Research Statement

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1 Doctoral Research Accomplishments

The research that has been completed during my Ph.D (doctorate) at Imperial College, University of London, is involved into the following main areas of interest in Theoretical Physics:

1.1 Generalised Spacetime Quantum Theory and the Decoherent Histories Approach to Quantum Field Theory

Reference [1]: The Decoherent Histories approach is a recently developed formulation of Quantum Theory, which aims to assign probabilities to the histories of a closed quantum system. The advantage of this approach is that it does not rely on notions of measurement or on an external classical domain and permits predictions to be made in genuinely closed systems, like the whole universe. Moreover the decoherent histories formalism, in its path-integral form, provides a spacetime formulation of Quantum theory which permits the calculation of probabilities not only for alternatives defined at definite moments of time as in conventional Quantum theory, but more general ones. Using the above framework I have studied in detail the problem of defining probabilities for alternatives of a quantum system fixed by spacetime regions. Probabilities for spacetime coarse grainings have thus been calculated for a variety of spacetime problems. The results of my research have enhanced significantly the framework of *Generalised Spacetime Quantum Theory* and permit the further study of a wide range of theoretical problems. Concerning, in particular, Quantum Cosmology, there is a fundamental necessity to understand why, on sufficiently large scales Quantum theory admits an emergent description of the universe involving only a small number of dynamical variables, the "hydrodynamic variables", which obey deterministic evolution equations, and are related to the existence of conservation laws as well. Concerning Quantum Field theory it would be an appealing idea to study the possibility of defining probabilities for spacetime alternatives. An example is provided by field averages over spacetime regions with extent both in space and time. Concerning Quantum Gravity the generalisation provided by the consideration of probabilities for spacetime alternatives, may be central for the construction of a quantum theory of gravity in which there is no well-defined notion of time, namely spacetime alternatives may be the natural observables for which the theory makes predictions.

1.2 Symmetries and Exact Solutions in General Relativity

References [2], [4]: The basic tool to simplify Einstein's field equations, in search of exact solutions, has been the introduction of spacetime symmetries. The latter form Lie algebras of vector fields on the spacetime manifold which are invariant vector fields of certain geometrical objects on this manifold. Motivated by this fact, I have developed a method to write the field equations for general matter, in a form that fully incorporates the character of a symmetry on spacetime. The method has been expressed in a covariant formalism using the framework of congruences permitting the introduction of arbitrary reference frames. The basic notion on which it is based is that of the geometrisation of a general symmetry. As a special application I have considered the case of a spacelike Conformal Killing vector symmetry in spacetime, regarding various types of matter fields, like perfect fluids and anisotropic matter. In relation to the search of *exact solutions in General Relativity, by introducing spacetime symmetries*, the framework I have constructed allows the study of general spacetime symmetries by means of their geometrisation, namely the expression of the symmetries as necessary and sufficient conditions on the geometry of the integral lines of the field generating the symmetry. Moreover, using the framework of the symmetries-incorporated Einstein's field equations, it has been made possible to look for solutions which by construction comply with the particular collineations employed. Furthermore we are now able to examine what types of matter can be present in spacetime, in case a symmetry has been imposed. The results may finally find fruitful applications in the construction of realistic cosmological models.

1.3 Covariant kinematics of relativistic strings and branes

References [3], [5], [6]: The idea of matter of multidimensional objects is a generalisation of the notion of point particles to matter, whose elementary constituents are extensive objects, called branes, like strings and higher dimensional branes. The main motivation for studying these models originates from Cosmology, where it has been assumed that the behaviour of the various topological defects is relevant to structure formation in the early universe, with interesting cosmological and astrophysical implications. In the above context, I have studied the geometry of deformations of brane worlsheet congruences in spacetime and I have also constructed a fully covariant kinematical framework for classical relativistic branes as well as multidimensional fluids. In relation to *relativistic brane theory*, it is of particular interest to apply the kinematical results to specific brane models and study their behaviour. Since these models may have relevance for structure formation in the early universe their properties deserve a special cosmological and astrophysical interest. Moreover, the main motivation for a proper kinematical description originates from a clear analogy. To be concrete, it is well established today that the proof of the existence of spacetime singularities in General Relativity relies on the consequences obtained from Raychaudhuri equations for geodesic congruences. In brane theories the notion of the point particle and the associated with it worldline, gets replaced by the notion of extensive objects with their corresponding worldsheets. Thus, in principle, it would be possible from the generalised Raychaudhuri equations for brane worldsheet congruences to arrive at analogous singularity theorems in Classical relativistic Brane theory.

