Minimal SU(5) Grand Unification

D. Katsinis

University of Athens

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Ingredients of SM

Principles:
- Locality
- Quantum Mechanics
- Poincare Invariance

Structure:
- Gauge Theory
- SSB

QFT

Renormalizability

Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!
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\[ \text{QFT} \]

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Must include every gauge invariant operators up to dim 4!
RGEs govern parameters evolution!
EW Theory and QCD constitute SM.

- **Gauge Group**: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- **Particles**: belong to representations of the Gauge Group.
- **Force Carriers**: belong to the adjoint rep of the GG.
- **Flavor Sector**: CKM Mixing (CP Violations), GIM Mechanism (No FCNC), Yukawa Couplings break $[U(3)]^5$.
- **Higgs Sector**: Breaks GG to $SU(3)_C \times U(1)_{em}$
  Custodial SU(2) Symmetry $\rightarrow \rho = 1$.
- B & L are accidental symmetries, but B+L is broken by Sphaleron.
- $\theta_{QCD}$
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Open Questions

- Why 5 different reps for the particles?
- Why so many (19/26/28) parameters?
- Gauge Hierarchy Problem: EW vs Planck Scale?
- Neutrino Mass?
- Dark Matter?
- Gravity?
- Is SM an Effective Field Theory?
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GUT Ingredients

- The choice of a simple GG allows only 1 coupling constant!
- Particles belong to less representations!
- Mass relations!
- Proton decay!
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This assignment cancels all triangle anomalies! Unification requires that the whole multiplet must transform according to the same Poincare rep.
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Charge Conjugation

Charge conjugations transforms righthanded particles to lefthanded antiparticles $\psi^c_{L,R} = C \overline{\psi}_{R,L}^T$.

$$\chi_R \rightarrow \left(1 + \frac{i}{2} \epsilon \cdot \sigma - \frac{1}{2} n \cdot \sigma \right) \chi_R$$

$$\chi^c_L = i \sigma^2 \chi^*_R \rightarrow \left(1 + \frac{i}{2} \epsilon \cdot \sigma + \frac{1}{2} n \cdot \sigma \right) i \sigma^2 \chi^*_R$$

$$\overline{\psi}^c_L \gamma^\mu \chi^c_L = -\psi^T_R C^{-1} \gamma^\mu C \overline{\chi}^T_R = \psi^T_R (\gamma^\mu)^T \overline{\chi}^T_R = -\overline{\chi} R \gamma^\mu \psi_R$$

$$\overline{\psi}^c_L \partial^\mu \psi^c_L = \overline{\psi}^c_L \gamma^\mu \left( \partial^\mu \psi^c_L \right) = - \left( \partial^\mu \overline{\psi}_R \right) \gamma^\mu \psi_R = \overline{\psi}_R \partial^\mu \psi_R$$

$$\overline{\chi}_R \chi_L = \left(C^T \left(C \overline{\chi}^T_R \right) \right)^T \chi_L = \chi^c_L C \chi_L$$

(1)
SU(5) is the only simple rank 4 group that can be broken to SM!

\[
\begin{pmatrix}
G_{\mu}^{11} - \frac{2}{\sqrt{30}} B_{\mu} & G_{\mu}^{12} & G_{\mu}^{13} & X_{\mu}^{1} & Y_{\mu}^{1} \\
G_{\mu}^{21} & G_{\mu}^{22} - \frac{2}{\sqrt{30}} B_{\mu} & G_{\mu}^{23} & X_{\mu}^{2} & Y_{\mu}^{2} \\
G_{\mu}^{31} & G_{\mu}^{32} & G_{\mu}^{33} - \frac{2}{\sqrt{30}} B_{\mu} & X_{\mu}^{3} & Y_{\mu}^{3} \\
X_{\mu}^{1\dagger} & X_{\mu}^{2\dagger} & X_{\mu}^{3\dagger} & \frac{1}{\sqrt{2}} \left( A_{\mu}^{3\dagger} + \sqrt{\frac{3}{5}} B_{\mu} \right) & W_{\mu}^{\dagger} \\
Y_{\mu}^{1\dagger} & Y_{\mu}^{2\dagger} & Y_{\mu}^{3\dagger} & \frac{1}{\sqrt{2}} \left( -A_{\mu}^{3\dagger} + \sqrt{\frac{3}{5}} B_{\mu} \right)
\end{pmatrix}
\]

\[24 = (8, 1)_0 \oplus (3, 2)_{-5/3} \oplus (\bar{3}, 2)_{+5/3} \oplus (1, 3)_0 \oplus (1, 1)_0\]

