Σύγχρονες Τεχνικές Υπολογισμού Πλατών Σκέδασης στην Θεωρία Πεδίου

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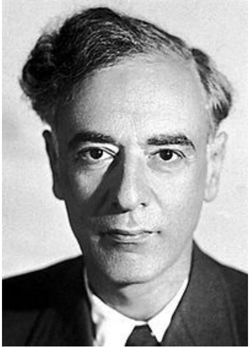
Τομέας Πυρηνικής Φυσικής και Στοιχειωδών Σωματιδίων
Τμήμα Φυσικής
Εθνικό & Καποδιστριακό Πανεπιστημίο Αθηνών

Αθήνα, 2014

A method is more important than a discovery, since the right method can lead to new and even more important discoveries.

Lev Landau



How to Calculate Scattering Amplitudes?

Feynman Rules For:

- Matter Fields
- Gauge Bosons Gauge Redundancy $\rightarrow$ Fadeev Popov Ghosts
- Higgs R_ξ Gauge $\rightarrow$ Goldstone Bosons
- Counterterms

$$gg \rightarrow ng \tag{1}$$

n	2	3	4	5	6	7	8
#	4	25	220	2485	34300	559405	10525900

Textbook Methods are very inefficient!

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Something goes Wrong...

A Small Part of Tree Level 5 Gluon Amplitude

$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$

The search for efficient methods led to a real breakthrough. A small taste is given in the following pages.

Outline:

- Intro to Scattering Amplitudes
- All tree level Amplitudes in $\mathcal{N} = 4$ SYM

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Spinor Helicity Formalism

We work in the chiral rep of Dirac Matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

where $\sigma^\mu = (1, \vec{\sigma})$, $\overline{\sigma}^\mu = (1, -\vec{\sigma})$, with $Tr\left[\sigma^\mu \overline{\sigma}^\nu\right] = n^{\mu\nu}$

The Chiral Dirac spinors are

$$U_R(p) = \begin{pmatrix} 0 \\ u_R(p) \end{pmatrix}, \quad U_L(p) = \begin{pmatrix} u_L(p) \\ 0 \end{pmatrix} \tag{3}$$

Massless Weyl spinors satisfy the equations:

$$p \cdot \sigma u_R = 0, \quad p \cdot \overline{\sigma} u_L = 0 \tag{4}$$

and are related upon Charge Conjugation:

$$u_R = \imath \sigma^2 u_L^* \tag{5}$$

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Spinor Helicity Formalism 1

For Massless Particles Helicity is a good Quantum Number.
We use the shorthands:

$$\overline{U}_L(p) = \langle p, \quad \overline{U}_R(p) = [p, \quad U_L(p) = p], \quad U_R(p) = p\rangle. \qquad (6)$$

We have the following Lorentz invariant products

$$\overline{U}_L(p)U_R(q) = \langle pq\rangle, \quad \overline{U}_R(p)U_L(q) = [pq], \quad \langle pq\rangle^* = [qp] \qquad (7)$$

and completeness relations

$$p\rangle[p = U_R(p)\overline{U}_R(p) = \not{p}\frac{1-\gamma^5}{2}, \quad p]\langle p = U_L(p)\overline{U}_L(p) = \not{p}\frac{1+\gamma^5}{2}. \qquad (8)$$

Spinor products relation to momentum is

$$\langle pq\rangle[pq] = |\langle pq\rangle|^2 = |[pq]|^2 = 2p\cdot q, \qquad (9)$$

Both products are antisymmetric

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Spinor Helicity Formalism 2

Currents take the form

$$\overline{U}_L(p)\gamma^\mu U_L(q) = u_L^\dagger(p)\overline{\sigma}^\mu u_L(q) = \langle p\gamma^\mu q]$$
$$\overline{U}_R(p)\gamma^\mu U_R(q) = u_R^\dagger(p)\sigma^\mu u_R(q) = [p\gamma^\mu q\rangle \tag{11}$$

These expressions can be inverted

$$\langle p\gamma^\mu q] = [q\gamma^\mu p\rangle \quad \langle p\gamma^\mu q]^* = \langle q\gamma^\mu p] \tag{12}$$

Using Fierz identity

$$(\overline{\sigma}^\mu)_{ab}(\overline{\sigma}_\mu)_{cd} = 2\left(\imath\sigma^2\right)_{ac}\left(\imath\sigma^2\right)_{bd} \tag{13}$$

we can simplify current products

$$\langle p\gamma^\mu q]\langle k\gamma_\mu \ell] = 2\langle pk\rangle[\ell q], \quad \langle p\gamma^\mu q][k\gamma_\mu \ell\rangle = 2\langle p\ell\rangle[kq] \tag{14}$$

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Spinor Helicity Formalism - Polarization Vectors

Polarization vectors have the following form:

$$e_R^{*\mu}(k) = \frac{1}{\sqrt{2}} \frac{\langle r\gamma^\mu k]}{\langle rk\rangle}, \quad e_L^{*\mu}(k) = -\frac{1}{\sqrt{2}} \frac{[r\gamma^\mu k\rangle}{[rk]} \tag{15}$$

r is a reference momentum. It is trivial to verify that the satisfy the relations:

$$e_R^{*\mu}(k) = e_L^\mu(k) \quad k_\mu e_{L,R}^\mu(k) = 0 \quad e_{L,R}^{*\mu}(k) e_{L,R\mu}^*(k) = 0 \quad e_{L,R}^{*\mu}(k) e_{L,R\mu}(k) = -1 \tag{16}$$

The choice of reference vector corresponds to gauge fixing

$$e_R^{*\mu}(k;r) - e_R^{*\mu}(k;s) = \sqrt{2} \frac{\langle sr\rangle}{\langle rk\rangle\langle sk\rangle} k^\mu \tag{17}$$

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Massless particles $\to p^2 = 0$ Nonlinear Constraint!

We linearize it! $SO(1,3,\mathbb{R}) \simeq SL(2,\mathbb{C})$

$$p^\mu \quad \to \quad p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} \quad det(p_{a\dot{a}}) = p^2 = 0 \tag{18}$$

P-matrix is expressed as $p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$, where $(\lambda_a, \tilde{\lambda}_{\dot{a}}) \in \mathbb{C}^2 \times \mathbb{C}^2$.

The spinors are scaling redundant

$$(\lambda_a, \tilde{\lambda}_{\dot{a}}) \quad \to \quad (t\lambda_a, t^{-1}\tilde{\lambda}_{\dot{a}})\ t \in \mathbb{C}^* \tag{19}$$

thus they can be taken in $\mathbb{CP}^1 \times \mathbb{CP}^1$.