2 Postdoctoral Research Accomplishments

The research that has been completed during my postdoctoral period is involved into the following main areas of interest in Theoretical Physics:

2.1 Foundations of Quantum Physics

References [13], [17]: A fundamental methodological concept of physics is grounded on the idea that the advancement of our knowledge about the functioning of the universe as a whole has to proceed by compartmentalization. The classical dogma adopts in addition the idea that the analysis of nature is achieved through the description of interactions of independently existing objects supplemented by the belief that this description is in accordance with the empirical facts. In sharp contrast the establishment of quantum theory as the best fundamental theory of the material world has proved that the classical dogma is inadequate, since it predicts that individual objects cannot have existence in an absolute sense, or else, do not exist in perfect isolation. The non-separability of nature is described by Einstein-Podolsky-Rosen correlations, which exist even in absence of any interactions, making all subsystems of the universe inextricably entangled. The classical dogma was methodologically reductionistic and philosophically atomistic. Its purpose was the explanation of complex phenomena in terms of few elementary objects and their interactions. Quantum theory denies the atomistic world view and describes the universe as a whole. Conceptually, a move in the direction of replacing the classical dogma with a view in harmony with quantum theory, is the replacement of the traditional difference between whole and part with that between system and environment in conjunction with the notion of observer. This move essentially signifies the re-formulation of what has been conceived as the difference between whole and part in a theory of systems differentiation. Accordingly, the universe as a differentiated system is no longer simply composed of a certain number of independently existing parts and the relations (interactions) between them; rather, it is composed of a relatively large number of operationally employed system/environment differences, objectified in the terms of generalized reference frames by observers, which each, along different cutting lines, reconstruct the whole universe as the unity of subsystem and environment. In this sense, it arises an understanding of the possibility of employment of varying and even complementary viewpoints within subsystem differentiation. The conclusion stated, finds a precise mathematical formulation in terms of the categorical notion of adjunction, and essentially corresponds to a principle of complementarity interpreted in a generalized Bohrian sense.

2.2 Quantum Event and Quantum Observable Structures

References [7], [8], [9], [16], [22]: At the formal level the transition from classical to quantum physics essentially involves the transition from Boolean event structures to non-Boolean event structures, or else from those which do, to those which do not admit two-valued homomorphisms. Observables schematize the quantum event structure and play the role of coordinatizing objects in the attempt to probe the quantum world. Technically speaking, Boolean algebras of observables function as generalized reference frames, relative to which the measurement results are being coordinatized, establishing simultaneously, a properly defined contextual perspective on the structure of quantum events. Philosophically speaking, we can assert that the quantum world is being perceived through Boolean reference frames, regulated by measurement procedures of a variety of observers, which interlock to form a coherent picture in a non-trivial way. The main guiding idea, referring to the constitution of the universe on the basis of part-whole relations according to the quantum theoresis of reality, is based on the existence of partial structural congruences between the quantum and Boolean kinds of observable structure. Technically, it is being implemented by the functioning of objects belonging to the Boolean species of observable structure as modeling figures for probing the objects belonging to the quantum species of observable structure. Furthermore, it is being concretely modeled by defining appropriate functors interrelating the quantum observable structure of the universe with the partial recognition mechanisms introduced by Boolean reference frames of observers, who probe its global content by filtering the information collected in terms of observable attributes, through local Boolean abstraction contexts of measurement. The mathematical syntactic language which is appropriately suited to formulate and support these claims is category theory, and especially the theory of sheaves. Category theory provides a general theoretical framework for dealing with systems emphasizing, in particular, their mutual relations and transformations. The concept of sheaf expresses essentially gluing compatibility conditions, namely the way by which partial data can be collated meaningfully in a global sense.

2.3 Complex Systems Theories

References [14], [15], [31], [32]: Recently there has been a considerable interest in the foundational issues related with the modeling and comprehension of complex systems in the physical sciences. I argue that the resolution of these issues necessitates the adoption of a simple but prevailing epistemological principle. According to this principle, the analysis of a complex system, and the consequent comprehension of its behavior, may be successfully performed in terms of interlocking families of simple, sufficiently understood partially or locally defined systems, which are constrained to satisfy certain appropriate compatibility relations. The simple systems may be conceived as localization devices, as information filters or as modes of perception of the complex objects, the internal structure and functioning of which, will be hopefully recovered by the interconnecting machinery governing the local objects. This point of view effectively necessitates a contextual scheme for the modeling of a complex system, as an interconnected family of simple ones interlocking in a non-trivial fashion, where the contexts are specified by the qualitative features of the simple systems. In order to explicate such a modeling scheme for complex systems, a suitable mathematical language has to be used. The language of Category theory proves to be appropriate for the implementation of this idea in a universal way. The conceptual essence of this scheme is the development of a sheaf theoretical perspective on the study of complex systems related with the modeling of physical information processes.