The non-diagonal parts give raise to new interactions curried by the leptoquarks \(X^{+4/3}, Y^{-1/3}!!!\)
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X^{1\dagger}_{\mu} & X^{2\dagger}_{\mu} & X^{3\dagger}_{\mu} & \frac{1}{\sqrt{2}} (A^3_{\mu} + \sqrt{3/5} B_{\mu}) & W^\dagger_{\mu} \\
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The non-diagonal parts give raise to new interactions curried by the leptoquarks \( X^{+4/3}, Y^{-1/3} \)!!!
Particle Content

Simplest rep:

\[
\bar{5} = (\bar{3}, 1)_{\frac{2}{3}} \oplus (1, 2)_{-1} \quad 10 = (\bar{3}, 1)_{-\frac{4}{3}} \oplus (3, 2)_{\frac{1}{3}} \oplus (1, 1)_{2}
\]

Particles:

\[
Q_L (3, 2)_{\frac{1}{3}} \quad L_L (1, 2)_{-1} \quad u^c_L (\bar{3}, 1)_{-\frac{4}{3}} \quad d^c_L (\bar{3}, 1)_{\frac{2}{3}} \quad \ell^c_L (1, 1)_{2}
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\[
\psi_5 = \begin{pmatrix}
d^1_L \\ d^2_L \\ d^3_L \\ \ell_L \\ -\nu_L
\end{pmatrix} \quad \psi_{10} = \begin{pmatrix}
0 & u^3_L & -u^2_L & -u^1_L & -d^1_L \\
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### Charge Quantization

\( \hat{Q} \text{ is traceless} \implies \hat{Q} \psi_5 = 0 \implies 3 \hat{Q} d^c_L + \hat{Q} \ell_L = 0 \)

Charge Quantization: \( \hat{Q} d = \frac{1}{N_c} \hat{Q} \ell \)

\[
\hat{Q} = \text{diag} \left( -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0 \right)
\]

\[
\hat{Q} = \hat{l}_3 + \frac{\hat{Y}}{2} \implies \hat{Y} = \frac{1}{3} \text{diag}(-2, -2, -2, 3, 3)
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\]
The Fields Transform as

\[ \psi_5 \rightarrow \psi'_5 = (I - \nu \theta^a t^a) \psi_5 \quad \psi_\bar{5} \rightarrow \psi'_\bar{5} = (I + \nu \theta^a t^a T) \psi_\bar{5} \]

\[ \psi^{ij} \rightarrow \psi^{ij'} = \left[ \delta^i_k \delta^j_\ell - \nu \theta^a \left( \delta^j_\ell t^{ai}_k + \delta^i_k t^{aj}_\ell \right) \right] \psi^{k\ell}_10 \]

As a result the Gauge Interactions are

\[ \mathcal{L}_5 = \bar{\psi}_5 \phi \psi_5 = \bar{\psi}_5 \phi \psi_5 - \frac{g_5}{\sqrt{2}} \bar{\psi}_5 \psi^T \psi_5, \]

\[ \mathcal{L}_{10} = \frac{\nu}{2} \text{Tr} \left[ \bar{\psi}_{10} \phi \psi_{10} \right] + \frac{g_5}{\sqrt{2}} \text{Tr} \left[ \bar{\psi}_{10} \psi \psi_{10} \right] \]
Gauge Invariance

The Fields Transform as

\[ \psi_5 \rightarrow \psi'_5 = (\mathcal{I} - \imath \theta^a t^a) \psi_5 \quad \psi_\bar{5} \rightarrow \psi'_\bar{5} = (\mathcal{I} + \imath \theta^a t^{aT}) \psi_\bar{5} \]

\[ \psi^{ij}_{10} \rightarrow \psi'^{ij}_{10} = \left[ \delta^i_k \delta^j_\ell - \imath \theta^a \left( \delta^j_\ell t^a_k + \delta^i_k t^a_\ell \right) \right] \psi^{k\ell}_{10} \]

