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The Social Ideologies of School Mathematics Applications: A Case Study of the Elementary School Textbooks

DIMITRIS CHASSAPIS

1. The mapping of mathematical constructs to real world situations

It can be maintained that any mathematical construct is defined in the context of a mathematical theory--a set of axioms and a number of deduced theorems--in which it is embedded and as a consequence, the propositions of a mathematical theory function as meaning postulates for any construct embedded in that theory. Thus any mathematical construct acquires its specific mathematical meaning from a particular mathematical theory.

The construct of addition, for example, is defined within a theory of numbers founded on Peano's axiomatics by the following two propositions: $a + 0 = x$ and $a + f(x) = f(a+x)$, where $f(x)$ is the successor of the number x. These two propositions, on the one hand define generally the sum of any two numbers and assert on the other hand certain propositions--usually called properties or laws--about the addition of numbers. That is, they define what is meant by the arithmetical operation of addition according to that theory: addition is an operation on numbers that satisfies these two propositions. This is the specific mathematical meaning of addition within that mathematical theory.

Mathematical constructs however, just as many other mental constructs, are used to describe or are applied, as it has prevailed to be called, to real world situations. Any such mathematical description of or any such mathematical application to any real world situation may be considered as a micro-theory of that particular aspect of the real world, in which case any implicated mathematical statement may be interpreted as a statement about the concerned aspect of the real world.

The mathematical construct of addition on integers for instance, may be applied to and considered as a reasonable micro-theory of simple financial transactions describing income or credit by positive integers and expense or debt by negative integers. In a such case any mathematical statement implied by that particular application of addition to a particular situation of financial transactions may be interpreted as a statement about that particular aspect of the social reality.

A description of a real world situation in terms of a mathematical construct or an application of a mathematical construct to a real world situation is set up on the basis of a mapping between that construct and the real world situation.

Every such mapping though, is indispensably mediated by a class of non-mathematical concepts that circumscribes the class of real world situations to which the mathematical construct is applied and by a set of associated linguistic and more generally symbolic expressions that signify these concepts. The non-mathematical concepts in association to their symbolic expressions assign another non-mathematical meaning and simultaneously specify a reference of the mathematical construct within the particular description or application. This meaning may be considered as constituting the applicational or the referential meaning of the mathematical construct to the real world.

The mathematical construct of addition, as an example, may be used to describe or may be applied to a class of real world situations circumscribed by concepts of change, combine or compare. Particular instances of these concepts (e.g., the growth of a particular quantity as an instance of change, the union of two particular quantities as an instance of combination or the difference of two particular quantities as an instance of comparison) in conjunction to their signifying symbolic-linguistic expressions attribute to the applied mathematical construct of addition another meaning differentiated from its mathematical one, specifying at the same time particular references of addition within its particular applications to particular real world situations. Anyone of such meanings constitutes an applicational or referential meaning of the mathematical construct of addition on numbers.

The referential meanings assigned to the mathematical construct of addition by the different instances of the concepts of change, combine or compare are not however in every aspect identical. When addition, as an example, is applied to and interpreted as describing the growth process of a population group over a given time interval, the first addend describes and refers to the population size and the second addend to its increment at the given time interval. As a consequence the addends in this case are not actually interchangeable since the growth of a population size X by an increment Y is not the same process as the growth of a population size Y by an increment X, the rates of growth being for instance not identical. This means that in such a case addition cannot be commutative although the mathematical construct of addition is by definition commutative. In contrast, when addition is applied to and interpreted as describing the process of uniting two population groups, the first addend describes and refers to the size of anyone while the second addend to the size of the other population group, respectively. In such a case the process of uniting a population size X with a population size Y is identical to uniting Y with X, the addends may thus be interchangeable and the addition can be commutative, conformably to its mathematical definition.

Therefore, a particular meaning of the growth or the union concept specifies how the applied mathematical construct of addition refers to and acquires meaning by a particular mapping to a particular real world situation.

In summary, It may be held that any mathematical construct beyond its specific mathematical meaning can acquire multiple referential meanings as it is assigned several different mappings in several different applications to several different real world situations.

2. The real world situations utilised in school mathematics applications

Adopting the view that real world situations acquire their meanings by the implicated human activities which are always meaningful since intentional, there may be defended that any real world situation and its representation bear meanings that are never value-free thus ideologically neutral. One step further, it may be claimed that the selected real world situations and consequently the associated references of the mathematical constructs to the specific aspects of the real world, which are used as examples, applications, questions or problems-to-be-solved in the teaching of school mathematics are never value-free and bear - in any feasible case - a more or less definite, even not clear, ideological orientation. They thus assign to the mathematical constructs corresponding ideologically oriented referential meanings.