For real momenta the spinors are connected via

$$\lambda^*_{\dot{a}} = \pm \tilde{\lambda}_{\dot{a}} \tag{20}$$

We have two types of invariant products

$$\langle \lambda \lambda' \rangle = \epsilon_{\alpha\beta} \lambda^\alpha \lambda'^\beta, \qquad [\tilde{\lambda}\tilde{\lambda}'] = \epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}^{\dot{\alpha}} \tilde{\lambda}'^{\dot{\beta}} \tag{21}$$

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Polarization Vectors

As polarization vectors must satisfy $p^\mu \epsilon_\mu = 0$ and gauge redundancy $\epsilon^\mu \;\to\; \epsilon^\mu + cp^\mu$, we guess

$$\epsilon^+_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu \lambda \rangle}, \qquad \epsilon^-_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda} \tilde{\mu}]} \tag{22}$$

Under the transformation $\delta \mu_a = \alpha \mu_a + \beta \lambda_a$ polarization vectors transform as

$$\delta \epsilon^-_{a\dot{a}} = \frac{\beta}{1+\alpha} \frac{\lambda_a \tilde{\lambda}_{\dot{a}}}{\langle \mu \lambda \rangle} \tag{23}$$

Under the scalling $(t\lambda_a, t^{-1}\tilde{\lambda}_{\dot{a}})$ $t \in \mathbb{C}^*$ polarization vectors scale as

$$(\epsilon^-, \epsilon^+) \quad \to \quad (t^2 \epsilon^-, t^{-2} \epsilon^+) \tag{24}$$

In general

$$\left(\lambda^a \frac{\partial}{\partial \lambda^a} - \tilde{\lambda}^{\dot{a}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{a}}} \right) \psi(\lambda, \tilde{\lambda}) = -2h \psi(\lambda, \tilde{\lambda}) \tag{25}$$

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Scattering Amplitudes

Scattering Amplitudes for Yang Mills theoty can be expresssed as

$$A = (2\pi)^4 g^{n-2} \delta^4 \left(\sum_i \lambda_i^a \tilde{\lambda}_i^{\dot{a}} \right) \mathcal{A}(\lambda_i, \tilde{\lambda}_i, h_i) \tag{26}$$

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for every particle i.

Color Ordering

The interacting Lagrangian of a SU(N) gauge theory is

$$\mathcal{L}^I = -\imath g \overline{\psi} \not{G}^a t^a \psi + \frac{g}{2}\left(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu}\right) f^{abc} A^b_\mu A^c_\nu - \frac{g^2}{4} f^{abc} f^{ade} A^{b\mu} A^{c\nu} A^d_\mu A^e_\nu \tag{28}$$

The generators satisfy Lie algebra

$$\left[t^a, t^b\right] = \imath f^{abc} t^c, \text{with}\, Tr\left[t^a t^b\right] = \frac{1}{2}\delta^{ab} \tag{29}$$

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$$-gf^{abc} = \imath\frac{g}{\sqrt{2}} Tr\left[T^aT^bT^c - T^aT^cT^b\right]$$
$$-\imath g^2 f^{abe}f^{cde} = \imath\frac{g^2}{2} Tr\left[\left[T^a, T^b\right]\left[T^c, T^d\right]\right] \tag{31}$$

Thus color order Feynman rules read

$$\imath\frac{g}{\sqrt{2}} Tr\left[T^aT^bT^c\right]\left(g^{\mu\nu}(1-2)^\lambda + g^{\nu\lambda}(2-3)^\mu + g^{\lambda\mu}(3-1)^\nu\right) \tag{32}$$

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Tree Level Amplitude can be expressed as

$$\mathcal{A}^{tree}_{tot}(\{k_i, \epsilon_i, a_i\}) = \sum_{\sigma \in S_n/Z_n} Tr[T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \ldots T^{a_{\sigma(n)}}] \mathcal{A}^{tree}_n(\sigma(1), \ldots, \sigma(n)) \quad (34)$$

While 1−Loop Amplitude can be expressed as

$$\mathcal{A}^{1-loop} = g^2 N \Big\{ \sum_{\sigma \in S_n/Z_n} Tr[T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}] \mathcal{A}^{1-loop}_n(\sigma(1), \ldots, \sigma(n))$$
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Partial Amplitudes allow as to use effective SUSY!

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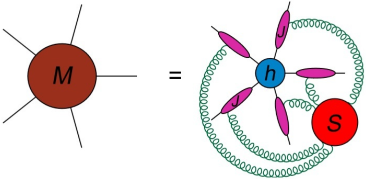
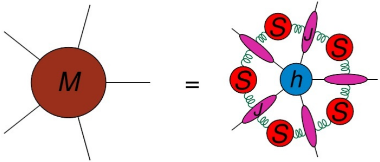
IR-singularities Factorize in planar limit



Factorization of soft and collinear singularities.



Soft-collinear factorization in the planar limit.

Identities for Partial Amplitudes

- Cyclicity: $\mathcal{A}_n(1,2,\dots,n) = \mathcal{A}_n(n,1,\dots,n-1)$ $(n-1)!$
- Color Reversal: $\mathcal{A}_n(1,2,\dots,n-1,n) = (-)^n \mathcal{A}_n(n,n-1,\dots,2,1)$
- $U(1)$ Decoupling: $\sum_{\sigma cyclic} \mathcal{A}(1,\sigma(2),\sigma(3),\dots,\sigma(n)) = 0$
- Kleiss - Kuijf: $\mathcal{A}_n(1,\{\alpha\},n,\{\beta\}) = (-)^{n_\beta} \sum_{\sigma\in OP(\{\alpha\},\{\beta^T\})} \mathcal{A}(1,\sigma,n)$ $(n-2)!$
- BCJ: $0 = I_4 = \mathcal{A}(2,4,3,1)(s_{43}+s_{41}) + \mathcal{A}(2,3,4,1)s_{41}$ $0 = I_5 = \mathcal{A}(2,4,3,5,1)(s_{43}+s_{45}+s_{41}) + \mathcal{A}(2,3,4,5,1)(s_{45}+s_{41}) + \mathcal{A}(2,3,5,4,1)s_{41}$ $(n-3)!$

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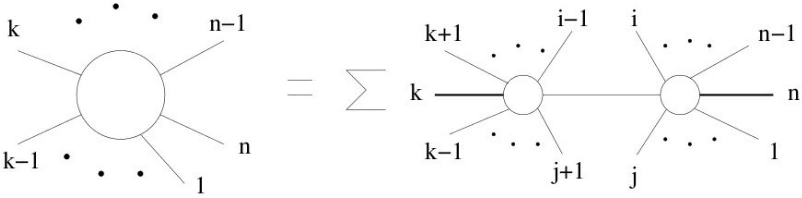
BCFW 1

Scattering Amplitudes factorize as:

$$\mathcal{A}_n = \sum \acute{\mathcal{A}}^{h}_{r+1} \frac{1}{P_r^2} \acute{\mathcal{A}}^{-h}_{n-r+1}, \tag{36}$$



BCFW 2

We deform the momenta as

$$\begin{aligned}\lambda_k \to \hat{\lambda}_k(z) &= \lambda_k - z\lambda_n \\ \tilde{\lambda}_n \to \hat{\tilde{\lambda}}_n(z) &= \tilde{\lambda}_n + z\tilde{\lambda}_k\end{aligned} \tag{37}$$

where z is a complex variable. Thus

$$\begin{aligned}p_k(z) &= (\lambda_k - z\lambda_n)\,\tilde{\lambda}_k \\ p_n(z) &= \lambda_n\left(\tilde{\lambda}_n + z\tilde{\lambda}_k\right) \\ p_s(z) &= p_s \quad s \neq n, k\end{aligned} \tag{38}$$

The transfered momentum is

$$P_{ij}(z) = p_i + \cdots + p_j \quad \to \quad P_{ij}(z) = P_{ij} + z\lambda_n\tilde{\lambda}_k \tag{39}$$