2.4 Topos-Theoretic Methods in Quantum Physics

References [11], [12], [18], [19], [24]: The fascinating theory of topoi is becoming ever more important in theoretical physics. This fits very well with modern physics increasingly becoming about local structures. Einstein's general relativity made geometry (and gravity) local by using differential geometry. Quantum theory too benefits from such a move. In particular I demonstrate one interesting way of using topoi in quantum theory. It has been known for a long time that quantum logics differ from ordinary, classical or Boolean logic. Motivated by this fact I attempt to shed some interesting light on this difference. For this purpose I show that quantum logics can be formulated in terms of local Boolean logics. As a starting point, we have quantum event algebras, which are posets endowed with a maximum element and an orthocomplentation relation. The first step is the construction of an adjunctive correspondence between Boolean presheaves of (classical) event algebras and the aforementioned quantum event algebras. By endowing the base category of Boolean event algebras with a Grothendieck topology, this leads to an equivalence between the category of quantum event algebras and a Grothendieck topos. Hence, I conclude that the real significance of a quantum structure proves to be not at the level of events, but at the level of gluing together appropriately compatible observational contexts. Furthermore, on the basis of this adjunctive correspondence, I prove the existence of an object of truth values in the category of quantum logics, characterized as subobject classifier. This classifying object plays the equivalent role that the two-valued Boolean truth values object plays in classical events structures. I construct the object of quantum truth values explicitly, and then, demonstrate its functioning for the valuation of propositions in a typical quantum measurement situation.

2.5 Topological Localization and Modern Differential Geometry in Quantum Field Theory and Quantum Gravity

References [10], [20], [21], [23], [25], [26], [27], [28], [29], [30]: Physical observation presupposes, at the fundamental level, the existence of a localization process for extracting information related with the local behaviour of a physical system. On the basis of a localization process it is possible to discern observable events and assign an individuality to them. Generally, a localization process is being co-implied by the preparation of suitable local reference domains for measurement. These domains identify concretely the kind of reference loci used for observation of events. The methodology of observation is being effectuated by the functioning of events-registering measurement devices, that operate locally within the contexts of the prepared reference loci. In this general setting, it is important to notice that registering an event, that has been observed in the context of a reference locus, is not always equivalent to conferring a numerical identity to it, by means of a real value corresponding to a physical attribute. On the contrary, the latter is only a limited case of the localization process, when, in particular, it is assumed that all reference loci can be contracted to points. This is exactly the crucial assumption underlying the employment of the set-theoretic structure of the real line as a model of the physical “continuum". The semantics of the latter is associated with the codomain of valuation of physical attributes used for registering events. It is instructive to clarify that the set-theoretic model of the real line stands for a structure of points that are independent and possess the property of sharp distinguishability. I study the possibility of mathematically implementing a general localization process referring to physical observation, that is not necessarily based on the existence of an underlying structure of points on the real line. For this purpose, the focus is shifted from point-set to topological and sheaf-theoretic localization models of ordered global quantum events structures. From a physical point of view, this move reflects the appropriate generalization of the arithmetics, or sheaves of coefficients, that have to be used in the transition from the classical to the quantum regime. The appropriate framework to accommodate structure sheaves of the above form is Modern Differential Geometry. The latter is an extension of classical Differential Geometry according to which, instead of smooth functions, one starts with a general sheaf of algebras. The important thing is that these sheaves of algebras, which in my scheme correspond to quantum observables, can be interrelated with appropriate differentials, interpreted as Leibniz sheaf morphisms. This interpretation is suited to the development of Differential Geometry in the quantum regime, and moreover, it provides a natural framework for studying the dynamics of quantum gravity.

References

[1] E. Zafiris and J. J. Halliwell, *'Decoherent Histories Approach to the Arrival Time Problem'* Phys. Rev. **D57** 3351 (1998), quant-ph/9706045.

[2] E. Zafiris, *'Incorporation of Spacetime Symmetries in Einstein's Field Equations'*, J. Math. Phys. **38** (11), 5854, (1997), gr-qc/9710103.

[3] E. Zafiris, *'Kinematical Approach to Brane Worldsheet Deformations in Spacetime'* Annals of Physics **264** (1), 75 (1998).

[4] E. Zafiris, *'Irreducible Decomposition of Einstein's Equations in Spacetimes with Symmetries'* Annals of Physics **263** (2), 155 (1998).

[5] E. Zafiris, *'Covariant Generalisation of Codazzi-Raychaudhuri and Area Change Equations for Relativistic Branes'*, Journal of Geometry and Physics **27** (3), 292 (1998), hep-th/9710169.

[6] E. Zafiris, *'Generalised Raychaudhuri and Area Change Equations for Classical Brane Models in Space-time with Torsion'*, Phys. Rev. **D58**, 043509 (1998).

[7] E. Zafiris, *'On Quantum Event Structures. Part I: The Categorical Scheme'*, Foundations of Physics Letters **14**(2), (2001).

