Exact Simulation of the Stationary Distribution of M/G/c and Other Queues

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Abstract

We present two exact simulation algorithms for the stationary distribution of customer delay for first-in-first-out (FIFO) multi-server queues in which the arrival process is Poisson at rate λ, and the service times $\{S_n\}$ are general iid (with finite mean $0 < E(S) = 1/\mu < \infty$); the M/G/c model. We assume that the service-time distribution $G(x) = P(S \leq x),\ x \geq 0$, and its corresponding equilibrium distribution $G_e(x) = \mu \int_0^x P(S > y)dy$ are such that samples of them can be simulated.

Our first algorithm (from [3]) is in continuous time and is only for the special case when $\rho = \lambda/\mu < 1$ (super stable case). This algorithm involves the general method of dominated coupling from the past (DCFTP) and we use the single-server queue operating under the *processor sharing (PS)* discipline as an upper bound. The algorithm is shown to have finite expected termination time if and only if service times have finite second moment.

Our second algorithm (from [2]) is for the general case of $\rho < c$. Here we use discrete-time processes and basic regenerative simulation, in which as regeneration points, we use return visits to state 0 of a corresponding random assignment (RA) model which serves as a sample-path upper bound.

Both algorithms yield, as output, a stationary copy of the entire Kiefer-Wolfowitz workload vector. We also discuss recent results on extending the first algorithm to more complex models such as tandem queues, and even general route queueing networks as in [1].

References

[1] K. Sigman (2011). Using the M/G/1 Queue Under Processor Sharing for Exact Simulation of Queues. (Submitted.)

[2] K. Sigman (2011). Exact simulation of the stationary distribution of the FIFO M/G/c queue: The general case of $\rho < c$. *Queueing Systems.* **70**, (To appear.)

[3] K. Sigman (2011). Exact simulation of the stationary distribution of the FIFO M/G/c queue. *Journal of Applied Probability.* **Special Volume 48A: New Frontiers in Applied Probability**, 209-216.