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$$z_{ij} = \frac{P^2_{ij}}{\langle\lambda_n|P_{ij}|\tilde{\lambda}_k]} \tag{40}$$

A complex function with simple poles can be expanded as

$$\mathcal{A}_n(z) = \sum_{i,j} \frac{c_{ij}}{z - z_{ij}}, \tag{41}$$

As a result the Scattering Amplitude is

$$\mathcal{A}_n(0) = -\sum_{i,j} \frac{c_{ij}}{z_{ij}}, \tag{42}$$

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$$\mathcal{A}_n(0) = -\sum_{i,j} \frac{c_{ij}}{z_{ij}}, \tag{42}$$

where

$$c_{ij} = -\sum_h \acute{\mathcal{A}}^h(p_j(z), \ldots, p_i(z)) \frac{1}{\langle\lambda_n|P_{ij}|\tilde{\lambda}_k]} \acute{\mathcal{A}}^{-h}(p_{i+1}(z) \ldots p_{j-1}(z))\Big|_{z=z_{ij}} \tag{43}$$

BCFW 4

We conclude that

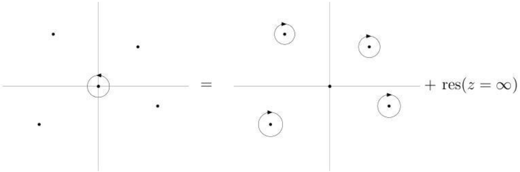
$$\mathcal{A}_n = \sum_{i,j} \sum_h \mathcal{A}^h(p_j(z), \ldots, p_i(z)) \frac{1}{P_{ij}} \mathcal{A}^{-h}(p_{i+1}(z), \ldots, p_{j-1}(z)) \Big|_{z=z_{ij}} \tag{44}$$

The above process is equivalent to calculating

$$I = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{\mathcal{A}_n(z)}{z} dz \tag{45}$$

Thus

$$I_0 = -I_{\text{poles}} + res(\infty) \tag{46}$$



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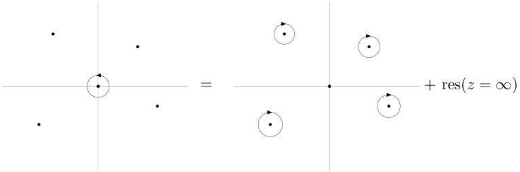

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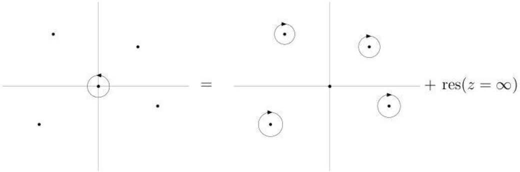
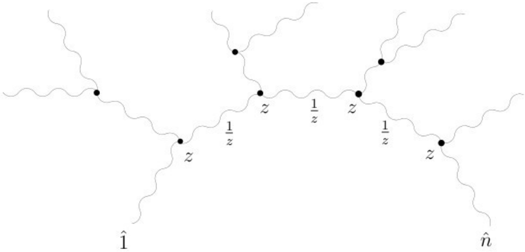


Large z Naive 1

Deformed propagators contribute as $1/z$ and 3-point vertices contribute as z. Worst diagramm scales as z.



Large z Naive 2

Polarization vectors bahave as

$$\epsilon^{a\dot{a}}_{k,+} = \frac{\tilde{\lambda}^{\dot{a}}_k \mu^a}{\langle \hat{\lambda}_k(z)\mu \rangle} \sim \frac{1}{z} \qquad \epsilon^{a\dot{a}}_{n,+} = \frac{\hat{\tilde{\lambda}}^{\dot{a}}_n(z)\mu^a}{\langle \hat{\lambda}_n \mu \rangle} \sim z$$
$$\epsilon^{a\dot{a}}_{k,-} = \frac{\hat{\lambda}^a_k(z)\tilde{\mu}^{\dot{a}}}{\left[\tilde{\lambda}_k \tilde{\mu}\right]} \sim z \qquad \epsilon^{a\dot{a}}_{n,-} = \frac{\lambda^a_n \tilde{\mu}^{\dot{a}}}{\left[\hat{\tilde{\lambda}}_n(z)\tilde{\mu}\right]} \sim \frac{1}{z} \tag{47}$$

Thus

$$\mathcal{A}(+-) \sim \frac{1}{z} \quad \mathcal{A}(++) \sim z \quad \mathcal{A}(--) \sim z \quad \mathcal{A}(-+) \sim z^3 \tag{48}$$

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Large z Background Field Method 1

We expand the field as $A_\mu = B_\mu + a_\mu$. We add the term $(D_\mu a^\mu)^2$ in order to fix the gauge.

$$\mathcal{L}^{aa} = -\frac{1}{4} Tr \left[D_\mu a_\nu D^\mu a^\nu \right] + \frac{\imath}{2} Tr \left[F_B^{\mu\nu} [a_\mu, a_\nu] \right] \tag{49}$$

Propagators of a_μ contribute as $1/z$ and $Ba\partial a$ vertices contribute as z. There is a broken Spin - Lorentz symmetry

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The Amplitude can be expanded as

$$\mathcal{A}^{ab} = n^{ab} \left(cz + d + \dots \right) + A^{ab} + \frac{1}{z} B^{ab} + \dots \tag{51}$$

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Using Weyl - van der Waerden formulation we get

$$\mathcal{A}^{a\dot{a}b\dot{b}} = \epsilon^{ab}\epsilon^{\dot{a}\dot{b}}\,(cz + d + \dots) + \left(\epsilon^{ab}\tilde{s}^{\dot{a}\dot{b}} + \epsilon^{\dot{a}\dot{b}}s^{ab}\right) + \frac{1}{z}B^{a\dot{a}b\dot{b}} + \dots \tag{52}$$

For $(++)$ we have

$$\frac{\left[\tilde{\lambda}_k\hat{\tilde{\lambda}}_n(z)\right] s^{ab}\mu_a\mu_b}{\left\langle\hat{\lambda}_k(z)\mu\right\rangle\langle\lambda_n\mu\rangle} \tag{53}$$

Improved behavior is

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MHV Amplitudes

The MHV Amplitude is given by Parke Taylor formula

$$A_n^{MHV}(1^+,\ldots,i^-,\ldots,j^-,\ldots,n^+) = \delta(p)\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle} \quad (55)$$

For real momenta there is no 3-point Scattering

$$p_1 + p_2 + p_3 = 0 \leftrightarrow \lambda_1^a\tilde{\lambda}_1^{\dot{a}} + \lambda_2^a\tilde{\lambda}_2^{\dot{a}} + \lambda_3^a\tilde{\lambda}_3^{\dot{a}} = 0 \quad (56)$$

But for complex momenta

$$\langle 12\rangle[12] = \langle 23\rangle[23] = \langle 31\rangle[31] = 0 \quad (57)$$

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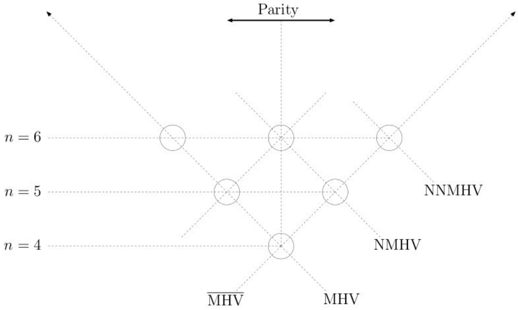
Amplitudes Classification



