Minimal SU(5) Grand Unification

D. Katsinis

University of Athens

June , 2011

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!

RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!

RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!

RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!

RGEs govern parameters evolution!

Ingredients of SM

Principles:

- Locality
- Quantum Mechanics
- Poincare Invariance

} QFT

Structure:

- Gauge Theory
- SSB

} Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Particle Content

EW Theory and QCD constitute SM.

- Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Particles: belong to representations of the Gauge Group.
- Force Carriers: belong to the adjoint rep of the GG.
- Flavor Sector: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- Higgs Sector: Breaks GG to $SU(3)_C \times U(1)_{em}$ Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- θ_{QCD}

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?

GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!

GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!

GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!

GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!

GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!

Particle Content

Multiplet	Color	W. Isospin	Hypercharge
Q_L	3	2	+1/3
L_L	1	2	-1
u_L^c	$\overline{3}$	1	-4/3
d_L^c	$\overline{3}$	1	+2/3
ℓ_L^c	1	1	+2

This assignment cancels all triangle anomalies!
Unification requires that the whole multiplet must transform according to the same Poincare rep.

Particle Content

Multiplet	Color	W. Isospin	Hypercharge
Q_L	3	2	+1/3
L_L	1	2	-1
u_L^c	$\overline{3}$	1	-4/3
d_L^c	$\overline{3}$	1	+2/3
ℓ_L^c	1	1	+2

This assignment cancels all triangle anomalies!

Unification requires that the whole multiplet must transform according to the same Poincare rep.

Particle Content

Multiplet	Color	W. Isospin	Hypercharge
Q_L	3	2	+1/3
L_L	1	2	-1
u_L^c	$\overline{3}$	1	-4/3
d_L^c	$\overline{3}$	1	+2/3
ℓ_L^c	1	1	+2

This assignment cancels all triangle anomalies!
Unification requires that the whole multiplet must transform according to the same Poincare rep.

Charge Conjugation

Charge conjugations transforms righthanded particles to lefthanded antiparticles $\Psi^c_{L,R} = \mathcal{C}\overline{\Psi}^T_{R,L}$.

$$\chi_R \to \left(1 + \frac{\imath}{2}\epsilon\cdot\sigma - \frac{1}{2}n\cdot\sigma\right)\chi_R$$

$$\chi^c_L = \imath\sigma^2\chi^*_R \to \left(1 + \frac{\imath}{2}\epsilon\cdot\sigma + \frac{1}{2}n\cdot\sigma\right)\imath\sigma^2\chi^*_R$$

$$\overline{\psi^c}_L\gamma^\mu\chi^c_L = -\psi^T_R\mathcal{C}^{-1}\gamma^\mu\mathcal{C}\overline{\chi}^T_R = \psi^T_R\left(\gamma^\mu\right)^T\overline{\chi}^T_R = -\overline{\chi}_R\gamma^\mu\psi_R$$

$$\overline{\psi^c}_L\partial\!\!\!/\psi^c_L = \overline{\psi^c}_L\gamma^\mu\left(\partial_\mu\psi^c_L\right) = -\left(\partial_\mu\overline{\psi}_R\right)\gamma^\mu\psi_R = \overline{\psi}_R\partial\!\!\!/\psi_R$$

$$\overline{\chi}_R\chi_L = \left(\mathcal{C}^T\left(\mathcal{C}\overline{\chi}^T_R\right)\right)^T\chi_L = \chi^{cT}_L\mathcal{C}\chi_L \qquad (1)$$

Gauge Sector

SU(5) is the only simple rank 4 group that can be broken to SM!

$$\begin{pmatrix} G^{11}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{12}_\mu & G^{13}_\mu & X^1_\mu & Y^1_\mu \\ G^{21}_\mu & G^{22}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{23}_\mu & X^2_\mu & Y^2_\mu \\ G^{31}_\mu & G^{32}_\mu & G^{33}_\mu - \frac{2}{\sqrt{30}}B_\mu & X^3_\mu & Y^3_\mu \\ X^{1\dagger}_\mu & X^{2\dagger}_\mu & X^{3\dagger}_\mu & \frac{1}{\sqrt{2}}\left(A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) & W_\mu \\ Y^{1\dagger}_\mu & Y^{2\dagger}_\mu & Y^{3\dagger}_\mu & W^\dagger_\mu & \frac{1}{\sqrt{2}}\left(-A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) \end{pmatrix}$$

$$24 = (8,1)_0 \oplus (3,2)_{-5/3} \oplus (\bar{3},2)_{+5/3} \oplus (1,3)_0 \oplus (1,1)_0$$

The non-diagonal parts give raise to new interactions curried by the leptoquarks $X^{+4/3}$, $Y^{-1/3}$!!!

Gauge Sector

SU(5) is the only simple rank 4 group that can be broken to SM!

$$\begin{pmatrix} G^{11}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{12}_\mu & G^{13}_\mu & X^1_\mu & Y^1_\mu \\ G^{21}_\mu & G^{22}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{23}_\mu & X^2_\mu & Y^2_\mu \\ G^{31}_\mu & G^{32}_\mu & G^{33}_\mu - \frac{2}{\sqrt{30}}B_\mu & X^3_\mu & Y^3_\mu \\ X^{1\dagger}_\mu & X^{2\dagger}_\mu & X^{3\dagger}_\mu & \frac{1}{\sqrt{2}}\left(A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) & W_\mu \\ Y^{1\dagger}_\mu & Y^{2\dagger}_\mu & Y^{3\dagger}_\mu & W^\dagger_\mu & \frac{1}{\sqrt{2}}\left(-A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) \end{pmatrix}$$

$$24 = (8,1)_0 \oplus (3,2)_{-5/3} \oplus (\overline{3},2)_{+5/3} \oplus (1,3)_0 \oplus (1,1)_0$$

The non-diagonal parts give raise to new interactions curried by the leptoquarks $X^{+4/3}$, $Y^{-1/3}$!!!

Gauge Sector

SU(5) is the only simple rank 4 group that can be broken to SM!

$$\begin{pmatrix} G^{11}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{12}_\mu & G^{13}_\mu & X^1_\mu & Y^1_\mu \\ G^{21}_\mu & G^{22}_\mu - \frac{2}{\sqrt{30}}B_\mu & G^{23}_\mu & X^2_\mu & Y^2_\mu \\ G^{31}_\mu & G^{32}_\mu & G^{33}_\mu - \frac{2}{\sqrt{30}}B_\mu & X^3_\mu & Y^3_\mu \\ X^{1\dagger}_\mu & X^{2\dagger}_\mu & X^{3\dagger}_\mu & \frac{1}{\sqrt{2}}\left(A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) & W_\mu \\ Y^{1\dagger}_\mu & Y^{2\dagger}_\mu & Y^{3\dagger}_\mu & W^\dagger_\mu & \frac{1}{\sqrt{2}}\left(-A^3_\mu + \sqrt{\frac{3}{5}}B_\mu\right) \end{pmatrix}$$

$$24 = (8,1)_0 \oplus (3,2)_{-5/3} \oplus (\bar{3},2)_{+5/3} \oplus (1,3)_0 \oplus (1,1)_0$$

The non-diagonal parts give raise to new interactions curried by the leptoquarks $X^{+4/3}$, $Y^{-1/3}$!!!