[8] E. Zafiris, *'On Quantum Event Structures. Part II: Interpretational Aspects'*, Foundations of Physics Letters **14**(2), (2001).

[9] E. Zafiris, *'Probing Quantum Structure Through Boolean Localization Systems'*, International Journal of Theoretical Physics **39**, (12) (2000).

[10] E. Zafiris, *'Boolean Coverings of Quantum Observable Structure: A Setting for an Abstract Differential Geometric Mechanism '*, Journal of Geometry and Physics **50**, 99 (2004), math-ph/0306045.

[11] E. Zafiris, *'On Quantum Event Structures. Part III: Object of Truth Values '*, Foundations of Physics Letters **17**(5), (2004).

[12] E. Zafiris, *'Quantum Event Structures from the perspective of Grothendieck Topoi'*, Foundations of Physics **34**(7), (2004).

[13] E. Zafiris, *'Interpreting Observables in a Quantum World from the Categorial Standpoint '*, International Journal of Theoretical Physics **43**, (1) (2004).

[14] E. Zafiris, *'Complex Systems from the perspective of Category Theory: I. Functioning of the Adjunction Concept '*, Axiomathes **15**, (1) (2005), PITT-PHIL-SCI 1236.

[15] E. Zafiris, *'Complex Systems from the perspective of Category Theory: II. Covering Systems and Sheaves '*, Axiomathes **15**, (2) (2005), PITT-PHIL-SCI 1237.

[16] E. Zafiris, *'Functorial Transfer of Quantum Information into Boolean Localization Systems'*, submitted for publication to Applied Categorical Structures (2006).

[17] E. Zafiris, *'Category-Theoretic Analysis of the Notion of Complementarity for Quantum Systems'*, International Journal of General Systems, **35**, (1) (2006).

[18] E. Zafiris, *'Topos Theoretical Reference Frames on the Category of Quantum Observables'*, quant-ph/0202057.

[19] E. Zafiris, *'Categorical Foundations of Quantum Logics and Their Truth Values Structures '*, quant-ph/0402174.

[20] E. Zafiris, *'Quantum Observables Algebras and Abstract Differential Geometry '*, gr-qc/0405009.

[21] E. Zafiris, *'Quantum Observables Algebras and Abstract Differential Geometry: The Topos-Theoretic Dynamics of Diagrams of Commutative Algebraic Localizations'*, International Journal of Theoretical Physics, International Journal of Theoretical Physics **46**, (2) (2007).

[22] E. Zafiris, *'Sheaf-Theoretic Representation of Quantum Measure Algebras'*, Journal of Mathematical Physics **47**, 092103 (2006).

[23] E. Zafiris, *'Generalized Topological Covering Systems on Quantum Events Structures'*, Journal of Physics A: Mathematical and General **39**, (2006).

[24] E. Zafiris, *'Topos-Theoretic Classification of Quantum Events Structures in terms of Boolean Reference Frames'*, International Journal of Geometric Methods in Modern Physics, **3** (8) (2006).

[25] E. Zafiris, *'A Sheaf-Theoretic Topos Model of the Physical "Continuum" and its Cohomological Observable Dynamics'*, International Journal of General Systems, **38**, (1) (2009).

[26] E. Zafiris, and A. Mallios, *'Topos-theoretic Relativization of Physical Representability and Quantum Gravity'*, gr-qc/0610113.

[27] E. Zafiris and A. Mallios, *'The Homological Kahler-De Rham Differential Mechanism: I. Application in General Theory of Relativity'*, Advances in Mathematical Physics, doi:10.1155/2011/191083 (2011).

[28] E. Zafiris and A. Mallios, *'The Homological Kahler-De Rham Differential Mechanism: II. Sheaf-Theoretic Localization of Quantum Dynamics'*, Advances in Mathematical Physics, doi:10.1155/2011/189801 (2011).

[29] E. Zafiris, *'Boolean Information Sieves: A Local-to-Global Approach to Quantum Information '*, International Journal of General Systems **39**, (8) (2010).

[30] E. Zafiris, *'A Categorical Perspective on Connections with Application in the Formulation of Functorial Physical Dynamics'*, invited paper in the edited collection 'Emerging Topics on Differential Geometry and Graph Theory', Lucas Bernard and Francois Roux (ed.), Nova Science (2010).

[31] E. Zafiris, *' Categorical Modelling of Natural Complex Systems. Part I: Functorial Process of Representation'*, invited paper for the Journal of the International Institute for General Systems Studies, Advances in Systems Science and Applications (2) (2008).

[32] E. Zafiris, *' Categorical Modelling of Natural Complex Systems. Part II: Functorial Process of Localization-Globalization'*, invited paper for the Journal of the International Institute for General Systems Studies, Advances in Systems Science and Applications (3) (2008).