The ideological orientations of the referential meanings assigned to the mathematical construct of addition, for instance, when applied to and interpreted as describing a growth process of profit in a situation of commercial dealings or a growth process of nuclear waste in a situation of environmental pollution are not the same. The two situations highlight different aspects of human activity, and implicitly emphasise different attitudes and patterns of thinking towards human activities, support different life values and ultimately transmit different social ideologies.

From this point of view, school mathematics, just as many other school subjects, may not be considered as an ideologically and hence a socially neutral subject of knowledge which is derived from a conformably neutral scientific mathematical activity. It has to be conceived as a school subject composed of selected mathematical topics bearing ideologically oriented referential meanings assigned to mathematical constructs by their selected mappings in selected applications to selected real world situations.

In this sense, school mathematics along with other school subjects advocates the dominant system of values and patterns of thought in a particular society; it is used to a considerable extent as a vehicle for the indoctrination of children in a specific societal ideology.

3. The case of the Greek elementary school mathematics.

The mathematics curriculum for the Greek elementary school is centrally prescribed in extensive detail by the Ministry of Education. The present curriculum includes the study of the natural numbers and fractions with the their four fundamental operations and the associated algorithms plus the elements of geometry and the basics of measurement. Applications in both mathematical and everyday life activities are emphasised for any topic. A unique mathematics textbook playing essentially the role of a workbook is used for each of the six elementary school grades accompanied by a compatible teacher's book dictating in detail every teaching unit, its content, teaching method and learning tools. Both of them are produced and established at a national level by a State institution. Since the contents along with teaching methods and materials are centrally prescribed and controlled the teaching of mathematics in the Greek elementary schools may be considered as uniform in any of its main aspects.

The related question encountered in this paper is twofold. First, which are the prominent characteristics of the real world situations prevailing in the teaching of the Greek elementary school mathematics as references of the mathematical constructs to the real world and second, which is the social ideology, if any, advocated by the meanings these references assign to the mathematical constructs.

In other words which aspects of the real world are selected and in which patterns are they structured and nominated by the Greek elementary school as the prominent real world objects of mathematical activity.

Passing over the presentation of detailed quantitative data, a content analysis of the Greek elementary school mathematics textbooks yields the conclusion that two classes of situations are prevailing in the examples, applications and problems-to-be-solved as references of the mathematical constructs to the real world :

(1) artificial and socially indefinite situations coming out of a supposed childish and ostensibly abstract natural or social world. This world is made up of material objects, (mostly playthings and pieces of furniture), plants (mostly flowers and vegetables), fruits (mostly apples and oranges) and animals (mostly birds and cats) that situated side by side in collections out of any meaningful real-world context are at the immediate disposal of any mathematical activity incited usually for its own sake by the question “How many or How much ?”. Besides, the coins consist an indispensable element of this world being indirectly attributed an almost natural existence and their manipulation is introduced as an outstanding reference of the mathematical constructs as early as on the first teaching units of the 2nd elementary school grade.

The human beings involved in this world are vaguely indicated by ordinary personal names (e.g., George, Helen), family relatives (e.g., mother, brother) or sex identifications (e.g., a girl. a boy) who detached from any social setting are engaged in counting, comparing, measuring or calculating activities that, as a rule, have no clear reason beyond mathematics itself.

This class of situations is the prevalent reference of the mathematical constructs in the lower grades of the Greek elementary school being the context of more than 85% of the examples, applications and problems included in the mathematics textbooks for the first and second elementary school grades. The number of this kind of situations included in the elementary school mathematics textbooks gradually decreases as the school grade ascends and are substituted by

(2) financial and especially commercial situations devoid of any pertinent social relationships. Buying and selling of commodities, money returns and payments, business profit and loss accounts, individual incomes and expenses, householding expenditure, consumption bills, funds debit and credit and any kind of relevant financial transactions compose the prevailing references of the mathematical constructs in the upper grades of the elementary school. Even calculations of interest on capital are included as a distinct teaching unit in the mathematics textbooks for the 6th elementary school grade.