Particle Content

Simplest rep:

$\overline{5} = (\overline{3},1)_{+2/3} \oplus (1,2)_{-1}$ $\quad$ $10 = (\overline{3},1)_{-4/3} \oplus (3,2)_{+1/3} \oplus (1,1)_{+2}$

Particles:

$Q_L\ (3,2)_{+1/3}$ $L_L\ (1,2)_{-1}$ $u^c_L\ (\overline{3},1)_{-4/3}$ $d^c_L\ (\overline{3},1)_{+2/3}$ $\ell^c_L\ (1,1)_{+2}$

$$\psi_{\overline{5}} = \begin{pmatrix} d_L^{1c} \\ d_L^{2c} \\ d_L^{3c} \\ \ell_L \\ -\nu_L \end{pmatrix} \qquad \psi_{10} = \begin{pmatrix} 0 & u_L^{3c} & -u_L^{2c} & -u_L^1 & -d_L^1 \\ -u_L^{3c} & 0 & u_L^{1c} & -u_L^2 & -d_L^2 \\ u_L^{2c} & -u_L^{1c} & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & -\ell_L^c \\ d_L^1 & d_L^2 & d_L^3 & \ell_L^c & 0 \end{pmatrix}$$

Particle Content

Simplest rep:

$$\overline{5} = (\overline{3},1)_{+2/3} \oplus (1,2)_{-1} \qquad 10 = (\overline{3},1)_{-4/3} \oplus (3,2)_{+1/3} \oplus (1,1)_{+2}$$

Particles:

$Q_L\ (3,2)_{+1/3}$ $L_L\ (1,2)_{-1}$ $u_L^c\ (\overline{3},1)_{-4/3}$ $d_L^c\ (\overline{3},1)_{+2/3}$ $\ell_L^c\ (1,1)_{+2}$

$$\psi_{\overline{5}} = \begin{pmatrix} d_L^{1c} \\ d_L^{2c} \\ d_L^{3c} \\ \ell_L \\ -\nu_L \end{pmatrix} \qquad \psi_{10} = \begin{pmatrix} 0 & u_L^{3c} & -u_L^{2c} & -u_L^1 & -d_L^1 \\ -u_L^{3c} & 0 & u_L^{1c} & -u_L^2 & -d_L^2 \\ u_L^{2c} & -u_L^{1c} & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & -\ell_L^c \\ d_L^1 & d_L^2 & d_L^3 & \ell_L^c & 0 \end{pmatrix}$$

Particle Content

Simplest rep:

$$\overline{5} = (\overline{3}, 1)_{+2/3} \oplus (1, 2)_{-1} \qquad 10 = (\overline{3}, 1)_{-4/3} \oplus (3, 2)_{+1/3} \oplus (1, 1)_{+2}$$

Particles:

$Q_L\ (3, 2)_{+1/3}\ L_L\ (1, 2)_{-1}\ u_L^c\ (\overline{3}, 1)_{-4/3}\ d_L^c\ (\overline{3}, 1)_{+2/3}\ \ \ell_L^c\ (1, 1)_{+2}$

$$\psi_{\overline{5}} = \begin{pmatrix} d_L^{1c} \\ d_L^{2c} \\ d_L^{3c} \\ \ell_L \\ -\nu_L \end{pmatrix} \qquad \psi_{10} = \begin{pmatrix} 0 & u_L^{3c} & -u_L^{2c} & -u_L^1 & -d_L^1 \\ -u_L^{3c} & 0 & u_L^{1c} & -u_L^2 & -d_L^2 \\ u_L^{2c} & -u_L^{1c} & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & -\ell_L^c \\ d_L^1 & d_L^2 & d_L^3 & \ell_L^c & 0 \end{pmatrix}$$

Charge Quantization

$\hat{Q}$ is traceless $\Rightarrow$ $\hat{Q}\psi_{\overline{5}} = 0$ $\Rightarrow$ $3\hat{Q}d_L^c + \hat{Q}\ell_L = 0$

Charge Quantization: $\hat{Q}d = \frac{1}{N_c}\hat{Q}\ell$

$$\hat{Q} = diag\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\right)$$

$$\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2} \quad \Rightarrow \quad \hat{Y} = \frac{1}{3}diag(-2, -2, -2, 3, 3)$$

Charge Quantization

$\hat{Q}$ is traceless $\Rightarrow$ $\hat{Q}\psi_{\overline{5}} = 0$ $\Rightarrow$ $3\hat{Q}d_L^c + \hat{Q}\ell_L = 0$

Charge Quantization: $\hat{Q}d = \frac{1}{N_c}\hat{Q}\ell$

$$\hat{Q} = diag\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\right)$$

$$\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2} \quad \Rightarrow \quad \hat{Y} = \frac{1}{3} diag(-2, -2, -2, 3, 3)$$

Charge Quantization

$\hat{Q}$ is traceless $\Rightarrow$ $\hat{Q}\psi_{\overline{5}} = 0$ $\Rightarrow$ $3\hat{Q}d_L^c + \hat{Q}\ell_L = 0$

Charge Quantization: $\hat{Q}d = \frac{1}{N_c}\hat{Q}\ell$

$$\hat{Q} = diag\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\right)$$

$$\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2} \quad \Rightarrow \quad \hat{Y} = \frac{1}{3}diag(-2, -2, -2, 3, 3)$$

Charge Quantization

$\hat{Q}$ is traceless $\Rightarrow$ $\hat{Q}\psi_{\overline{5}} = 0$ $\Rightarrow$ $3\hat{Q}d_L^c + \hat{Q}\ell_L = 0$

Charge Quantization: $\hat{Q}d = \frac{1}{N_c}\hat{Q}\ell$

$$\hat{Q} = diag\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\right)$$

$$\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2} \quad \Rightarrow \quad \hat{Y} = \frac{1}{3}diag(-2, -2, -2, 3, 3)$$

Charge Quantization

$\hat{Q}$ is traceless $\Rightarrow$ $\hat{Q}\psi_{\overline{5}} = 0$ $\Rightarrow$ $3\hat{Q}d^c_L + \hat{Q}\ell_L = 0$

Charge Quantization: $\hat{Q}d = \frac{1}{N_c}\hat{Q}\ell$

$$\hat{Q} = diag\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\right)$$

$$\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2} \quad \Rightarrow \quad \hat{Y} = \frac{1}{3} diag(-2, -2, -2, 3, 3)$$

Gauge Invariance

The Fields Transform as

$$\psi_5 \to \psi_5' = \left(\mathcal{I} - \imath\theta^a t^a\right)\psi_5 \qquad \psi_{\overline{5}} \to \psi_{\overline{5}}' = \left(\mathcal{I} + \imath\theta^a t^{aT}\right)\psi_{\overline{5}}$$

$$\psi_{10}^{ij} \to \psi_{10}^{ij\prime} = \left[\delta_k^i\delta_\ell^j - \imath\theta^a\left(\delta_\ell^j t_k^{ai} + \delta_k^i t_\ell^{aj}\right)\right]\psi_{10}^{k\ell}$$

As a result the Gauge Interactions are

$$\mathcal{L}_{\overline{5}} = \imath\overline{\psi}_{\overline{5}}\not{D}\psi_{\overline{5}} = \imath\overline{\psi}_{\overline{5}}\not{\partial}\psi_{\overline{5}} - \frac{g_5}{\sqrt{2}}\overline{\psi}_{\overline{5}}\not{V}^T\psi_{\overline{5}},$$

$$\mathcal{L}_{10} = \frac{\imath}{2}Tr\left[\overline{\psi}_{10}\not{\partial}\psi_{10}\right] + \frac{g_5}{\sqrt{2}}Tr\left[\overline{\psi}_{10}\not{V}\psi_{10}\right]$$

