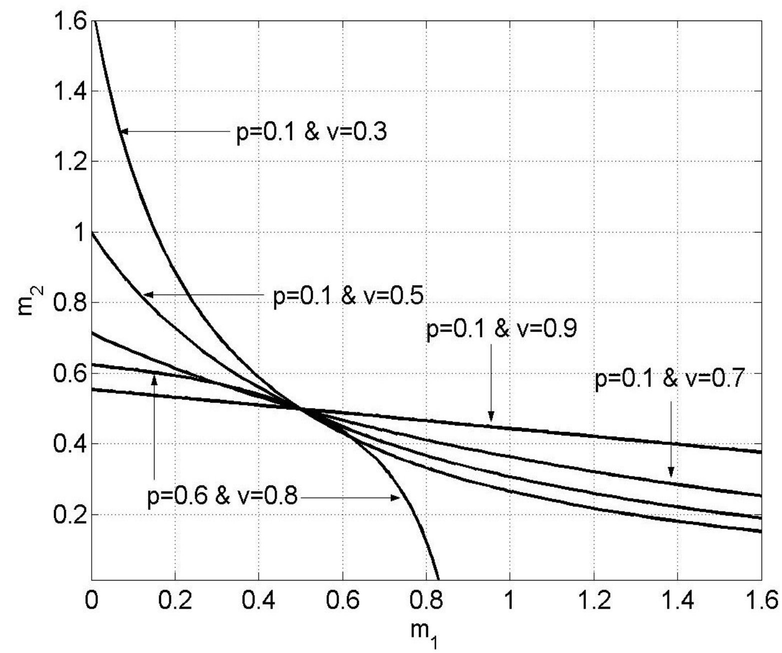
CONSERVATION OF MEASURE	MULTIFRACTAL CASCADES	MULTIPLIERS m_o, m_1
$m_0 + m_1 = 1$	Restricted	Fixed
$pm_0 + (1-p)\cdot m_1 = 1/2$	Independent conservative	Fixed probability
$4(1-p-v)m_0 m_1 + 2(pm_0 + vm_1) = 1$	Dependent	Markovian memory in multipliers

The Cascade Multipliers: m_1, m_2

$$4(1-p-\nu)m_0 m_1 + 2(p m_0 + \nu m_1) = 1$$



(A) Symmetric case: $p = \nu$



(B) Asymmetric case: $p \neq \nu$

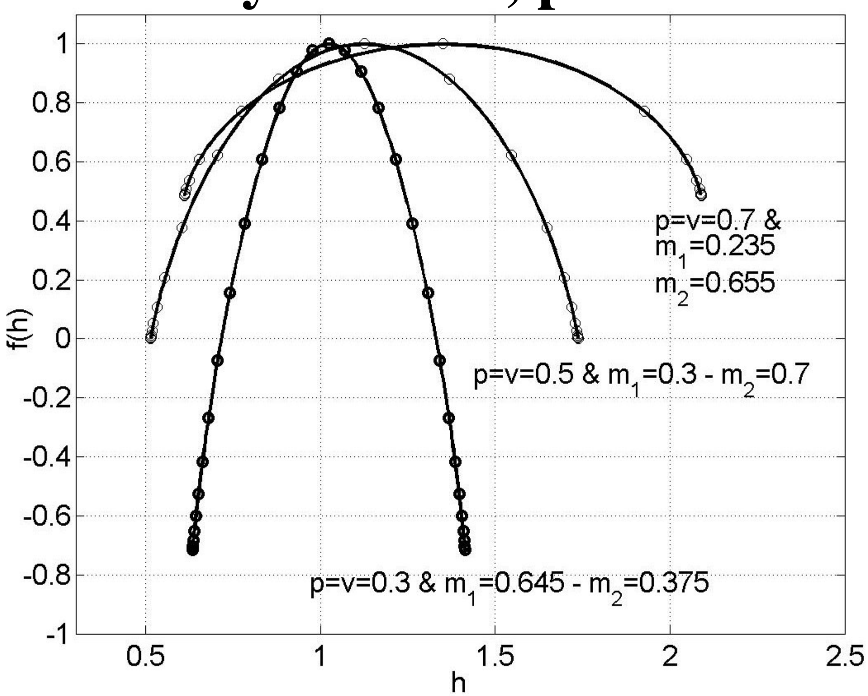
~~$\begin{cases} m_1 < 0.5 \\ m_2 < 0.5 \end{cases}$~~

Examples of allowed pairs of multipliers for different Markov self-transitions probabilities.