As a result the Gauge Interactions are

\[ \mathcal{L}_5 = \bar{\psi}_5 \mathcal{D}_\psi \psi_5 = \bar{\psi}_5 \mathcal{D}_\psi \psi_\bar{5} - \frac{g_5}{\sqrt{2}} \bar{\psi}_5 \psi^T \psi_5, \]

\[ \mathcal{L}_{10} = \frac{\imath}{2} \text{Tr} \left[ \bar{\psi}_{10} \mathcal{D}_\psi \psi_{10} \right] + \frac{g_5}{\sqrt{2}} \text{Tr} \left[ \bar{\psi}_{10} \psi \psi_{10} \right] \]
We separate the gauge bosons as

\[ \mathcal{V}_\mu = G_\mu + A_\mu + B_\mu + \chi_\mu + \mathcal{V}_\mu \]

\[ \mathcal{L}_I = \mathcal{L}_I^{QCD} + \mathcal{L}_I^{SU(2)} + \mathcal{L}_I^{U(1)} + \mathcal{L}_I^X + \mathcal{L}_I^Y \]

\[ \mathcal{L}_I^{U(1)} = \sqrt{\frac{3}{5}} \frac{g_5}{2} \left( \frac{1}{3} (\bar{u}_L \mathcal{B}_L u_L + \bar{d}_L \mathcal{B}_d_L) + \frac{4}{3} \bar{u}_R \mathcal{B}_u_R - \frac{2}{3} \bar{d}_R \mathcal{B}_d_R - (\bar{\ell}_L \mathcal{B}_L \ell_L + \bar{\nu}_L \mathcal{B}_\nu_L) - 2 \ell_R \mathcal{B}_R \ell_R \right) \]

\[ \mathcal{L}_I^{SU(2)} = \frac{g_5}{2} \left( \bar{u}_L \mathcal{A}_3 u_L - \bar{d}_L \mathcal{A}_3 d_L + \sqrt{2} \bar{d}_L \mathcal{W}^\dag u_L + \sqrt{2} \bar{d}_L \mathcal{W} d_L + \bar{\nu}_L \mathcal{A}_3 \nu_L - \bar{e}_L \mathcal{A}_3 e_L + \sqrt{2} \bar{e}_L \mathcal{W}^\dag \nu_L + \sqrt{2} \bar{e}_L \mathcal{W} e_L \right) \]

\[ \mathcal{L}_I^{SU(3)} = \frac{g_5}{\sqrt{2}} \left( \bar{d}^1 \mathcal{G}^{11} d^1 + \bar{d}^1 \mathcal{G}^{12} d^2 + \bar{d}^1 \mathcal{G}^{13} d^3 + \bar{u}^2 \mathcal{G}^{21} u^1 + \bar{u}^2 \mathcal{G}^{22} u^2 + \bar{u}^2 \mathcal{G}^{23} u^3 + \bar{u}^3 \mathcal{G}^{31} u^1 + \bar{u}^3 \mathcal{G}^{32} u^2 + \bar{u}^3 \mathcal{G}^{33} u^3 \right) \]