Gauge Invariance

The Fields Transform as

$$\psi_5 \to \psi_5' = \left(\mathcal{I} - \imath\theta^a t^a\right)\psi_5 \qquad \psi_{\overline{5}} \to \psi_{\overline{5}}' = \left(\mathcal{I} + \imath\theta^a t^{aT}\right)\psi_{\overline{5}}$$

$$\psi_{10}^{ij} \to \psi_{10}^{ij\prime} = \left[\delta_k^i\delta_\ell^j - \imath\theta^a\left(\delta_\ell^j t_k^{ai} + \delta_k^i t_\ell^{aj}\right)\right]\psi_{10}^{k\ell}$$

As a result the Gauge Interactions are

$$\mathcal{L}_{\overline{5}} = \imath\overline{\psi}_{\overline{5}}\not{D}\psi_{\overline{5}} = \imath\overline{\psi}_{\overline{5}}\not{\partial}\psi_{\overline{5}} - \frac{g_5}{\sqrt{2}}\overline{\psi}_{\overline{5}}\not{V}^T\psi_{\overline{5}},$$

$$\mathcal{L}_{10} = \frac{\imath}{2}Tr\left[\,\overline{\psi}_{10}\not{\partial}\psi_{10}\,\right] + \frac{g_5}{\sqrt{2}}Tr\left[\,\overline{\psi}_{10}\not{V}\psi_{10}\,\right]$$

Coupling Constants

We separate the gauge bosons as

$$\mathcal{V}_\mu = \mathcal{G}_\mu + \mathcal{A}_\mu + \mathcal{B}_\mu + \mathcal{X}_\mu + \mathcal{Y}_\mu$$

$$\mathcal{L}_I = \mathcal{L}_I^{QCD} + \mathcal{L}_I^{SU(2)} + \mathcal{L}_I^{U(1)} + \mathcal{L}_I^{X} + \mathcal{L}_I^{Y}$$

$$\mathcal{L}_I^{U(1)} = \sqrt{\frac{3}{5}}\frac{g_5}{2}\Big(\frac{1}{3}\left(\bar{u}_L \not{B} u_L + \bar{d}_L \not{B} d_L\right) + \frac{4}{3}\bar{u}_R \not{B} u_R - \frac{2}{3}\bar{d}_R \not{B} d_R - \left(\bar{\ell}_L \not{B} \ell_L + \overline{\nu}_L \not{B} \nu_L\right) - 2\ell_R \not{B} \bar{\ell}_R\Big)$$

$$\mathcal{L}_I^{SU(2)} = \frac{g_5}{2}\Big(\overline{u}_L \not{A}^3 u_L - \overline{d}_L \not{A}^3 d_L + \sqrt{2}\overline{d}_L \not{W}^\dagger u_L + \sqrt{2}\overline{u}_L \not{W} d_L + \overline{\nu}_L \not{A}^3 \nu_L - \overline{e}_L \not{A}^3 e_L + \sqrt{2}\overline{e}_L \not{W}^\dagger \nu_L + \sqrt{2}\overline{\nu}_L \not{W} e_L\Big)$$

$$\begin{aligned}\mathcal{L}_I^{SU(3)} = \frac{g_5}{\sqrt{2}}\Big(&\overline{d}^1 \not{G}^{11} d^1 + \overline{d}^1 \not{G}^{12} d^2 + \overline{d}^1 \not{G}^{13} d^3 + \overline{d}^2 \not{G}^{21} d^1 + \overline{d}^2 \not{G}^{22} d^2 + \overline{d}^2 \not{G}^{23} d^3 \\ &+\overline{d}^3 \not{G}^{31} d^1 + \overline{d}^3 \not{G}^{32} d^2 + \overline{d}^3 \not{G}^{33} d^3 + \overline{u}^1 \not{G}^{11} u^1 + \overline{u}^1 \not{G}^{12} u^2 + \overline{u}^1 \not{G}^{13} u^3 \\ &+\overline{u}^2 \not{G}^{21} u^1 + \overline{u}^2 \not{G}^{22} u^2 + \overline{u}^2 \not{G}^{23} u^3 + \overline{u}^3 \not{G}^{31} u^1 + \overline{u}^3 \not{G}^{32} u^2 + \overline{u}^3 \not{G}^{33} u^3\Big)\end{aligned}$$

We predict

$$g_5 = \sqrt{\frac{5}{3}}g' = g = g_s \qquad \tan^2\theta_W = 0.6$$

Coupling Constants

We separate the gauge bosons as

$$\mathcal{V}_\mu = \mathcal{G}_\mu + \mathcal{A}_\mu + \mathcal{B}_\mu + \mathcal{X}_\mu + \mathcal{Y}_\mu$$

$$\mathcal{L}_I = \mathcal{L}_I^{QCD} + \mathcal{L}_I^{SU(2)} + \mathcal{L}_I^{U(1)} + \mathcal{L}_I^{X} + \mathcal{L}_I^{Y}$$

$$\mathcal{L}_I^{U(1)} = \sqrt{\frac{3}{5}}\frac{g_5}{2}\Big(\frac{1}{3}\left(\bar{u}_L \not{B} u_L + \bar{d}_L \not{B} d_L\right) + \frac{4}{3}\bar{u}_R \not{B} u_R - \frac{2}{3}\bar{d}_R \not{B} d_R - \left(\bar{\ell}_L \not{B} \ell_L + \overline{\nu}_L \not{B} \nu_L\right) - 2\ell_R \not{B} \bar{\ell}_R\Big)$$

$$\mathcal{L}_I^{SU(2)} = \frac{g_5}{2}\Big(\overline{u}_L \not{A}^3 u_L - \overline{d}_L \not{A}^3 d_L + \sqrt{2}\overline{d}_L \not{W}^\dagger u_L + \sqrt{2}\overline{u}_L \not{W} d_L + \overline{\nu}_L \not{A}^3 \nu_L - \overline{e}_L \not{A}^3 e_L + \sqrt{2}\overline{e}_L \not{W}^\dagger \nu_L + \sqrt{2}\overline{\nu}_L \not{W} e_L\Big)$$

$$\begin{aligned}\mathcal{L}_I^{SU(3)} = \frac{g_5}{\sqrt{2}}\Big(&\overline{d}^1 \not{G}^{11} d^1 + \overline{d}^1 \not{G}^{12} d^2 + \overline{d}^1 \not{G}^{13} d^3 + \overline{d}^2 \not{G}^{21} d^1 + \overline{d}^2 \not{G}^{22} d^2 + \overline{d}^2 \not{G}^{23} d^3 \\ &+ \overline{d}^3 \not{G}^{31} d^1 + \overline{d}^3 \not{G}^{32} d^2 + \overline{d}^3 \not{G}^{33} d^3 + \overline{u}^1 \not{G}^{11} u^1 + \overline{u}^1 \not{G}^{12} u^2 + \overline{u}^1 \not{G}^{13} u^3 \\ &+ \overline{u}^2 \not{G}^{21} u^1 + \overline{u}^2 \not{G}^{22} u^2 + \overline{u}^2 \not{G}^{23} u^3 + \overline{u}^3 \not{G}^{31} u^1 + \overline{u}^3 \not{G}^{32} u^2 + \overline{u}^3 \not{G}^{33} u^3\Big)\end{aligned}$$

We predict

$$g_5 = \sqrt{\frac{5}{3}} g' = g = g_s \qquad \tan^2\theta_W = 0.6$$

The Running Couplings

Problem:

$$\tan^2\theta_W = 0.30073 \pm 0.00025 \text{ @ } M_Z$$

Not really... The prediction refers to GUT scale!
We ignore threshold corrections and assume desert! Then 1-loop RGEs for $SU(N)$:

$$\frac{1}{g_i^2(\mu)} = \frac{1}{g_i^2(Q)} + b_i \log\left(Q^2/\mu^2\right) \qquad b_N = \frac{1}{(4\pi)^2}\left[-\frac{11}{3}N + \frac{4}{3}n_g\right]$$

$$b_1 = \frac{1}{4\pi^2} \quad b_2 = -\frac{5}{24\pi^2} \quad b_3 = -\frac{7}{16\pi^2}$$

$$g_1 = g' = 0.357 \quad g_2 = g = 0.652 \quad g_3 = g_s = 1.221 \text{ @ } M_Z$$