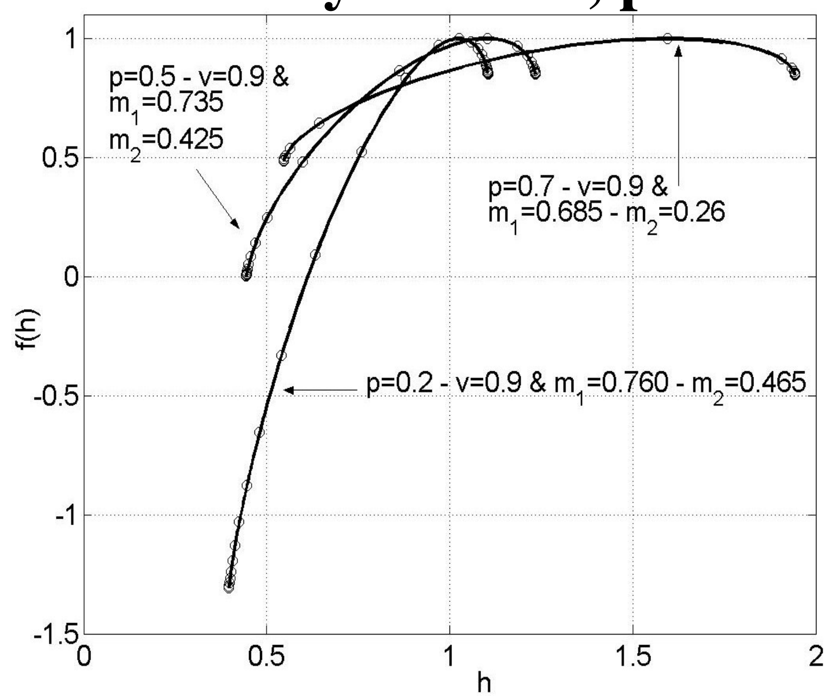
~~$\begin{cases} m_1 > 1 \\ m_2 > 1 \end{cases}$~~

f(h) Spectrum & Markovian Memory[a]