We predict

\[ g_5 = \sqrt{\frac{5}{3}} g' = g = g_s \quad \tan^2 \theta_W = 0.6 \]
We separate the gauge bosons as
\[ \mathcal{V}_\mu = G_\mu + A_\mu + B_\mu + \chi_\mu + Y_\mu \]
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\[ \mathcal{L}_I^{SU(1)} = \sqrt{\frac{3}{5}} \frac{g_5}{2} \left( \frac{1}{3} (\bar{u}_L B u_L + \bar{d}_L B d_L) + \frac{4}{3} \bar{u}_R B u_R - \frac{2}{3} \bar{d}_R B d_R - (\bar{\ell}_L B \ell_L + \bar{\nu}_L B \nu_L) - 2 \ell_R B \bar{\ell}_R \right) \]
\[ \mathcal{L}_I^{SU(2)} = \frac{g_5}{2} \left( \bar{u}_L A^3 u_L - \bar{d}_L A^3 d_L + \sqrt{2} \bar{d}_L W^\dagger u_L + \sqrt{2} \bar{u}_L W d_L + \bar{\nu}_L A^3 \nu_L - \bar{\ell}_L A^3 e_L + \sqrt{2} \bar{\nu}_L W^\dagger \nu_L + \sqrt{2} \bar{\ell}_L W e_L \right) \]
\[ \mathcal{L}_I^{SU(3)} = \frac{g_5}{\sqrt{2}} \left( \bar{d}^1 \bar{d}^{11} d^1 + \bar{d}^1 \bar{d}^{12} d^2 + \bar{d}^1 \bar{d}^{13} d^3 + \bar{d}^2 \bar{d}^{21} d^1 + \bar{d}^2 \bar{d}^{22} d^2 + \bar{d}^2 \bar{d}^{23} d^3 + \bar{d}^3 \bar{d}^{31} d^1 + \bar{d}^3 \bar{d}^{32} d^2 + \bar{d}^3 \bar{d}^{33} d^3 + \bar{u}^1 \bar{u}^{11} u^1 + \bar{u}^1 \bar{u}^{12} u^2 + \bar{u}^1 \bar{u}^{13} u^3 + \bar{u}^2 \bar{u}^{21} u^1 + \bar{u}^2 \bar{u}^{22} u^2 + \bar{u}^2 \bar{u}^{23} u^3 + \bar{u}^3 \bar{u}^{31} u^1 + \bar{u}^3 \bar{u}^{32} u^2 + \bar{u}^3 \bar{u}^{33} u^3 \right) \]
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The Running Couplings

Problem:

\[ \tan^2 \theta_W = 0.30073 \pm 0.00025 \ @ \ M_Z \]

Not really... The prediction refers to GUT scale!
We ignore threshold corrections and assume desert! Then 1-loop RGEs for SU(N):

\[
\frac{1}{g_i^2(\mu)} = \frac{1}{g_i^2(Q)} + b_i \log \left( \frac{Q^2}{\mu^2} \right) \quad b_N = \frac{1}{(4\pi)^2} \left[ -\frac{11}{3} N + \frac{4}{3} n_g \right]
\]

\[
b_1 = \frac{1}{4\pi^2} \quad b_2 = -\frac{5}{24\pi^2} \quad b_3 = -\frac{7}{16\pi^2}
\]

\[
g_1 = g' = 0.357 \quad g_2 = g = 0.652 \quad g_3 = g_s = 1.221 @ M_Z
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Coupling Unification

- Why Unification?
- Building SU(5)
- Higgs Sector
- Fermion Mass
- B Violation
- Summary
- Refs

The graph illustrates the coupling unification of SU(3), SU(2), and U(1) groups as a function of energy (GeV). The coupling constants are represented on the vertical axis, with SU(3) increasing, SU(2) decreasing, and U(1) remaining constant. The horizontal axis represents energy in GeV, ranging from 100 to 10^20. The graph shows the unification of these couplings at around 10^16 GeV.
Why Unification?

Building SU(5)

Higgs Sector

Fermion Mass

B Violation

Summary

Refs

\[ \tan^2 \theta_W \]
Let's sum up!

- Particles
- Coupling Constants
- Gauge Bosons
- We have to break SU(5)....
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- Particles
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Our Goals

- SU(5) Breaking
- $SU(2) \times U(1)$ Breaking
- Give correct mass to known particles
- Get rid of unobserved particles
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The 24-Higgs

We introduce 24 Higgs fields in the adjoint rep of SU(5). We define a $5 \times 5$ traceless matrix $\Sigma$ transforming according to $5 \times \overline{5} = 24 \oplus 1$.

$$\Sigma = 2\phi^a t^a$$

$$\mathcal{L}_{\text{kin}}^\Sigma = \frac{1}{4} \text{Tr} \left[ (\mathcal{D}_\mu \Sigma)^\dagger \mathcal{D}^\mu \Sigma \right] \quad \mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i \frac{g_5}{\sqrt{2}} [\mathcal{V}_\mu, \Sigma]$$
The VEV must have the form

$$\langle \Sigma \rangle = v \ diag \left( 1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right)$$