The Running Couplings

Problem:

$$\tan^2\theta_W = 0.30073 \pm 0.00025 \text{ @ } M_Z$$

Not really... The prediction refers to GUT scale!
We ignore threshold corrections and assume desert! Then 1-loop RGEs for $SU(N)$:

$$\frac{1}{g_i^2(\mu)} = \frac{1}{g_i^2(Q)} + b_i \log\left(Q^2/\mu^2\right) \quad b_N = \frac{1}{(4\pi)^2}\left[-\frac{11}{3}N + \frac{4}{3}n_g\right]$$

$$b_1 = \frac{1}{4\pi^2} \quad b_2 = -\frac{5}{24\pi^2} \quad b_3 = -\frac{7}{16\pi^2}$$

$$g_1 = g' = 0.357 \quad g_2 = g = 0.652 \quad g_3 = g_s = 1.221 \text{@ } M_Z$$

The Running Couplings

Problem:

$$\tan^2\theta_W = 0.30073 \pm 0.00025 \ @ \ M_Z$$

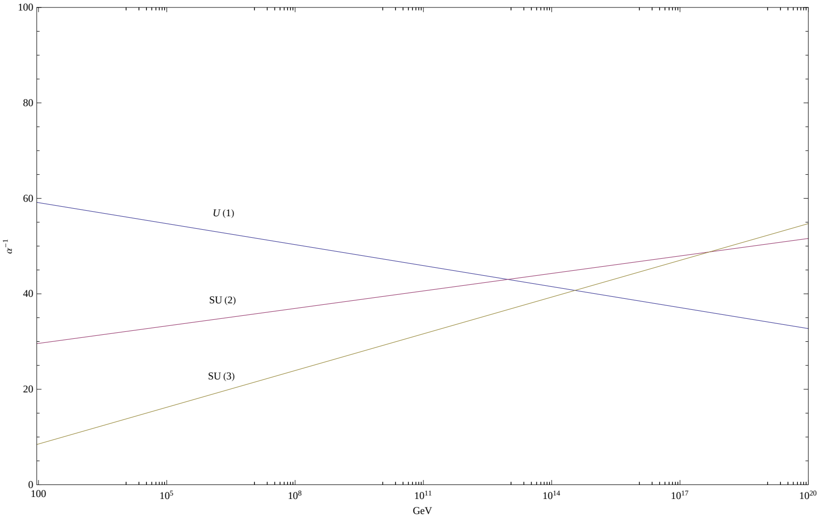
Not really... The prediction refers to GUT scale!
We ignore threshold corrections and assume desert! Then 1-loop RGEs for $SU(N)$:

$$\frac{1}{g_i^2(\mu)} = \frac{1}{g_i^2(Q)} + b_i \log\left(Q^2/\mu^2\right) \qquad b_N = \frac{1}{(4\pi)^2}\left[-\frac{11}{3}N + \frac{4}{3}n_g\right]$$

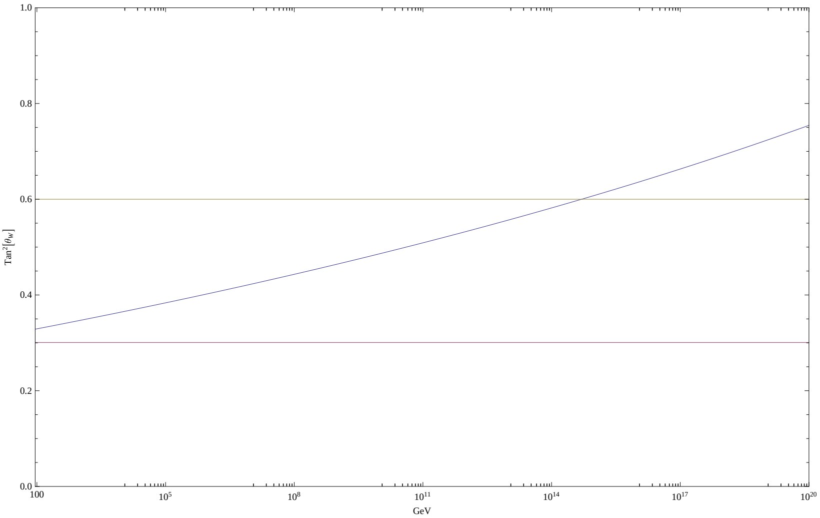
$$b_1 = \frac{1}{4\pi^2} \quad b_2 = -\frac{5}{24\pi^2} \quad b_3 = -\frac{7}{16\pi^2}$$

$$g_1 = g' = 0.357 \quad g_2 = g = 0.652 \quad g_3 = g_s = 1.221 @ \ M_Z$$

Coupling Unification



$\tan^2 \theta_W$


1.0
0.8
0.6
0.4
0.2
0.0
Tan$^2[\theta_W]$
100
10^5
10^8
10^{11}
10^{14}
10^{17}
10^{20}
GeV

Lets sum up!

- Particles
- Coupling Constants
- Gauge Bosons
- We have to break SU(5)....

Lets sum up!

- Particles
- Coupling Constants
- Gauge Bosons
- We have to break SU(5)....

Lets sum up!

- Particles
- Coupling Constants
- Gauge Bosons
- We have to break SU(5)....

Lets sum up!

- Particles
- Coupling Constants
- Gauge Bosons
- We have to break SU(5)....

Our Goals

- SU(5) Breaking
- $SU(2) \times U(1)$ Breaking
- Give correct mass to known particles
- Get rid of unobserved particles

Our Goals

- SU(5) Breaking
- $SU(2) \times U(1)$ Breaking
- Give correct mass to known particles
- Get rid of unobserved particles

Our Goals

- SU(5) Breaking
- $SU(2) \times U(1)$ Breaking
- Give correct mass to known particles
- Get rid of unobserved particles

Our Goals

- SU(5) Breaking
- $SU(2) \times U(1)$ Breaking
- Give correct mass to known particles
- Get rid of unobserved particles

The 24-Higgs

We introduce 24 Higgs fields in the adjoint rep of SU(5).
We define a 5×5 traceless matrix Σ transforming according to $5 \times \overline{5} = 24 \oplus 1$.

$$\Sigma = 2\phi^a t^a$$

$$\mathcal{L}^{\Sigma}_{kin} = \frac{1}{4} Tr \left[\left(\mathcal{D}_\mu \Sigma\right)^\dagger \mathcal{D}^\mu \Sigma \right] \quad \mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - \imath \frac{g_5}{\sqrt{2}} \left[\mathcal{V}_\mu, \Sigma\right]$$

SU(5) Breaking

The VEV must have the form

$$\langle\Sigma\rangle = v\ diag\left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right)$$

Gauge Bosons Mass Matrix

$$m^2_{ab} V^a_\mu V^{\mu b} = \frac{g_5^2}{8} Tr\left[\left[\mathcal{V}_\mu, \langle\Sigma\rangle\right]\left[\mathcal{V}^\mu, \langle\Sigma\rangle\right]\right]$$

As a result we get

$$m_X^2 = m_Y^2 = \frac{25}{8} g_5^2 v^2$$

This mass has to be at the GUT scale!