The 4-point MHV Amplitude

We express $\mathcal{A}_4^{MHV}$ as

$$\mathcal{A}_4^{MHV}(1^-,2^+,3^-,4^+) = \mathcal{A}_3^{\overline{MHV}}(\hat{1}^-,2^+,P^+)\frac{1}{P^2}\mathcal{A}_3^{MHV}(-P^-,3^-,\hat{4}^+) \tag{60}$$

with $\hat{1}\rangle = 1\rangle - zn\rangle, \quad \hat{4}] = 4] + z1]$ and $P^2 = \langle 12\rangle[12]$

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$$\mathcal{A}_4^{MHV}(1^-,2^-,3^+,4^+) = \delta(p)\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{62}$$

Cyclicity and Kleiss - Kuijf allow as to conclude that

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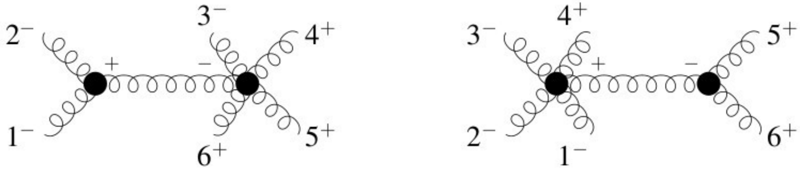
Efficiency Comparison

Efficiency for $\mathcal{A}_6(1^+, 2^+, 3^+, 4^-, 5^-, 6^-)$

- Traditional 220 diagramms
- Color Ordering 36 diagramms
- On-Shell Methods 2 diagramms



Precision Needed

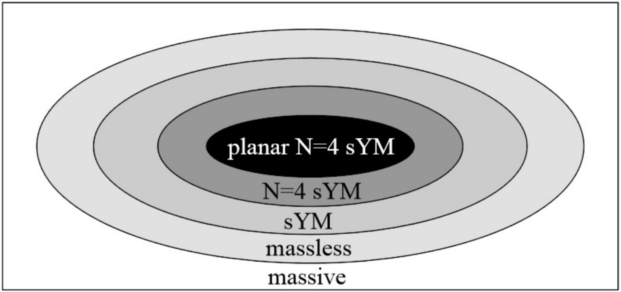
Efficient techniques at loop Level allowed many NLO and NNLO calculations for LHC and will be vital for FCC.

Method	Current relative precision	Future relative precision
e^+e^- evt shapes	expt $\sim 1\%$ (LEP) thry ~ 1–3% (NNLO+up to N^{3}LL, n.p. signif.) [27]	$< 1\%$ possible (ILC/TLEP) $\sim 1\%$ (control n.p. via Q^2-dep.)
e^+e^- jet rates	expt $\sim 2\%$ (LEP) thry $\sim 1\%$ (NNLO, n.p. moderate) [28]	$< 1\%$ possible (ILC/TLEP) $\sim 0.5\%$ (NLL missing)
precision EW	expt $\sim 3\%$ (R_Z, LEP) thry $\sim 0.5\%$ (N^{3}LO, n.p. small) [9, 29]	0.1% (TLEP [10]), 0.5% (ILC [11]) $\sim 0.3\%$ (N^{4}LO feasible, $\sim$ 10 yrs)
τ decays	expt $\sim 0.5\%$ (LEP, B-factories) thry $\sim 2\%$ (N^{3}LO, n.p. small) [8]	$< 0.2\%$ possible (ILC/TLEP) $\sim 1\%$ (N^{4}LO feasible, $\sim$ 10 yrs)
ep colliders	~ 1–2% (pdf fit dependent) [30, 31], (mostly theory, NNLO) [32, 33]	0.1% (LHeC + HERA [23]) $\sim 0.5\%$ (at least N^{3}LO required)
hadron colliders	$\sim 4\%$ (Tev. jets), $\sim 3\%$ (LHC $t\bar{t}$) (NLO jets, NNLO $t\bar{t}$, gluon uncert.) [17, 21, 34]	$< 1\%$ challenging (NNLO jets imminent [22])
lattice	$\sim 0.5\%$ (Wilson loops, correlators, ...) (limited by accuracy of pert. th.) [35–37]	$\sim 0.3\%$ ($\sim$ 5 yrs [38])

from Snowmass FCC-QCD report '13

Simplicity

Following Landau's quote we will go on to discussing even more important discoveries.



Hierarchy of simplicity in scattering amplitudes for various types of gauge theory.

$\mathcal{N} = 4$ SUSY

We use Nair's On Shell Superspace Formulation

$$\Phi(p,n) = G^+(p) + n^A\Gamma_A(p) + \frac{1}{2!}n^A n^B S_{AB}(p) + \frac{1}{3!}n^A n^B n^C \epsilon_{ABCD}\overline{\Gamma}_D(p) + \frac{1}{4!}n^A n^B n^C n^D \epsilon_{ABCD} G^-(p), \tag{64}$$

Helicity generator takes the form

$$h = \frac{1}{2}\left[-\lambda^a \frac{\partial}{\partial \lambda^a} + \tilde{\lambda}^{\dot{a}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{a}}} + n^A \frac{\partial}{\partial n^A}\right], \tag{65}$$

CPT invariance allows us to write

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$$\overline{\Phi}(p,\overline{n}) = G^{-}(p) + \overline{n}^{A}\overline{\Gamma}_{A}(p) + \frac{1}{2!}\overline{n}^{A}\overline{n}^{B}S_{AB}(p) + \frac{1}{3!}\overline{n}^{A}\overline{n}^{B}\overline{n}^{C}\epsilon_{ABCD}\Gamma_{D}(p) + \frac{1}{4!}\overline{n}^{A}\overline{n}^{B}\overline{n}^{C}\overline{n}^{D}\epsilon_{ABCD}G^{+}(p) \tag{66}$$

$\mathcal{N} = 4$ SUSY Generators

SUSY generators have the form

$$p^{a\dot{a}} = \sum_i \lambda_i^a \tilde{\lambda}_i^{\dot{a}}, \qquad q^{aA} = \sum_i \lambda_i^a n_i^A, \qquad \bar{q}^{\dot{a}A} = \sum_i \tilde{\lambda}_i^{\dot{a}} \frac{\partial}{\partial n_i^A} \quad (67)$$

Lorentz and $SU(4)$ generators are

$$M_{ab} = \lambda_{(a} \frac{\partial}{\partial \lambda^{b)}}, \quad M_{\dot{a}\dot{b}} = \tilde{\lambda}_{(\dot{a}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{b})}}, \quad R^A_B = n^A \frac{\partial}{\partial n^B} - \frac{1}{4} \delta^A_B n^C \frac{\partial}{\partial n^C} \quad (68)$$

The Amplitudes are annihilated by the generators of Dilations,

$$d = \frac{1}{2} \sum_i \left[\lambda_i^a \frac{\partial}{\partial \lambda_i^a} + \tilde{\lambda}_i^{\dot{a}} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{a}}} \right] \quad (69)$$