Gauge Bosons Mass Matrix

$$m_{ab}^2 \mathcal{V}_\mu^a \mathcal{V}^{\mu b} = \frac{g_5^2}{8} Tr \left[ [\mathcal{V}_\mu, \langle \Sigma \rangle] [\mathcal{V}^\mu, \langle \Sigma \rangle] \right]$$

As a result we get

$$m_X^2 = m_Y^2 = \frac{25}{8} g_5^2 v^2$$

This mass has to be at the GUT scale!
SU(5) Breaking

The VEV must have the form

$$\langle \Sigma \rangle = v \ diag \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right)$$

Gauge Bosons Mass Matrix

$$m_{ab}^2 V^a_\mu V^{\mu b} = \frac{g_5^2}{8} \text{Tr} \left[[V_\mu, \langle \Sigma \rangle] [V^{\mu}, \langle \Sigma \rangle]\right]$$

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The 24-potential

The potential is

\[ V(\Sigma) = \frac{\mu^2}{2} \text{Tr} [\Sigma^2] + \frac{a}{4} \left[ \text{Tr} [\Sigma^2] \right]^2 + \frac{b}{2} \text{Tr} [\Sigma^4] \]

\[ a > 0, \quad 15a + 7b > 0, \quad \mu^2 < 0 \]

At minimum

\[ V(\langle \Sigma \rangle) = \frac{15}{2} \left( \frac{\mu^2}{2} v^2 + \frac{15}{8} av^4 + \frac{7}{8} bv^4 \right) \]

This leads to

\[ \frac{\partial}{\partial v} V(\langle \Sigma \rangle) = 0 \quad \Rightarrow \quad \mu^2 + \frac{15}{2} av^2 + \frac{7}{2} bv^2 = 0 \]
The 24-potential

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\[ V(\Sigma) = \frac{\mu^2}{2} \text{Tr} [\Sigma^2] + \frac{a}{4} \left( \text{Tr} [\Sigma^2] \right)^2 + \frac{b}{2} \text{Tr} [\Sigma^4] \]

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We write:

\[
\Sigma = \langle \Sigma \rangle + \left( \begin{array}{cccc}
H_{11}^8 & H_{12}^8 & H_{13}^8 & \tilde{H}_1^X \\
H_{21}^8 & H_{22}^8 + \sqrt{\frac{2}{15}} H_0 & H_{23}^8 & \tilde{H}_2^X \\
H_{31}^8 & H_{32}^8 & H_{33}^8 + \sqrt{\frac{2}{15}} H_0 & \tilde{H}_3^X \\
\tilde{H}_1^X \dagger & \tilde{H}_2^X \dagger & \tilde{H}_3^X \dagger & \frac{1}{\sqrt{2}} H_Z - \sqrt{\frac{3}{10}} H_0 \\
\tilde{H}_1^Y \dagger & \tilde{H}_2^Y \dagger & \tilde{H}_3^Y \dagger & - \frac{1}{\sqrt{2}} H_Z - \sqrt{\frac{3}{10}} H_0
\end{array} \right)
\]

The mass spectrum is:

\[
m_8^2 = \frac{5}{2} b v^2 \quad m_Z^2 = m_\pm^2 = 10 b v^2 \quad m_0^2 = -2 \mu^2
\]

\(\tilde{H}_X^X \& \tilde{H}_Y^Y\) are would be Goldstone Bosons absorbed by \(X\&Y\) Gauge Bosons.
24-Higgs Masses

We write:

\[ \Sigma = \langle \Sigma \rangle + \begin{pmatrix}
H_{11}^8 + \sqrt{\frac{2}{15}} H_0 & H_{12}^8 & H_{13}^8 & \tilde{H}_1^X & \tilde{H}_1^Y \\
H_{21}^8 & H_{22}^8 + \sqrt{\frac{2}{15}} H_0 & H_{23}^8 & \tilde{H}_2^X & \tilde{H}_2^Y \\
H_{31}^8 & H_{32}^8 & H_{33}^8 + \sqrt{\frac{2}{15}} H_0 & \tilde{H}_3^X & \tilde{H}_3^Y \\
\tilde{H}_1^X & \tilde{H}_2^X & \tilde{H}_3^X & \frac{1}{\sqrt{2}} H_Z - \sqrt{\frac{3}{10}} H_0 & H^+ \\
\tilde{H}_1^Y & \tilde{H}_2^Y & \tilde{H}_3^Y & \frac{1}{\sqrt{2}} H_Z - \sqrt{\frac{3}{10}} H_0 & H^- 
\end{pmatrix} \]