SU(5) Breaking

The VEV must have the form

$$\langle \Sigma \rangle = v \, diag \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right)$$

Gauge Bosons Mass Matrix

$$m^2_{ab} V^a_\mu V^{\mu b} = \frac{g_5^2}{8} Tr \left[\left[\mathcal{V}_\mu, \langle \Sigma \rangle \right] \left[\mathcal{V}^\mu, \langle \Sigma \rangle \right] \right]$$

As a result we get

$$m^2_X = m^2_Y = \frac{25}{8} g_5^2 v^2$$

This mass has to be at the GUT scale!

SU(5) Breaking

The VEV must have the form

$$\langle\Sigma\rangle = v\ diag\left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right)$$

Gauge Bosons Mass Matrix

$$m^2_{ab} V^a_\mu V^{\mu b} = \frac{g_5^2}{8} Tr\left[\left[\mathcal{V}_\mu, \langle\Sigma\rangle\right]\left[\mathcal{V}^\mu, \langle\Sigma\rangle\right]\right]$$

As a result we get

$$m^2_X = m^2_Y = \frac{25}{8} g_5^2 v^2$$

This mass has to be at the GUT scale!

The 24-potential

The potential is

$$V(\Sigma) = \frac{\mu^2}{2} Tr\left[\Sigma^2\right] + \frac{a}{4}\left[Tr\left[\Sigma^2\right]\right]^2 + \frac{b}{2} Tr\left[\Sigma^4\right]$$

$$a > 0, \quad 15a + 7b > 0, \quad \mu^2 < 0$$

At minimum

$$V(\langle\Sigma\rangle) = \frac{15}{2}\left(\frac{\mu^2}{2} v^2 + \frac{15}{8} a v^4 + \frac{7}{8} b v^4\right)$$

This leads to

$$\frac{\partial}{\partial v} V(\langle\Sigma\rangle) = 0 \quad \Rightarrow \quad \mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 = 0$$

The 24-potential

The potential is

$$V(\Sigma) = \frac{\mu^2}{2} Tr\left[\Sigma^2\right] + \frac{a}{4}\left[Tr\left[\Sigma^2\right]\right]^2 + \frac{b}{2} Tr\left[\Sigma^4\right]$$

$$a > 0, \quad 15a + 7b > 0, \quad \mu^2 < 0$$

At minimum

$$V(\langle\Sigma\rangle) = \frac{15}{2}\left(\frac{\mu^2}{2} v^2 + \frac{15}{8} a v^4 + \frac{7}{8} b v^4\right)$$

This leads to

$$\frac{\partial}{\partial v} V(\langle\Sigma\rangle) = 0 \quad \Rightarrow \quad \mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 = 0$$

The 24-potential

The potential is

$$V(\Sigma) = \frac{\mu^2}{2} Tr\left[\Sigma^2\right] + \frac{a}{4}\left[Tr\left[\Sigma^2\right]\right]^2 + \frac{b}{2} Tr\left[\Sigma^4\right]$$

$$a > 0, \quad 15a + 7b > 0, \quad \mu^2 < 0$$

At minimum

$$V(\langle\Sigma\rangle) = \frac{15}{2}\left(\frac{\mu^2}{2} v^2 + \frac{15}{8} a v^4 + \frac{7}{8} b v^4\right)$$

This leads to

$$\frac{\partial}{\partial v} V(\langle\Sigma\rangle) = 0 \quad \Rightarrow \quad \mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 = 0$$

24-Higgs Masses

We write:

$$\Sigma = \langle\Sigma\rangle + \begin{pmatrix} H^8_{11} + \sqrt{\frac{2}{15}}H_0 & H^8_{12} & H^8_{13} & \tilde{H}^X_1 & \tilde{H}^Y_1 \\ H^8_{21} & H^8_{22} + \sqrt{\frac{2}{15}}H_0 & H^8_{23} & \tilde{H}^X_2 & \tilde{H}^Y_2 \\ H^8_{31} & H^8_{32} & H^8_{33} + \sqrt{\frac{2}{15}}H_0 & \tilde{H}^X_3 & \tilde{H}^Y_3 \\ \tilde{H}^{X\dagger}_1 & \tilde{H}^{X\dagger}_2 & \tilde{H}^{X\dagger}_3 & \frac{1}{\sqrt{2}}H_Z - \sqrt{\frac{3}{10}}H_0 & H^+ \\ \tilde{H}^{Y\dagger}_1 & \tilde{H}^{Y\dagger}_2 & \tilde{H}^{Y\dagger}_3 & H^- & -\frac{1}{\sqrt{2}}H_Z - \sqrt{\frac{3}{10}}H_0 \end{pmatrix}$$

The mass spectrum is:

$$m_8^2 = \frac{5}{2}bv^2 \quad m_Z^2 = m_\pm^2 = 10bv^2 \quad m_0^2 = -2\mu^2$$

$\tilde{H}^X \& \tilde{H}^Y$ are would be Goldstone Bosons absorbed by $X \& Y$ Gauge Bosons.

24-Higgs Masses

We write:

$$\Sigma = \langle\Sigma\rangle + \begin{pmatrix} H^8_{11} + \sqrt{\frac{2}{15}}H_0 & H^8_{12} & H^8_{13} & \tilde{H}^X_1 & \tilde{H}^Y_1 \\ H^8_{21} & H^8_{22} + \sqrt{\frac{2}{15}}H_0 & H^8_{23} & \tilde{H}^X_2 & \tilde{H}^Y_2 \\ H^8_{31} & H^8_{32} & H^8_{33} + \sqrt{\frac{2}{15}}H_0 & \tilde{H}^X_3 & \tilde{H}^Y_3 \\ \tilde{H}^{X\dagger}_1 & \tilde{H}^{X\dagger}_2 & \tilde{H}^{X\dagger}_3 & \frac{1}{\sqrt{2}}H_Z - \sqrt{\frac{3}{10}}H_0 & H^+ \\ \tilde{H}^{Y\dagger}_1 & \tilde{H}^{Y\dagger}_2 & \tilde{H}^{Y\dagger}_3 & H^- & -\frac{1}{\sqrt{2}}H_Z - \sqrt{\frac{3}{10}}H_0 \end{pmatrix}$$

The mass spectrum is:

$$m_8^2 = \frac{5}{2}bv^2 \quad m_z^2 = m_\pm^2 = 10bv^2 \quad m_0^2 = -2\mu^2$$

$\tilde{H}^X \& \tilde{H}^Y$ are would be Goldstone Bosons absorbed by $X \& Y$ Gauge Bosons.

The 5-Higgs

The Higgs 5plet is $H = \begin{pmatrix} h^1 & h^2 & h^3 & h^+ & h^0 \end{pmatrix}^T$

VEV must have the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & v_0 \end{pmatrix}^T$

The potential is

$$V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0$$

Minimalization yields

$$V(\langle H \rangle) = \frac{1}{2}\mu_0^2 v_0^2 + \frac{\lambda}{4} v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0$$

As in SM $W\&Z$ Bosons acquire mass

$$m_W = m_Z \cos\theta_W = \frac{g v_0}{2}$$

Obviously this mass has to be at the EW scale!

The 5-Higgs

The Higgs 5plet is $H = \begin{pmatrix} h^1 & h^2 & h^3 & h^+ & h^0 \end{pmatrix}^T$

VEV must have the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & v_0 \end{pmatrix}^T$

The potential is

$$V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0$$

Minimalization yields

$$V(\langle H \rangle) = \frac{1}{2}\mu_0^2 v_0^2 + \frac{\lambda}{4} v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0$$

As in SM W&Z Bosons acquire mass

$$m_W = m_Z \cos\theta_W = \frac{g v_0}{2}$$

Obviously this mass has to be at the EW scale!