Symmetric, p=ν



Asymmetric, p≠ν

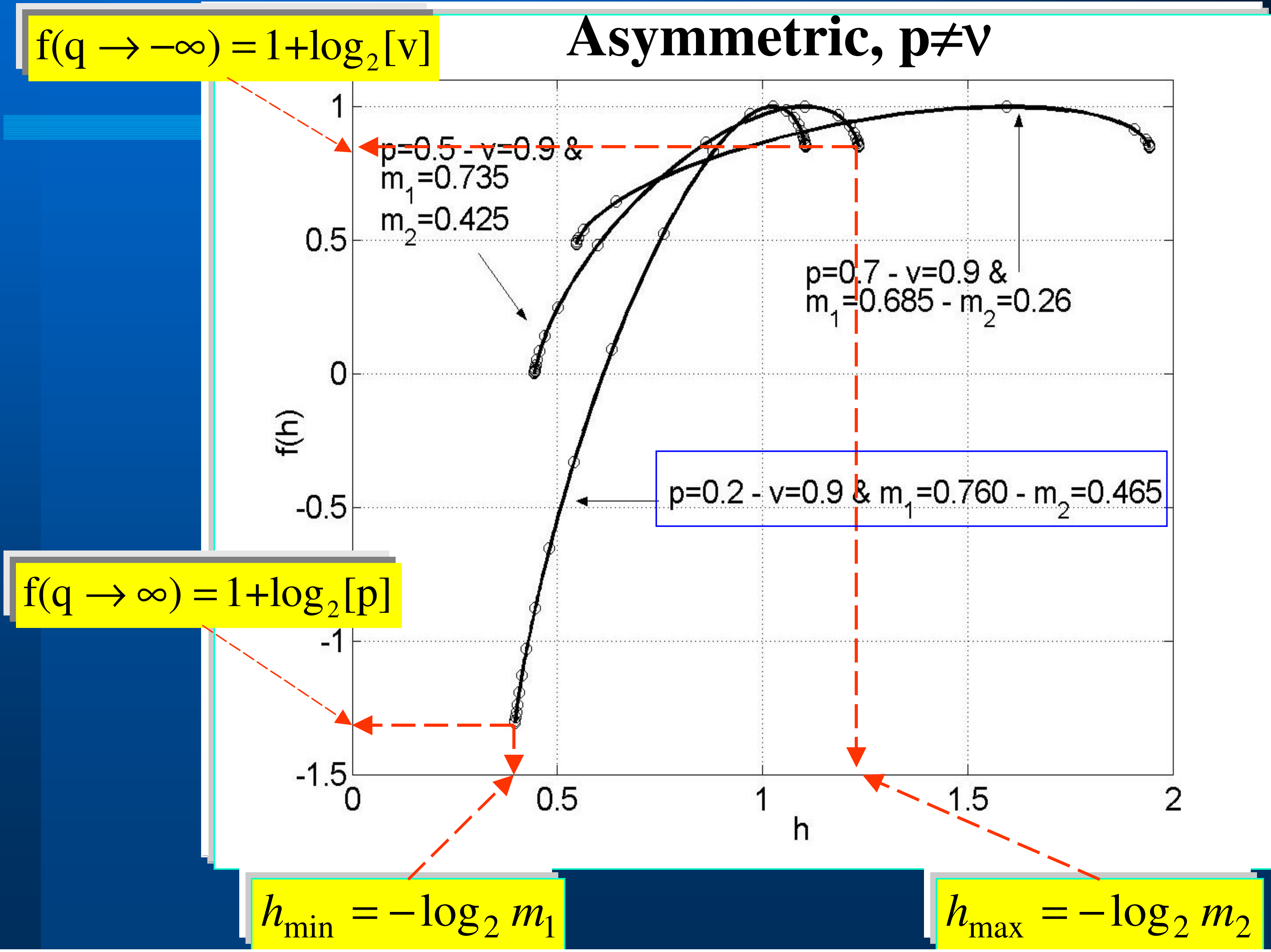


$$h(q) = -\frac{\log_2[m_1/m_2](pm_1^{\,q} - vm_2^{\,q})}{2\sqrt{(pm_1^{\,q} + vm_2^{\,q})^2 - 4(m_1m_2)^q(p+v-1)}} - \frac{\log_2[m_1m_2]}{2}$$