Special Conformal and Superconformal transformations

$$k_{a\dot{a}} = \sum_i \frac{\partial^2}{\partial \lambda_i^a \partial \tilde{\lambda}_i^{\dot{a}}}, \quad s_{aA} = \sum_i \frac{\partial^2}{\partial \lambda_i^a \partial n_i^A}, \quad \bar{s}^A_{\dot{a}} = \sum_i n_i^A \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{a}}} \quad (70)$$

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$\mathcal{N} = 4$ SUSY Amplitudes

The Full Superamplitude can be expanded as

$$\mathcal{A}(\Phi_1, \Phi_2, \ldots, \Phi_n) = (n_1)^4 (n_2)^4 A(-,-,+,\ldots,+) + \ldots \quad (71)$$

Taking into account all symmetries, the form of the Superamplitude is

$$\mathcal{A}(\Phi_i) = \frac{\delta^{(4)}(p)\delta^{(8)}(q)}{\langle 12\rangle \ldots \langle n1\rangle} \mathcal{P}_n(\lambda_i \tilde{\lambda}_i, n_i), \quad (72)$$

This relation holds for every theory at tree level! Effective SUSY!
The P factor can be expanded as

$$\mathcal{P}_n = \mathcal{P}_n^{(0)} + \mathcal{P}_n^{(4)} + \cdots + \mathcal{P}_n^{(4n-16)} \quad (73)$$

Each term corresponds to MHV, NHMV etc Amplitude.

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The 3 point MHV Amplitude is

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BCFW $\mathcal{N} = 4$ SUSY

In order to perform BCFW shift while preserving $\mathcal{N} = 4$ SUSY we must shift Grassmann parameters too. The correct shift is

$$\begin{aligned} \lambda_k &\to \hat{\lambda}_k(z) = \lambda_k - z\lambda_n \\ \tilde{\lambda}_n &\to \hat{\tilde{\lambda}}_n(z) = \tilde{\lambda}_n + z\tilde{\lambda}_k \\ n_n &\to \hat{n}_n(z) = n_n + zn_k \end{aligned} \tag{76}$$

The sum over the helicities of the intermediate state becomes integration into its Grassmann parameter. Thus

$$\mathcal{A}_n = \sum_i \int \frac{d\hat{n}_{P_i}}{P_i^2} \mathcal{A}_L(\hat{1}(z_{P_i}), \ldots, i-1, -\hat{P}(z)) \mathcal{A}_R(\hat{P}(z), i, \ldots, \hat{n}(z))\Big|_{z=z_i} \tag{77}$$

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We define

$$\lambda_i^a \tilde{\lambda}_i^{\dot{a}} = x_i^a - x_{i+1}^{\dot{a}} = p_i^{a\dot{a}} \qquad \lambda_i^a n_i^A = \theta_i^{aA} - \theta_{i+1}^{aA} = q_i^{aA} \tag{78}$$

The following shorthands are very usufull

$$x_{ij} = x_i - x_j = p_i + p_{i+1} + \cdots + p_{j-1} \qquad \theta_{ij} = \theta_i - \theta_j = q_i + q_{i+1} + \cdots + q_{j-1} \tag{79}$$

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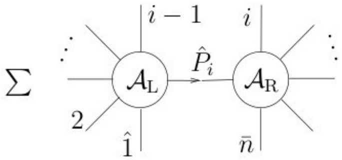
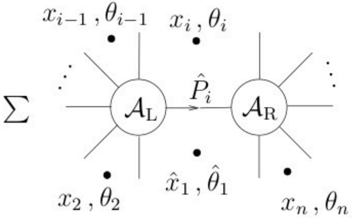


r.h.s. of on-shell recursion relation



dual variables

All Tree Level Amplitudes in $\mathcal{N} = 4$ SUSY

The Amplitude $\mathcal{A}_n^{N^p MHV}$ is given by the recursion

$$\mathcal{A}_n^{N^p MHV} = \int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(z_P) \mathcal{A}_{n-1}^{N^p MHV} + \sum_{m=0}^{p-1} \sum_i \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i^{N^m MHV}(z_{P_i}) \mathcal{A}_{n-i+2}^{N^{(p-m-1)} MHV}(z_{P_i}) \tag{82}$$

We are going to give a diagrammatic solution for to this recursion.

$\mathcal{A}_n^{MHV}$ in $\mathcal{N} = 4$ SUSY

$$\mathcal{A}_4^{MHV} = \int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(\hat{1}, 2, -\hat{P}) \mathcal{A}_3^{MHV}(\hat{P}, 3, \hat{4}) \tag{83}$$

$$\mathcal{A}_4^{MHV} = \int \frac{d^4 n_{\hat{P}}}{P^2} \frac{\delta^4(n_1[2\hat{P}] + n_2[\hat{P}1] + n_{\hat{P}}[12])}{[12][2\hat{P}][\hat{P}1]} \frac{\delta^8(\lambda_{\hat{P}} n_{\hat{P}} + \lambda_3 n_3 + \lambda_4 \hat{n}_4)}{\langle \hat{P}3\rangle\langle 34\rangle\langle 4\hat{P}\rangle} \tag{84}$$

The Grassmann integration sets $n_{\hat{P}} = -\frac{1}{[12]}\left(n_1[2\hat{P}] + n_2[\hat{P}1]\right)$ and gives a factor of $[12]^4$.
As in non-SUSY case we end up with

$$\mathcal{A}_4^{MHV} = \frac{\delta^8(q)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{85}$$

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$\mathcal{A}_n^{NMHV}$ in $\mathcal{N} = 4$ SUSY 1

$\mathcal{A}_n^{NMHV}$ can be constructed in 2 ways

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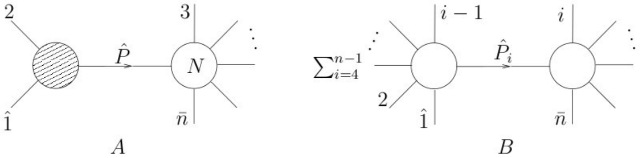

$\mathcal{A}_n^{NMHV}$ in $\mathcal{N}=4$ SUSY 2

We focus on the inhomogenius term

$$I_i = \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \frac{\delta^8(\hat{\lambda}_1 n_1 + \sum_2^{i-1} \lambda_j n_j - \lambda_{\hat{P}_i} n_{\hat{P}_i})}{\langle \hat{1}2\rangle\langle 23\rangle \dots \langle i-1\hat{P}_i\rangle\langle \hat{P}_i\hat{1}\rangle} \times \frac{\delta^8(\lambda_{\hat{P}_i} n_{\hat{P}_i} + \sum_i^{n-1} \lambda_j n_j + \lambda_n \hat{n}_n)}{\langle \hat{P}_i i\rangle\langle i i+1\rangle \dots \langle n\hat{P}_i\rangle} \tag{87}$$

One δ^8 can be exchanged for SUSY conservation, while the other gives

$$\delta^8\left(\hat{\lambda}_1 n_1 + \sum_2^{i-1} \lambda_j n_j - \lambda_{\hat{P}_i} n_{\hat{P}_i}\right) = \langle \hat{1}\hat{P}_i\rangle^4 \delta^4\left(n_1 + \sum_2^{i-1} \frac{\langle \hat{P}_i j\rangle}{\langle \hat{P}_i \hat{1}\rangle} n_j\right) \delta^4\left(\sum_2^{i-1} \frac{\langle \hat{1} j\rangle}{\langle \hat{1}\hat{P}_i\rangle} n_j - n_{\hat{P}_i}\right) \tag{88}$$