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The 5-Higgs

The Higgs 5plet is \( H = (h^1 \ h^2 \ h^3 \ h^+ \ h^0)^T \)

VEV must have the form \( \langle H \rangle = \frac{1}{\sqrt{2}} (0 \ 0 \ 0 \ 0 \ v_0)^T \)

The potential is

\[
V(H) = \mu_0^2 |H|^2 + \lambda |H|^4 \quad \mu_0^2 < 0, \quad \lambda > 0
\]

Minimalization yields

\[
V(\langle H \rangle) = \frac{1}{2} \mu_0^2 v_0^2 + \frac{1}{4} \lambda v_0^4, \quad \frac{\partial}{\partial v} V(\langle H \rangle) = 0 \quad \Rightarrow \quad \mu_0^2 + \lambda v_0^2 = 0
\]

As in SM \( W \& Z \) Bosons acquire mass

\[
m_W = m_Z \cos \theta_W = \frac{g v_0}{2}
\]

Obviously this mass has to be at the EW scale!
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We write:

\[
H = \begin{pmatrix}
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\tilde{H}^3 \\
H_4 \\
\frac{1}{\sqrt{2}} (v_0 + h) e^{i \frac{\zeta}{v_0}}
\end{pmatrix}
\]

Then \( m_h^2 = -\mu_0^2 \).

\( H_4 & \zeta \) are would be Goldstone Bosons absorbed by \( W & Z \) Gauge Bosons.
\( \tilde{H} \) is physical and remains massless.
The 5-Higgs Masses

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\( \tilde{H} \) is physical and remains massless.
It couldn’t be so easy...

We have broken SU(5) according to the pattern

\[
SU(5) \xrightarrow{GUT \text{ Scale}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{EW \text{ Scale}} SU(3) \times U(1)_{em}
\]

But...

- The 5’s color triplet is massless at tree level. Too rapid Proton Decay!!!
- Gauge Bosons couple $\Sigma$ to $H$ at 1-loop. Renormalizability requires coupling at tree level!

\[
V(\Sigma, H) = \alpha|H|^2 \text{Tr} \left[ \Sigma^2 \right] + \beta \bar{H} \Sigma^2 H
\]
It couldn’t be so easy...

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- Gauge Bosons couple $\Sigma$ to $H$ at 1-loop. Renormalizability requires coupling at tree level!

$$ V(\Sigma, H) = \alpha |H|^2 \text{Tr} \left[ \Sigma^2 \right] + \beta \bar{H} \Sigma^2 H $$
It couldn’t be so easy...

We have broken SU(5) according to the pattern

\[ SU(5) \xrightarrow{\text{GUT Scale}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{EW Scale}} SU(3) \times U(1)_{em} \]

But...

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The Total Potential

The total potential is:

\[ V (\Sigma, H) = \frac{\mu^2}{2} \text{Tr} \left[ \Sigma^2 \right] + \frac{a}{4} \left[ \text{Tr} \left[ \Sigma^2 \right] \right]^2 + \frac{b}{2} \text{Tr} \left[ \Sigma^4 \right] + \alpha |H|^2 \text{Tr} \left[ \Sigma^2 \right] \\
+ \beta \overline{H} \Sigma^2 H + \mu_0^2 |H|^2 + \lambda |H|^4 \]

The modified vacuum is:

\[ \langle \Sigma \rangle = \nu \text{diag} \left( 1, 1, 1, -\frac{3}{2} - \frac{1}{2} \epsilon, -\frac{3}{2} + \frac{1}{2} \epsilon \right) \quad \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \nu_0 \end{pmatrix} \]

where \( \epsilon = \frac{3}{20} \frac{\beta v_0^2}{b v^2} \) to order \( O(v_0^2/v^2) \).
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The modified Vacuum

The $\epsilon$ terms affect EW breaking but their result is negligible.