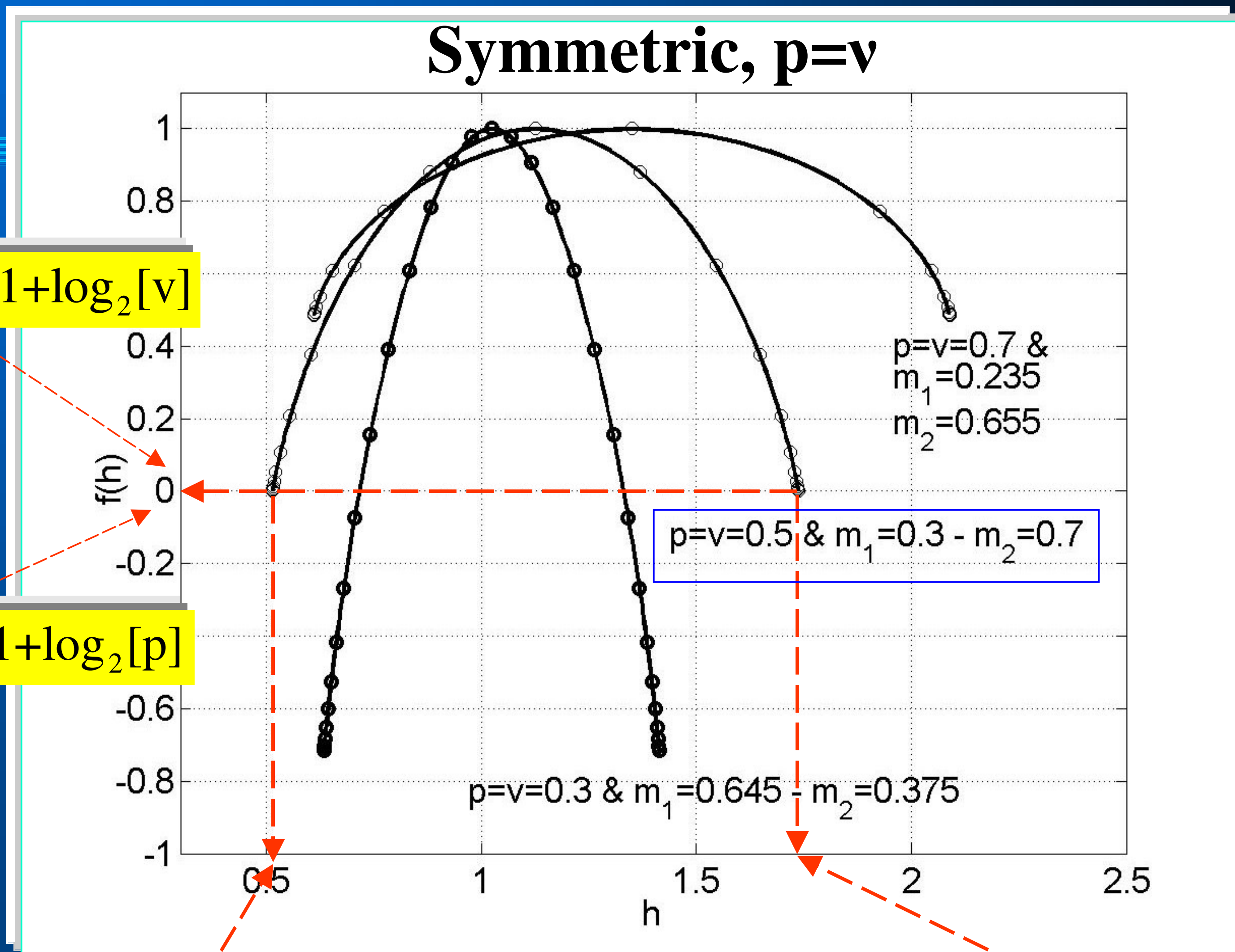
$$f(q) = qh(q) - \tau(q)$$

$$\tau(q) = -\log_2\left[pm_1^q + \nu m_2^q + \sqrt{(pm_1^q + \nu m_2^q)^2 - 4(m_1m_2)^q(p+\nu-1)}\right]$$

f(h) Spectrum & Markovian Memory[b]



f(h) Spectrum & Markovian Memory[c]



$f(q \to -\infty) = 1 + \log_2[v]$

$f(q \to \infty) = 1 + \log_2[p]$

$h_{\min} = -\log_2 m_1$

$h_{\max} = -\log_2 m_2$

Multifractality & Turbulence

A. Multifractals in turbulence

B. Markovian properties in turbulence…

C. …Observed in velocity increments…

…Application in multifractal spectrum

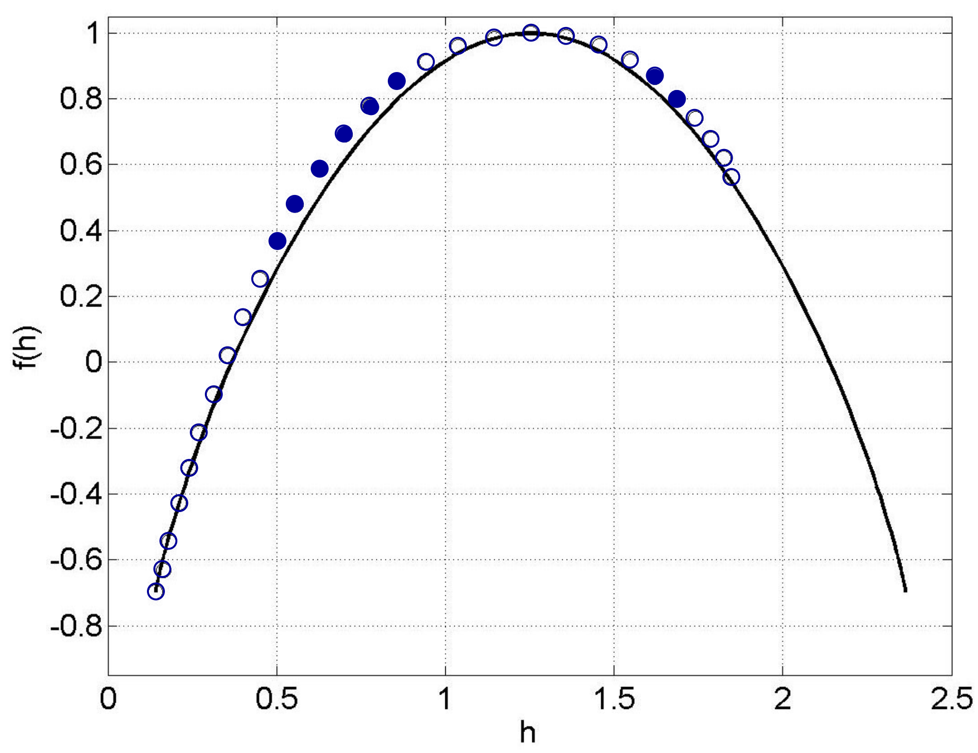
FOR MORE INFO...

A. B. B. Mandelbrot, J. Fluid. Mech. 62, 331 (1974)

B. R. Friedrich & J. Peinke, Physica D, 102, 14 (1997)

C. C. Renner, J. Peinke & R. Friedrich, J. Fluid Mech. 433 383 (2001)

Multifractal spectrum with Markovian Memory in Turbulence



m_1=0.907

m_2=0.194

$p = \nu = 0.308$

Anti-persistence

FOR MORE INFO...

M. Alber, S. Luck, C. Renner, J. Peinke:
arxivnlin.*CD/0007014* (2000)

Summary

- ***T***he shape of the multifractal spectrum depends on the Markovian transition matrix.
- ***M***easure convergence to a nonzero, finite, but not necessarily the initial, value.
- ***W***ider range of cascade multipliers.
- ***A***nti-persistent Markovian process underway in turbulence.

Vielen Dank für Ihre Aufmerksamkeit !

Thank you for your attention !

F. PALLIKARI WEBSITE… http://www.uoa.gr/~fpallik