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$\mathcal{A}_n^{NMHV}$ in $\mathcal{N} = 4$ SUSY 3

After some algebra and the observation that θ_1 can be exchanged for θ_n as $\langle n\theta_1\rangle = \langle n\theta_n\rangle$, we get the term

$$\delta^4\left(\langle n|x_{n2}x_{2i}|\theta_{in}\rangle + \langle n|x_{ni}x_{i2}|\theta_{2n}\rangle\right) \quad (89)$$

At the end of the day the inhomogenius term reads

$$\mathcal{A}_n^{MHV}\sum_{i=4}^{n-1} R_{n;2i} \quad (90)$$

where

$$R_{n;2i} = \frac{\langle 21\rangle\langle ii-1\rangle\delta^4\left(\langle n|x_{n2}x_{2i}|\theta_{in}\rangle + \langle n|x_{ni}x_{i2}|\theta_{2n}\rangle\right)}{x_{2i}^2\langle n|x_{ni}x_{i2}|2\rangle\langle n|x_{ni}x_{i2}|1\rangle\langle n|x_{n2}x_{2i}|i\rangle\langle n|x_{n2}x_{2i}|i-1\rangle} \quad (91)$$

$\mathcal{A}_5^{NMHV} = \mathcal{A}_5^{MHV} R_{5;24}$ which is compatible with $\mathcal{A}_5^{NMHV} = \mathcal{A}_5^{\overline{MHV}}$

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$$\mathcal{A}_n^{MHV}\sum_{i=4}^{n-1} R_{n;2i} \tag{90}$$

where

$$R_{n;2i} = \frac{\langle 21\rangle\langle i i-1\rangle\delta^4\left(\langle n|x_{n2}x_{2i}|\theta_{in}\rangle + \langle n|x_{ni}x_{i2}|\theta_{2n}\rangle\right)}{x_{2i}^2\langle n|x_{ni}x_{i2}|2\rangle\langle n|x_{ni}x_{i2}|1\rangle\langle n|x_{n2}x_{2i}|i\rangle\langle n|x_{n2}x_{2i}|i-1\rangle} \tag{91}$$

$\mathcal{A}_5^{NMHV} = \mathcal{A}_5^{MHV} R_{5;24}$ which is compatible with $\mathcal{A}_5^{NMHV} = \mathcal{A}_5^{\overline{MHV}}$

We generalize $R_{n;2i}$ as

$$R_{n;st} = \frac{\langle ss-1\rangle\langle tt-1\rangle\delta^4\left(\langle n|x_{ns}x_{st}|\theta_{tn}\rangle + \langle n|x_{nt}x_{ts}|\theta_{sn}\rangle\right)}{x_{st}^2\langle n|x_{nt}x_{ts}|s\rangle\langle n|x_{nt}x_{ts}|s-1\rangle\langle n|x_{ns}x_{st}|t\rangle\langle n|x_{ns}x_{st}|t-1\rangle} \tag{92}$$

$R_{n;st}$ is independant of n_{n-1}, n_1 and n_n.
It can be proved that

$$\delta^8(q)\sum_{s,t} R_{n;st} = \delta^8(q)\sum_{s,t} R_{\acute{n};st} \tag{93}$$

where the sum runs over all values of s and t with n, s, t (or $\acute{n}, s, t$) being ordered cyclic, with n and s (or $\acute{n}$ and s) and s and t being separated by at least two.

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We assume that $\mathcal{A}_n^{NMHV}$ is given by

$$\mathcal{A}_n^{NMHV} = \mathcal{A}_n^{MHV} \sum_{2 \leq s < t \leq n-1} \frac{\langle s s-1 \rangle \langle t t-1 \rangle \delta^4 \left(\langle n | x_{ns} x_{st} | \theta_{tn} \rangle + \langle n | x_{nt} x_{ts} | \theta_{sn} \rangle \right)}{x_{st}^2 \langle n | x_{nt} x_{ts} | s \rangle \langle n | x_{nt} x_{ts} | s-1 \rangle \langle n | x_{ns} x_{st} | t \rangle \langle n | x_{ns} x_{st} | t-1 \rangle} \tag{94}$$

Then

$$A = \int \frac{d^4 P}{P^2} \int d^4 n_{\hat{P}} \mathcal{A}_3^{\overline{MHV}}(z_P) \mathcal{A}_{n-1}^{MHV} \mathcal{P}_{n-1}^{NMHV}(\hat{P}, 3, \ldots, \hat{n}) \tag{95}$$

$\mathcal{A}_n^{NMHV}$ in $\mathcal{N}=4$ SUSY 5

We assume that $\mathcal{A}_n^{NMHV}$ is given by

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As $\mathcal{P}_{n-1}^{NMHV}$ is not affected by the integration

$$A = \mathcal{A}_n^{MHV} \mathcal{P}_{n-1}^{NMHV}(\hat{P}, 3, \ldots, \hat{n}) \tag{96}$$

3-point kinematics allows as to exchange $\langle \hat{P}|$ for $\langle 2|$, thus

$$A = \mathcal{A}_n^{MHV} \sum_{3 \leq s < t \leq n-1} R_{n;st} \tag{97}$$

Adding the inhomogenius term B we get

$$\mathcal{A}_n^{NMHV} = A + B = \mathcal{A}_n^{MHV} \sum_{2 \leq s < t \leq n-1} R_{n;st} \tag{98}$$

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The price of Simplicity

We have found that

$$\mathcal{P}_n^{NMHV} = \sum_{2\leq a_1<b_1\leq n-1} R_{n;a_1 b_1} \tag{99}$$

This expression is very elegant, but it contains nonlocal terms

$$\frac{1}{\langle n|x_{na}x_{ab}|b\rangle} \tag{100}$$

Even though the Theory remains local our result is not manifesty local. We have exchanged manifest locality for manifest Dual Superconformal Invariance.

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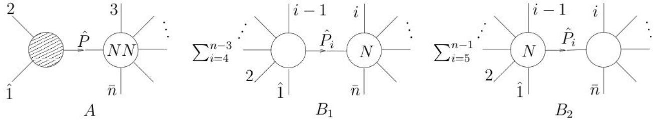
$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 1

$\mathcal{A}_n^{NNMHV}$ can be constructed in 3 ways

$$\mathcal{A}_n^{NNMHV} = \int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(z_P)\mathcal{A}_{n-1}^{NNMHV}(z_P)$$
$$+ \sum_{i=4}^{n-3} \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i^{MHV}(z_{P_i})\mathcal{A}_{n-i+2}^{NMHV}(z_{P_i})$$
$$+ \sum_{i=5}^{n-1} \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i^{NMHV}(z_{P_i})\mathcal{A}_{n-i+2}^{MHV}(z_{P_i}) \equiv A + B_1 + B_2 \tag{101}$$