The vacuum condition yields

\[
\mu^2 + \frac{15}{2} a v^2 + \frac{7}{2} b v^2 + \alpha v_0^2 + \frac{3}{10} \beta v_0^2 = 0
\]

\[
\mu_0^2 + \lambda v_0^2 + \frac{15}{2} \alpha v^2 + \left( \frac{9}{4} - \frac{3}{2} \epsilon \right) \beta v^2 = 0
\]

The cross-term saves Renormalizability, but the interaction between $\Sigma$ and $H$ requires extreme fine tuning in order to keep the SM’s Higgs at the EW scale. Hierarchy Problem appears again!
The $\epsilon$ terms affect EW breaking but their result is negligible. The vacuum condition yields

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The Modified Mass Spectrum is quite complicated.

- The already heavy Higgs $\tilde{H}^8$ & $H^\pm$ receive correction $\mathcal{O}(\nu_0^2)$.
- A combination of $h, H_Z$ & $H_0$ remains at EW Scale while the other two are at the GUT scale.
- A combination of $\tilde{H}$ & $\tilde{H}_Y$ is absorbed by $Y$ while the others becomes massive $\mathcal{O}(\nu^2)$. This suppresses Higgs mediated Proton Decay.
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Mass Terms

The particles transform according to $\overline{5}$&10. The product of these reps are

$$
\overline{5} \times \overline{5} = 10 \oplus 15, \quad \overline{5} \times 10 = 5 \oplus 45, \quad 10 \times 10 = \overline{5} \oplus 45 \oplus 50
$$

Only $5$&45 contain neutral components, thus couple to matter at tree level. $10$&50 could contribute only to mass renormalization.

In the Minimal SU(5) we use only 5-Higgs to generate mass for the fermions. The couplings are:

$$
\mathcal{L}_{\text{mass}} = Y_D^{ij} \psi_{5i\alpha}^T C \psi_{10j\alpha\beta} H_\beta + Y_U^{ij} \frac{1}{4} \epsilon_{\alpha\beta\gamma\delta} \psi_{10i\alpha\beta}^T C \psi_{10j\gamma\delta} H_\epsilon + H.C.
$$

$$
= -\frac{v_0}{2} Y_D^{ij} \left( \overline{d}_{iR} d_{jL} + \overline{\ell}_{iL} \ell_{jR} \right) - \frac{v_0}{\sqrt{2}} Y_U^{ij} \overline{u}_{iR} u_{jL} + H.C.
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where $Y^U = Y^{U\dagger}$.
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Mass Eigenstates

The Mass Terms can be written in the form

\[ \mathcal{L}_{\text{mass}} = \overline{d}'_L \mathcal{M}'_d d'_R + \overline{\ell}'_L \mathcal{M}'_\ell \ell'_R + \overline{u}'_L \mathcal{M}'_u u'_R + H.C. \]

where \( \mathcal{M}'_\ell = \mathcal{M}'_d \) & \( \mathcal{M}'_u = \mathcal{M}'_u \).

In terms of the diagonalization matrices this leads to

\[ V^\ell_{L,R} = V^d_{R,L} \quad V^u_R = V^u_L K, \]

where \( K \) a diagonal matrix containing phases.

The Mass Eigenstates are defined as:

\[ d_L = V^\dagger_{dL} d'_L \quad \ell_L = V^\dagger_{dR} \ell'_L \quad u_L = V^\dagger_{uL} u'_L \]

\[ d^c_L = V^\dagger_{dR} d^c'_L \quad \ell^c_L = V^\dagger_{dL} \ell^c'_L \quad u^c_L = K^* V^\dagger_{uL} u^c'_L \]
Mass Eigenstates

The Mass Terms can be written in the form

$$\mathcal{L}_{\text{mass}} = \overline{d}_L^\prime M'_d d_R^\prime + \overline{\ell}_L^\prime M'_\ell \ell_R^\prime + \overline{u}_L^\prime M'_u u_R^\prime + H.C.$$  

where $M'_\ell = M'^{\dagger}_d$ & $M'_u = M'^{\dagger}_u$.