The 5-Higgs

The Higgs 5plet is $H = \begin{pmatrix} h^1 & h^2 & h^3 & h^+ & h^0 \end{pmatrix}^T$

VEV must have the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & v_0 \end{pmatrix}^T$

The potential is

$$V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0$$

Minimalization yields

$$V(\langle H \rangle) = \frac{1}{2}\mu_0^2 v_0^2 + \frac{\lambda}{4} v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0$$

As in SM $W \& Z$ Bosons acquire mass

$$m_W = m_Z \cos\theta_W = \frac{g v_0}{2}$$

Obviously this mass has to be at the EW scale!

The 5-Higgs

The Higgs 5plet is $H = \begin{pmatrix} h^1 & h^2 & h^3 & h^+ & h^0 \end{pmatrix}^T$
VEV must have the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & v_0 \end{pmatrix}^T$
The potential is

$$V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0$$

Minimalization yields

$$V(\langle H \rangle) = \frac{1}{2}\mu_0^2 v_0^2 + \frac{\lambda}{4} v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0$$

As in SM W&Z Bosons acquire mass

$$m_W = m_Z \cos\theta_W = \frac{g v_0}{2}$$

Obviously this mass has to be at the EW scale!

The 5-Higgs

The Higgs 5plet is $H = \begin{pmatrix} h^1 & h^2 & h^3 & h^+ & h^0 \end{pmatrix}^T$
VEV must have the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & v_0 \end{pmatrix}^T$
The potential is

$$V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0$$

Minimalization yields

$$V(\langle H \rangle) = \frac{1}{2}\mu_0^2 v_0^2 + \frac{\lambda}{4} v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0$$

As in SM $W \& Z$ Bosons acquire mass

$$m_W = m_Z \cos\theta_W = \frac{g v_0}{2}$$

Obviously this mass has to be at the EW scale!

The 5-Higgs Masses

We write:

$$H = \begin{pmatrix} \tilde{H}^1 \\ \tilde{H}^2 \\ \tilde{H}^3 \\ H_4 \\ \frac{1}{\sqrt{2}}\left(v_0 + h\right) e^{\imath \frac{\zeta}{v_0}} \end{pmatrix}$$

Then $m_h^2 = -\mu_0^2$

$H_4 \& \zeta$ are would be Goldstone Bosons absorbed by $W \& Z$ Gauge Bosons.

$\tilde{H}$ is physical and remains massless.

The 5-Higgs Masses

We write:

$$H = \begin{pmatrix} \tilde{H}^1 \\ \tilde{H}^2 \\ \tilde{H}^3 \\ H_4 \\ \frac{1}{\sqrt{2}} \left(v_0 + h \right) e^{\imath \frac{\zeta}{v_0}} \end{pmatrix}$$

Then $m_h^2 = -\mu_0^2$

$H_4 \& \zeta$ are would be Goldstone Bosons absorbed by $W \& Z$ Gauge Bosons.

$\tilde{H}$ is physical and remains massless.

The 5-Higgs Masses

We write:

$$H = \begin{pmatrix} \tilde{H}^1 \\ \tilde{H}^2 \\ \tilde{H}^3 \\ H_4 \\ \frac{1}{\sqrt{2}}\left(v_0 + h\right) e^{\imath \frac{\zeta}{v_0}} \end{pmatrix}$$

Then $m_h^2 = -\mu_0^2$

$H_4 \& \zeta$ are would be Goldstone Bosons absorbed by $W \& Z$ Gauge Bosons.

$\tilde{H}$ is physical and remains massless.

The 5-Higgs Masses

We write:

$$H = \begin{pmatrix} \tilde{H}^1 \\ \tilde{H}^2 \\ \tilde{H}^3 \\ H_4 \\ \frac{1}{\sqrt{2}}\left(v_0 + h\right) e^{\imath \frac{\zeta}{v_0}} \end{pmatrix}$$

Then $m_h^2 = -\mu_0^2$

$H_4 \& \zeta$ are would be Goldstone Bosons absorbed by $W \& Z$ Gauge Bosons.

$\tilde{H}$ is physical and remains massless.

It couldn't be so easy...

We have broken SU(5) according to the pattern

$$SU(5) \xrightarrow{\text{GUT Scale}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{EW Scale}} SU(3) \times U(1)_{em}$$

But...

- The 5's color triplet is massless at tree level. Too rapid Proton Decay!!!
- Gauge Bosons couple Σ to H at 1-loop. Renormalizability requires coupling at tree level!

$$V(\Sigma, H) = \alpha |H|^2 Tr\left[\Sigma^2\right] + \beta \overline{H} \Sigma^2 H$$

It couldn't be so easy...

We have broken SU(5) according to the pattern

$$SU(5) \xrightarrow{\text{GUT Scale}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{EW Scale}} SU(3) \times U(1)_{em}$$

But...

- The 5's color triplet is massless at tree level. Too rapid Proton Decay!!!
- Gauge Bosons couple Σ to H at 1-loop. Renormalizability requires coupling at tree level!

$$V(\Sigma, H) = \alpha |H|^2 Tr\left[\Sigma^2\right] + \beta \overline{H} \Sigma^2 H$$

It couldn't be so easy...

We have broken SU(5) according to the pattern

$$SU(5) \xrightarrow{\text{GUT Scale}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{EW Scale}} SU(3) \times U(1)_{em}$$

But...

- The 5's color triplet is massless at tree level. Too rapid Proton Decay!!!
- Gauge Bosons couple Σ to H at 1-loop. Renormalizability requires coupling at tree level!

$$V(\Sigma, H) = \alpha |H|^2 Tr\left[\Sigma^2\right] + \beta \overline{H} \Sigma^2 H$$

The Total Potential

The total potential is:

$$V(\Sigma, H) = \frac{\mu^2}{2} Tr\left[\Sigma^2\right] + \frac{a}{4}\left[Tr\left[\Sigma^2\right]\right]^2 + \frac{b}{2} Tr\left[\Sigma^4\right] + \alpha |H|^2 Tr\left[\Sigma^2\right] + \beta \overline{H}\Sigma^2 H + \mu_0^2 |H|^2 + \lambda |H|^4$$

The modified vacuum is:

$$\langle\Sigma\rangle = v\, diag\left(1, 1, 1, -\frac{3}{2} - \frac{1}{2}\epsilon, -\frac{3}{2} + \frac{1}{2}\epsilon\right) \qquad \langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_0 \end{pmatrix}$$

where $\epsilon = \frac{3}{20}\frac{\beta v_0^2}{b v^2}$ to order $\mathcal{O}(v_0^2/v^2)$.

The Total Potential

The total potential is:

$$V(\Sigma, H) = \frac{\mu^2}{2} Tr\left[\Sigma^2\right] + \frac{a}{4}\left[Tr\left[\Sigma^2\right]\right]^2 + \frac{b}{2} Tr\left[\Sigma^4\right] + \alpha |H|^2 Tr\left[\Sigma^2\right] + \beta \overline{H}\Sigma^2 H + \mu_0^2 |H|^2 + \lambda |H|^4$$

The modified vacuum is:

$$\langle \Sigma \rangle = v\, diag\left(1, 1, 1, -\frac{3}{2} - \frac{1}{2}\epsilon, -\frac{3}{2} + \frac{1}{2}\epsilon\right) \qquad \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_0 \end{pmatrix}$$

where $\epsilon = \frac{3}{20}\frac{\beta v_0^2}{b v^2}$ to order $\mathcal{O}(v_0^2/v^2)$.