$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 2

We generalize R invariant as

$$R_{n;b_1a_1;b_2a_2;\dots b_ra_r;ab} = \frac{\langle a a-1\rangle\langle b b-1\rangle\delta^4\left(\langle\xi|x_{a_ra}x_{ab}|\theta_{ba_r}\rangle+\langle\xi|x_{a_rb}x_{ba}|\theta_{aa_r}\rangle\right)}{x_{ab}^2\langle\xi|x_{a_rb}x_{ba}|a\rangle\langle\xi|x_{a_rb}x_{ba}|a-1\rangle\langle\xi|x_{a_ra}x_{ab}|b\rangle\langle\xi|x_{a_ra}x_{ab}|b-1\rangle} \tag{102}$$

where

$$\langle\xi| = \langle n|x_{nb_1}x_{b_1a_1}x_{a_1b_2}\dots x_{b_ra_r} \tag{103}$$

We use expressesions like

$$\sum_{L\le a<b\le U} R^{l_1\dots l_p;u_1\dots n_q}_{n;b_1a_1;b_2a_2;\dots b_ra_r;ab} \tag{104}$$

At the boundaries:

$$\langle L-1| \to \langle n|x_{nl_1}x_{l_1l_2}\dots x_{l_{p-1}l_p} \quad \langle U| \to \langle n|x_{nu_1}x_{u_1u_2}\dots x_{u_{q-1}u_q} \tag{105}$$

$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 2

We generalize R invariant as

$$R_{n;b_1a_1;b_2a_2;\ldots b_ra_r;ab} = \frac{\langle a\,a-1\rangle\langle b\,b-1\rangle\delta^4\left(\langle\xi|x_{a_ra}x_{ab}|\theta_{ba_r}\rangle + \langle\xi|x_{a_rb}x_{ba}|\theta_{aa_r}\rangle\right)}{x_{ab}^2\langle\xi|x_{a_rb}x_{ba}|a\rangle\langle\xi|x_{a_rb}x_{ba}|a-1\rangle\langle\xi|x_{a_ra}x_{ab}|b\rangle\langle\xi|x_{a_ra}x_{ab}|b-1\rangle} \tag{102}$$

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$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N} = 4$ SUSY 2

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$$R_{n;b_1a_1;b_2a_2;\ldots b_ra_r;ab} = \frac{\langle aa-1\rangle\langle bb-1\rangle\delta^4\left(\langle\xi|x_{a_ra}x_{ab}|\theta_{ba_r}\rangle + \langle\xi|x_{a_rb}x_{ba}|\theta_{aa_r}\rangle\right)}{x_{ab}^2\langle\xi|x_{a_rb}x_{ba}|a\rangle\langle\xi|x_{a_rb}x_{ba}|a-1\rangle\langle\xi|x_{a_ra}x_{ab}|b\rangle\langle\xi|x_{a_ra}x_{ab}|b-1\rangle} \tag{102}$$

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$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 3

$$B_1 = \sum_{i=4}^{n-3} \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i^{MHV}(z_{P_i}) \mathcal{A}_{n-i+2}^{MHV}(z_{P_i}) \mathcal{P}_{n-i+2}^{NMHV}(z_{P_i}) \tag{106}$$

$$B_1 = \mathcal{A}_n^{MHV} \sum_{i=4}^{n-1} R_{n;2i} \mathcal{P}_{n-i+2}^{NMHV}(\hat{P}, \ldots, \hat{n}) \tag{107}$$

For $s = i$ the Amplitude depends on $\langle \hat{P}|$. Using $\langle n1\rangle[1\hat{P}]$ we observe that we can exchange this dependance for

$$\langle i-1| \rightarrow \langle n|x_{n2}x_{2i} \tag{108}$$

Thus

$$B_1 = \mathcal{A}_n^{MHV} \sum_{i=4}^{n-1} R_{n;2i} \sum_{i \leq s < t \leq n-1} R_{n;st}^{2i;0} \tag{109}$$

$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 3

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$$B_2 = \sum_{i=5}^{n-1} \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i^{MHV}(z_{P_i}) \mathcal{P}_i^{NMHV}(z_{P_i}) \mathcal{A}_{n-i+2}^{MHV}(z_{P_i}) \qquad (110)$$

$$B_2 = \mathcal{A}_n^{MHV} \sum_{i=4}^{n-1} R_{n;2i} \sum_{3\leq s<t\leq \hat{P}} R_{\hat{1};st}(2,\ldots,-\hat{P},\hat{1}) \qquad (111)$$

For $t=\hat{P}$ we use $\langle \hat{P}| \to \langle n|x_{n2}x_{2i}$.
As last particle is $\hat{1}$

$$\langle n| \to \langle n|x_{ni}x_{i2} \qquad (112)$$

Thus

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$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 4

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$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 4

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$\mathcal{A}_n^{NNMHV}$ in $\mathcal{N}=4$ SUSY 5

Inhomogenius terms add up to

$$B_1 + B_2 = \mathcal{A}_n^{MHV} \sum_{i=4}^{n-1} R_{n;2i} \left[\sum_{3 \le s < t \le i} R_{n;i2;st}^{0;2i} + \sum_{i \le s < t \le n-1} R_{n;st}^{2i;0} \right] \tag{114}$$

We generalize this term as

$$\mathcal{A}_n^{NNMHV} = \mathcal{A}_n^{MHV} \sum_{2 \le a_1 < b_1 \le n-1} R_{n;a_1 b_1}^{0;0} \left[\sum_{a_1+1 \le a_2 < b_2 \le b_1} R_{n;b_1 a_1;a_2 b_2}^{0;a_1 b_1} + \sum_{b_1 \le a_2 < b_2 \le n-1} R_{n;a_2 b_2}^{a_1 b_1;0} \right] \tag{115}$$

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Then the homogenius term is given by

$$\int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(z_P) \mathcal{A}_{n-1}^{MHV}(z_P) \mathcal{P}_{n-1}^{NNMHV}(\hat{P}, 3, \ldots, \hat{n}) \tag{116}$$

Thus

$$A = \mathcal{A}_n^{MHV} \sum_{3 \le a_1 < b_1 \le n-1} R_{n;a_1 b_1}^{0;0} \left[\sum_{a_1+1 \le a_2 < b_2 \le b_1} R_{n;b_1 a_1;a_2 b_2}^{0;a_1 b_1} + \sum_{b_1 \le a_2 < b_2 \le n-1} R_{n;a_2 b_2}^{a_1 b_1;0} \right] \tag{117}$$

Then the homogenius term is given by

$$\int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(z_P)\mathcal{A}_{n-1}^{MHV}(z_P)\mathcal{P}_{n-1}^{NNMHV}(\hat{P}, 3, \ldots, \hat{n}) \qquad (116)$$

Thus

$$A = \mathcal{A}_n^{MHV} \sum_{3 \leq a_1 < b_1 \leq n-1} R_{n;a_1 b_1}^{0;0} \left[\sum_{a_1+1 \leq a_2 < b_2 \leq b_1} R_{n;b_1 a_1;a_2 b_2}^{0;a_1 b_1} + \sum_{b_1 \leq a_2 < b_2 \leq n-1} R_{n;a_2 b_2}^{a_1 b_1;0} \right] \qquad (117)$$

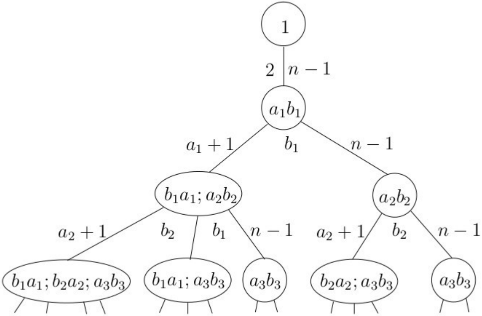
We observe that $B_1 + B_2$ is the homogenius term for $a_1 = 2$, thus

$$A + B_1 + B_2 = \mathcal{A}_n^{MHV} \sum_{2 \le a_1 < b_1 \le n-1} R^{0;0}_{n;a_1 b_1} \left[\sum_{a_1+1 \le a_2 < b_2 \le b_1} R^{0;a_1 b_1}_{n;b_1 a_1;a_2 b_2} + \sum_{b_1 \le a_2 < b_2 \le n-1} R^{a_1 b_1;0}_{n;a_2 b_2} \right] \tag{118}$$

This expression gives the correct result for $\mathcal{A}_6^{NNMHV}$, which concludes the proof.