In terms of the diagonalization matrices this leads to

$$V_{L,R}^\ell = V_{R,L}^d \quad V_{R}^u = V_{L}^u K,$$

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The Mass Eigenstates are defined as:

$$d_L = V_{dL}^\dagger d_L^\prime \quad \ell_L = V_{dR}^\dagger \ell_L^\prime \quad u_L = V_{uL}^\dagger u_L^\prime$$

$$d_L^c = V_{dR}^\dagger d_L^c \quad \ell_L^c = V_{dL}^\dagger \ell_L^c \quad u_L^c = K^* V_{uL}^\dagger u_L^c$$
Mass Eigenstates

The Mass Terms can be written in the form

\[ \mathcal{L}_{mass} = \overline{d'}_L M'_{d} d'_R + \overline{\ell'}_L M'_{\ell} \ell'_R + \overline{u'}_L M'_{u} u'_R + H.C. \]

where \( M'_{\ell} = M'_{d} \) & \( M'_{u} = M'_{u} \). In terms of the diagonalization matrices this leads to

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\[ d'_L = V^\dagger_{dR} d'_L \quad \ell'_L = V^\dagger_{dL} \ell'_L \quad u'_L = K^* V^\dagger_{uL} u'_L \]
The $\overline{5}$ is expressed as

\[
\begin{pmatrix}
d_L^c \\
\ell_L \\
-\nu_L
\end{pmatrix}
\]

and 10 is expressed as

\[
\begin{pmatrix}
V_{CKM}^\dagger K u_L^c \\
V_{CKM}^\dagger u_L \\
V_{CKM}^\dagger d_L \\
d_L \\
\ell_L^c \\
\ell_L
\end{pmatrix}
\]
Interactions of $X$ & $Y$ break the accidental symmetries $B$ & $L$, but leave $B - L$ unbroken.

\[ \mathcal{L}^X_i = \frac{g_5}{\sqrt{2}} \left( \bar{d}^i_L X^i e^c_L - \bar{e}^i_L X^i d^c_L + \epsilon_{ijk} \bar{u}^i_L K X^j u^k_L + H.C. \right) \]  
(2)

\[ \mathcal{L}^Y_i = \frac{g_5}{\sqrt{2}} \left( \bar{\nu}_L \gamma^i d^c_L - \bar{u}^i_L V^\dagger_{CKM} \gamma^i e^c_L + \epsilon_{ijk} \bar{u}^i_L K V^\dagger_{CKM} \gamma^j d^k_L + H.C. \right) \]  
(3)

$X$ & $Y$ Bosons are called Leptoquarks.
**X & Y Interactions 2**

\[ e^+ \rightarrow d^\alpha X_\alpha \]
\[ e^+ \rightarrow u^\alpha Y_\alpha \]
\[ \nu^c \rightarrow d^\alpha Y_\alpha \]
\[ d^\alpha \rightarrow u^\alpha Y_\alpha \]
\[ u^c \rightarrow X_\alpha \]
\[ u^c \rightarrow u^\gamma \]
\[ u^\beta \rightarrow u^\gamma \]
\[ u^\beta \rightarrow Y_\alpha \]

(-) lepto-quark

(-) diquark
Proton Decay

Proton Decay is the low energy remnant of many GUTs. In terms of SMs fields Proton Decay is a non-renormalizable dim 6 operator

\[ \mathcal{L}_{\text{eff}} = \frac{G_X}{\sqrt{2}} Q\gamma_{\mu} Qe\gamma_{\mu} Q \]

We estimate

\[ \tau_p \simeq \frac{M_X^4}{m_p^2} = \mathcal{O}(10^{30}) \text{ years} \]

Super-Kamiokande has set the following limits based on \( p \to e^+\pi^0 \) and \( p \to \mu^+\pi^0 \) decay modes

\[ 8.2 \times 10^{33} \text{ & } 6.6 \times 10^{33} \text{ years} \at 90\% \text{ c.l.} \]
Summary

- Minimal SU(5) is the prototype GUT.
- Probably its ruled out by the data.
- Gauge Hierarchy Problem. We need SuSy.
- Non Minimal models are viable.
- SuSy GUTs arise as low energy effective theories of string theory.
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Search for Proton Decay via $p \rightarrow e^+\pi^0$ and $p \rightarrow \mu^+\pi^0$ in a Large Water Cherenkov Detector: The Super-Kamiokande Collaboration, hep-ex/0903.0676v2