The modified Vacuum

The ϵ terms affect EW breaking but their result is negligible.
The vacuum condition yields

$$\mu^2 + \frac{15}{2}av^2 + \frac{7}{2}bv^2 + \alpha v_0^2 + \frac{3}{10}\beta v_0^2 = 0$$
$$\mu_0^2 + \lambda v_0^2 + \frac{15}{2}\alpha v^2 + \left(\frac{9}{4} - \frac{3}{2}\epsilon\right)\beta v^2 = 0$$

The cross-term saves Renormalizability, but the interaction between Σ and H requires extreme fine tuning in order to keep the SM's Higgs at the EW scale. Hierarchy Problem appears again!

The modified Vacuum

The ϵ terms affect EW breaking but their result is negligible.
The vacuum condition yields

$$\mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 + \alpha v_0^2 + \frac{3}{10} \beta v_0^2 = 0$$
$$\mu_0^2 + \lambda v_0^2 + \frac{15}{2} \alpha v^2 + \left(\frac{9}{4} - \frac{3}{2} \epsilon \right) \beta v^2 = 0$$

The cross-term saves Renormalizability, but the interaction between Σ and H requires extreme fine tuning in order to keep the SM's Higgs at the EW scale. Hierarchy Problem appears again!

The modified Vacuum

The ϵ terms affect EW breaking but their result is negligible. The vacuum condition yields

$$\mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 + \alpha v_0^2 + \frac{3}{10} \beta v_0^2 = 0$$
$$\mu_0^2 + \lambda v_0^2 + \frac{15}{2} \alpha v^2 + \left(\frac{9}{4} - \frac{3}{2} \epsilon \right) \beta v^2 = 0$$

The cross-term saves Renormalizability, but the interaction between Σ and H requires extreme fine tuning in order to keep the SM's Higgs at the EW scale. Hierarchy Problem appears again!

Modified Mass Spectrum

The Modified Mass Spectrum is quite complicated.

- The already heavy Higgs $\tilde{H}^8 \& H^{\pm}$ receive correction $\mathcal{O}(v_0^2)$.
- A combination of h, $H_Z \& H_0$ remains at EW Scale while the other two are at the GUT scale.
- A combination of $\tilde{H} \& \tilde{H}_Y$ is absorbed by Y while the others becomes massive $\mathcal{O}(v^2)$. This suppresses Higgs mediated Proton Decay.

Modified Mass Spectrum

The Modified Mass Spectrum is quite complicated.

- The already heavy Higgs $\tilde{H}^8$&$H^{\pm}$ receive correction $\mathcal{O}(v_0^2)$.
- A combination of h, H_Z&H_0 remains at EW Scale while the other two are at the GUT scale.
- A combination of $\tilde{H}$&$\tilde{H}_Y$ is absorbed by Y while the others becomes massive $\mathcal{O}(v^2)$. This suppresses Higgs mediated Proton Decay.

Modified Mass Spectrum

The Modified Mass Spectrum is quite complicated.

- The already heavy Higgs $\tilde{H}^8 \& H^{\pm}$ receive correction $\mathcal{O}(v_0^2)$.
- A combination of $h, H_Z \& H_0$ remains at EW Scale while the other two are at the GUT scale.
- A combination of $\tilde{H} \& \tilde{H}_Y$ is absorbed by Y while the others becomes massive $\mathcal{O}(v^2)$. This suppresses Higgs mediated Proton Decay.

Modified Mass Spectrum

The Modified Mass Spectrum is quite complicated.

- The already heavy Higgs $\tilde{H}^8$&$H^\pm$ receive correction $\mathcal{O}(v_0^2)$.
- A combination of h, H_Z&H_0 remains at EW Scale while the other two are at the GUT scale.
- A combination of $\tilde{H}$&$\tilde{H}_Y$ is absorbed by Y while the others becomes massive $\mathcal{O}(v^2)$. This suppresses Higgs mediated Proton Decay.

Mass Terms

The particles transform according to $\overline{5}$&10. The product of these reps are

$$\overline{5}\times\overline{5}=\overline{10}\oplus\overline{15},\quad \overline{5}\times 10=5\oplus\overline{45},\quad 10\times 10=\overline{5}\oplus 45\oplus 50$$

Only 5&45 contain neutral components, thus couple to matter at tree level. 10&50 could contribute only to mass renormalization.

In the Minimal SU(5) we use only 5-Higgs to generate mass for the fermions. The couplings are:

$$\mathcal{L}_{mass}=Y^D_{ij}\psi^T_{5i\alpha}\mathcal{C}\psi_{10j\alpha\beta}H_\beta+Y^U_{ij}\frac{1}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\psi^T_{10i\alpha\beta}\mathcal{C}\psi_{10j\gamma\delta}H_\epsilon+H.C.$$

$$=-\frac{v_0}{2}Y^D_{ij}\left(\overline{d}_{iR}d_{jL}+\overline{\ell}_{iL}\ell_{jR}\right)-\frac{v_0}{\sqrt{2}}Y^U_{ij}\overline{u}_{iR}u_{jL}+H.C.$$

where $Y^U=Y^{U\dagger}$

Mass Terms

The particles transform according to $\overline{5}$&10. The product of these reps are

$$\overline{5}\times\overline{5}=\overline{10}\oplus\overline{15},\quad \overline{5}\times 10=5\oplus\overline{45},\quad 10\times 10=\overline{5}\oplus 45\oplus 50$$

Only 5&45 contain neutral components, thus couple to matter at tree level. 10&50 could contribute only to mass renormalization.

In the Minimal SU(5) we use only 5-Higgs to generate mass for the fermions. The couplings are:

$$\mathcal{L}_{mass}=Y^D_{ij}\psi^T_{5i\alpha}\mathcal{C}\psi_{10j\alpha\beta}H_\beta+Y^U_{ij}\frac{1}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\psi^T_{10i\alpha\beta}\mathcal{C}\psi_{10j\gamma\delta}H_\epsilon+H.C.$$
$$=-\frac{v_0}{2}Y^D_{ij}\left(\overline{d}_{iR}d_{jL}+\overline{\ell}_{iL}\ell_{jR}\right)-\frac{v_0}{\sqrt{2}}Y^U_{ij}\overline{u}_{iR}u_{jL}+H.C.$$

where $Y^U=Y^{U\dagger}$

Mass Terms

The particles transform according to $\overline{5}$&10. The product of these reps are

$$\overline{5}\times\overline{5}=\overline{10}\oplus\overline{15},\quad \overline{5}\times 10=5\oplus\overline{45},\quad 10\times 10=\overline{5}\oplus 45\oplus 50$$

Only 5&45 contain neutral components, thus couple to matter at tree level. 10&50 could contribute only to mass renormalization.
In the Minimal SU(5) we use only 5-Higgs to generate mass for the fermions. The couplings are:

$$\begin{aligned}\mathcal{L}_{mass} &= Y^D_{ij}\psi^T_{5i\alpha}\mathcal{C}\psi_{10j\alpha\beta}H_\beta + Y^U_{ij}\frac{1}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\psi^T_{10i\alpha\beta}\mathcal{C}\psi_{10j\gamma\delta}H_\epsilon + H.C. \\ &= -\frac{v_0}{2}Y^D_{ij}\left(\overline{d}_{iR}d_{jL}+\overline{\ell}_{iL}\ell_{jR}\right)-\frac{v_0}{\sqrt{2}}Y^U_{ij}\overline{u}_{iR}u_{jL}+H.C.\end{aligned}$$

where $Y^U = Y^{U\dagger}$

Mass Eigenstates

The Mass Terms can be written in the form

$$\mathcal{L}_{mass} = \overline{d}'_L \mathcal{M}'_d d'_R + \overline{\ell}'_L \mathcal{M}'_\ell \ell'_R + \overline{u}'_L \mathcal{M}'_u u'_R + H.C.$$

where $\mathcal{M}'_\ell = \mathcal{M}'^{\dagger}_d$ & $\mathcal{M}'_u = \mathcal{M}'^{\dagger}_u$.