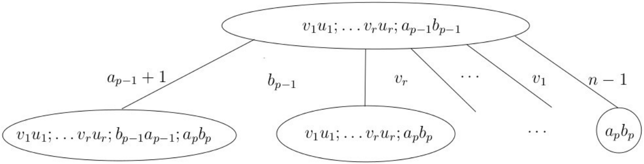
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 1

The general solution is given by the following diagramm



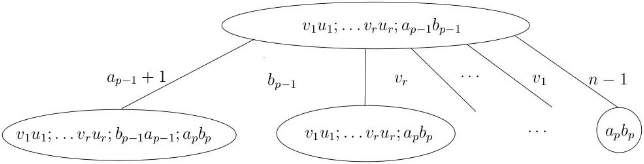
Each vertex correspond to an $R_{n;xx}^{xx;xx}$ term. The first line corresponds to $R_{n;a_1 b_1}^{0;0}$, the second to $R_{n;b_1 a_1;a_2 b_2}^{0;a_1 b_1}$ and $R_{n;a_2 b_2}^{a_1 b_1;0}$ etc. The number of vertices at each lines is given by the Catalan number $C(p) = \frac{(2p)!}{p!(p+1)!}$

$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 2



- $r+2$ vertices
- remove pair of indices
- summation limits
- right superscript $v_1 u_1; \dots v_r u_r; a_{p-1} b_{p-1}$
- left superscript of a given vertex coincides with the right superscript of the vertex to its left

$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 2



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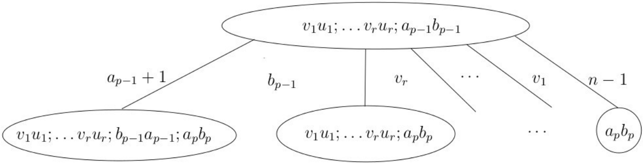
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 2



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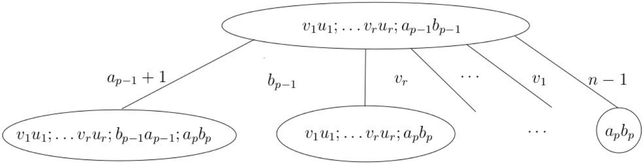
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 2



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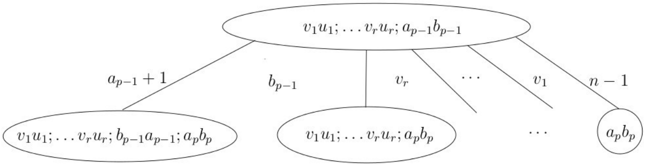
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N}=4$ SUSY 2



- $r+2$ vertices
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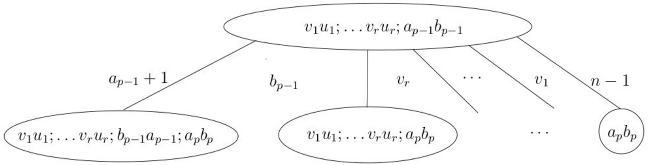
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N}=4$ SUSY 2



- $r+2$ vertices
- remove pair of indices
- summation limits
- right superscript $v_1 u_1; \dots v_r u_r; a_{p-1} b_{p-1}$
- left superscript of a given vertex coincides with the right superscript of the vertex to its left

$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N} = 4$ SUSY 3

We take all possible vertical path of nested sums. Then $\mathcal{P}_n$ is given by

$$\mathcal{P}_n = \sum \text{all vertical paths until line } n - 4 \tag{119}$$

There is only a path of zero length corresponding to vertex 1

$$\mathcal{P}_n^{MHV} = 1 \tag{120}$$

There is only a path of length one. The root correspond to 1 and first line vertex corresponds to $R_{n;a_1 b_1}$ summed over a_1, b_1 from 2 to $n - 1$

$$\mathcal{P}_n^{NMHV} = \sum_{2 \leq a_1 < b_1 \leq n-1} R_{n;a_1 b_1} \tag{121}$$

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$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N}=4$ SUSY 3

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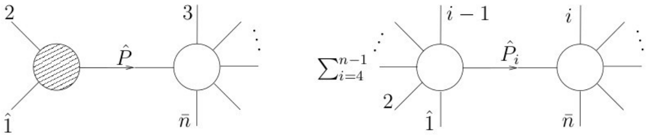
$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N}=4$ SUSY 4

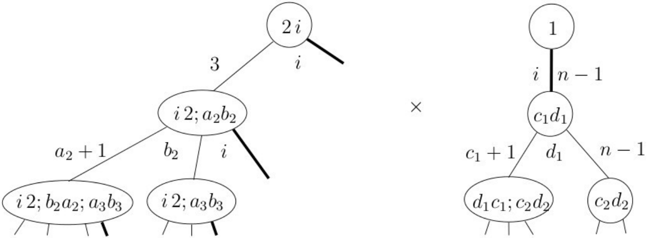
By induction it can be proved that

$$\mathcal{A}_n = \int \frac{d^4 n_{\hat{P}}}{P^2} \mathcal{A}_3^{\overline{MHV}}(z_P)\mathcal{A}_{n-1}(z_P) + \sum_{i=4}^{n-1} \int \frac{d^4 n_{\hat{P}_i}}{P_i^2} \mathcal{A}_i(z_{P_i})\mathcal{A}_{n-i+2}(z_{P_i}) \tag{122}$$



$\mathcal{A}_n^{N^P MHV}$ in $\mathcal{N}=4$ SUSY 5

$$\mathcal{P}_n = \mathcal{P}_{n-1}(\hat{P},3,\ldots,\hat{n}) + \sum_{i=4}^{n-1} R_{n;2i}\mathcal{P}_i(2,\ldots,-\hat{P}_i,\hat{1})\mathcal{P}_{n-i+2}(\hat{P}_i,i,\ldots,\hat{n}) \tag{123}$$



Further Developments

- Twistor Space / Grassmann Planes / Amplituhedron
- Integrability of non planar $\mathcal{N} = 4$ SYM ?
- Perturbative Finiteness of $\mathcal{N} = 8$ SUGRA ?
- Color - Kinematics Duality
- Wilson Loops - Scattering Amplitudes Correspondance
- Scattering Equations

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Sakurai Award



APS J.J Sakurai Award for 2014 was given to Zvi Bern, Lance J. Dixon, and David A. Kosower:
For pathbreaking contributions to the calculation of perturbative scattering amplitudes, which led to a deeper understanding of quantum field theory and to powerful new tools for computing QCD processes