In terms of the diagonalization matrices this leads to

$$V^{\ell}_{L,R} = V^{d}_{R,L} \quad V^{u}_{R} = V^{u}_{L} K,$$

where K a diagonal matrix containing phases.

The Mass Eigenstates are defines as:

$$d_L = V^{\dagger}_{dL} d'_L \quad \ell_L = V^{\dagger}_{dR} \ell'_L \quad u_L = V^{\dagger}_{uL} u'_L$$
$$d^c_L = V^{\dagger}_{dR} d^{c\prime}_L \quad \ell^c_L = V^{\dagger}_{dL} \ell^{c\prime}_L \quad u^c_L = K^* V^{\dagger}_{uL} u^{c\prime}_L$$

Mass Eigenstates

The Mass Terms can be written in the form

$$\mathcal{L}_{mass} = \overline{d}'_L \mathcal{M}'_d d'_R + \overline{\ell}'_L \mathcal{M}'_\ell \ell'_R + \overline{u}'_L \mathcal{M}'_u u'_R + H.C.$$

where $\mathcal{M}'_\ell = \mathcal{M}'^{\dagger}_d$ & $\mathcal{M}'_u = \mathcal{M}'^{\dagger}_u$.
In terms of the diagonalization matrices this leads to

$$V^\ell_{L,R} = V^d_{R,L} \quad V^u_R = V^u_L K,$$

where K a diagonal matrix containing phases.
The Mass Eigenstates are defines as:

$$d_L = V^\dagger_{dL} d'_L \quad \ell_L = V^\dagger_{dR} \ell'_L \quad u_L = V^\dagger_{uL} u'_L$$
$$d^c_L = V^\dagger_{dR} d^{c\prime}_L \quad \ell^c_L = V^\dagger_{dL} \ell^{c\prime}_L \quad u^c_L = K^* V^\dagger_{uL} u^{c\prime}_L$$

Mass Eigenstates

The Mass Terms can be written in the form

$$\mathcal{L}_{mass} = \overline{d}'_L \mathcal{M}'_d d'_R + \overline{\ell}'_L \mathcal{M}'_\ell \ell'_R + \overline{u}'_L \mathcal{M}'_u u'_R + H.C.$$

where $\mathcal{M}'_\ell = \mathcal{M}'^{\dagger}_d$ & $\mathcal{M}'_u = \mathcal{M}'^{\dagger}_u$.
In terms of the diagonalization matrices this leads to

$$V^\ell_{L,R} = V^d_{R,L} \quad V^u_R = V^u_L K,$$

where K a diagonal matrix containing phases.
The Mass Eigenstates are defines as:

$$\begin{aligned} d_L = V^\dagger_{dL} d'_L \quad \ell_L = V^\dagger_{dR} \ell'_L \quad u_L = V^\dagger_{uL} u'_L \\ d^c_L = V^\dagger_{dR} d^{c\prime}_L \quad \ell^c_L = V^\dagger_{dL} \ell^{c\prime}_L \quad u^c_L = K^* V^\dagger_{uL} u^{c\prime}_L \end{aligned}$$

Mixing

The $\overline{5}$ is expressed as

$$\begin{pmatrix} d_L^c \\ \ell_L \\ -\nu_L \end{pmatrix}$$

and 10 is expressed as

$$\begin{pmatrix} V_{CKM}^\dagger K u_L^c \\ V_{CKM}^\dagger u_L \\ d_L \\ \ell_L^c \end{pmatrix}$$

$X\&Y$ Interactions 1

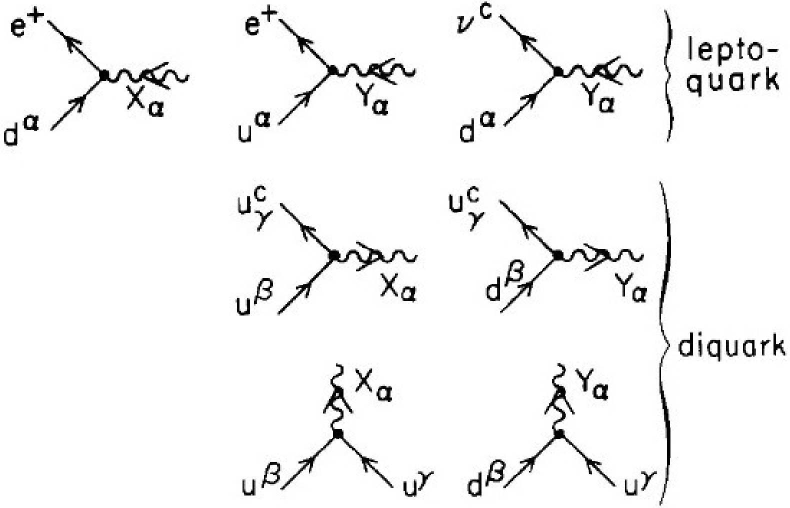
Interactions of $X\&Y$ break the accidental symmetries $B\&L$, but leave $B-L$ unbroken.

$$\mathcal{L}_I^X = \frac{g_5}{\sqrt{2}}\left(\overline{d}_L^i \not{X}^i e_L^c - \overline{e}_L^i \not{X}^i d_L^c + \epsilon_{ijk}\overline{u}_L^{ic} K \not{X}^j u_L^k + H.C.\right) \quad (2)$$

$$\mathcal{L}_I^Y = \frac{g_5}{\sqrt{2}}\left(\overline{\nu}_L \not{Y}^i d_L^{ic} - \overline{u}_L^i V_{CKM}^\dagger \not{Y}^i e_L^c + \epsilon_{ijk}\overline{u}_L^{ic} K V_{CKM}^\dagger \not{Y}^j d_L^k + H.C.\right) \quad (3)$$

$X\&Y$ Bosons are called Leptoquarks.

$X \& Y$ Interactions 2



Proton Decay

Proton Decay is the low energy remnant of many GUTs. In terms of SMs fields Proton Decay is a non-renormalizable dim 6 operator

$$\mathcal{L}_{eff} = \frac{G_X}{\sqrt{2}} \overline{Q}\gamma^\mu Q \overline{e}\gamma_\mu Q$$

We estimate

$$\tau_p \simeq \frac{M_X^4}{m_p^2} = \mathcal{O}(10^{30}) \; years$$

Super-Kamiokande has set the following limits based on $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ decay modes

$$8.2 \times 10^{33} \;\&\; 6.6 \times 10^{33} \quad years \quad @\, 90\% \; c.l. \tag{4}$$

Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.

Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.

Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.

Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.

Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.

1. The Standard Model: Alchemy and Astrology: J. Lykken, hep-ph/0609274.
2. Unity of All Elementary-Particle Forces: H. Georgi S.L. Glashow, PRL 32, (1974), pp 438-441.
3. Hierarchy of interaction in Unified Theories: H. Georgi, H. Quinn, S. Weinberg, PRL 33, (1974), pp 451-454.
4. Aspects of the unification of Strong, Weak and Electromagnetic Interactions: A. Buras, J. Ellis, M.K. Gaillard, D.V. Nanopoulos Nucl. Phys B 135, (1978), 66-92.
5. Symmetry Breaking Patterns in SU(5): M. Magg, Q. Shaffi, Z Phys C 4, (1980), 63-66.
6. Grand Unified Theories and Proton Decay: P. Langacker, Phys. Rep. 72, (1981), 185-385.
7. Grand Unified Theories: G. Ross (1985).
8. Search for Proton Decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in a Large Water Cherenkov Detector: The Super-Kamiokande Collaboration, hep-ex/0903.0676v2