These transactions derive as a rule from a socially abstract material production and distribution or rendering of professional services introduced by statements of the type “a Factory manufactures ..., a Farm produces ...”, “a Store sells ...”, or “Bottles are packed ...”, “Apples are sold ..., Drinks are canned ...”, to all appearances on their own beyond any human agent away from any space, time or social structure.

On the other hand, whenever persons are implicated in such situations they are either presented as socially indefinable agents (e.g. a Producer retails…., a Worker earns..., a Consumer spends...) or indicated by an occupational identity (e.g. a Bookseller, a Grocer, a Confectioner, a Craftsman). In that second case the induced image of the Greek economically active population is entirely distorted inasmuch as retail trading is implied as its dominant occupational activity.

Likewise money manipulations as references of the mathematical constructs belong to the standard repertoire of the examples, applications and problems included in the mathematics textbooks for the upper grades of the Greek elementary school, even in cases quite unrealistic and never justified by any reason.

The problem for example, “You had 8/10 of a ten-drachma coin and spent its 5/10. How many tenths of the ten-drachma coin have you got left?”, is included in the mathematics textbook for the 4th grade as an application of the decimal fractions and suchlike problems are not rare in the textbooks for other school grades.

The outlined class of financial and commercial situations constitutes the context of more than 70% of the examples, applications and problems included in the mathematics textbooks for the 5th and 6th Greek elementary school grades.

In conclusion it may be supported that the commercial market is covertly endorsed by the Greek elementary school mathematics textbooks as the prominent field of mathematical activity and moreover as the dominant aspect of the real social world.

4. The social ideology of the real world references of mathematical constructs

The foregoing references of the mathematical constructs to real world situations that are included in the elementary school mathematics textbooks being apparently not exclusively dictated by the mathematical activity per se may not be considered as ideologically neutral options. These options covertly reinforce and promote fundamental aspects of a specific ideological conception of the mathematical knowledge and its real world applications along with a specific image of worthy modes of human activity.

The real world situations included as references of the mathematical constructs In the Greek elementary school mathematical textbooks and as a consequence in the teaching of mathematics are embedded either in a socially indefinite context or whenever this context is defined it is mainly one of commercial transactions. These two classes of real world situations are in their turn implicitly presented as the principal objects of mathematical activity. In addition commercial transactions are covertly nominated as the dominant mode of human activity while commodity production and distribution are abstractly presented as impersonal and socially neutral therefore as essentially technical activities. The background of such options may be ascribed to a predominant ideological orientation in the Greek society that according to the relevant literature may be presumed as a version of a middle class mercantile ideology. These options in their turn assign ideologically conformable referential meanings to the applied mathematical constructs.

5. The school mathematics and its relation to mathematical knowledge.

Reflecting on It may be claimed that a traditional relation between school mathematics and mathematical knowledge that formally constitutes its subject matter is a practical relation of usage. This denotes that the relation between school mathematics and its subject matter has as its prime function not the learning of the mathematics subject matter itself but the assimilation of rules, procedures and approaches aiming at the establishment of a specific relationship between children and the mathematics subject matter. Anyway the declared objectives of elementary mathematics education in Greece and in many other countries point out this intention by various wordings. In the endorsed by the law Greek elementary school curriculum, for example, it is stated that "the broad aims of teaching mathematics are the systematic training of pupils in the rational thinking and the logical reasoning as well as their initiation in the deductive processes of mathematics, the advancement of their overall intellectual growth, the development of their abilities in conceiving quantities, properties and relations especially those that are necessary for the comprehension and solution of real world problems and the awareness of contemporary technological, economic and social reality, their familiarisation in putting clearly, precisely and neatly their thoughts into words and finally the development of their appreciation of the role that mathematics play in different sciences."

In my words the primary purpose of mathematics teaching in the Greek elementary school is the learning of rules and procedures for the appropriate usage of the mathematics subject matter. The headings of the teaching units included in the mathematics textbooks for the 5th and 6th elementary school grade emphasise this fact by their phrasing being invariably "How to or How we .... plus a verb of usage" as for instance, write, compare, form, find, measure, count, do, calculate, check, distinguish, solve, etc.

Such a relation of usage that the teaching of mathematics establishes with its subject matter, contributes of course to the construction of the mathematics knowledge. This knowledge however is primarily a practical knowledge of mathematics usage and alongside it is a scientific knowledge of the mathematics subject matter.

In my view this fact may be considered as the essential meaning of the conceptually vague Greek term mathematical "paideia" roughly translated as mathematical "enculturation"; a mathematical knowledge invested in a practical knowledge of its usage. For this among other reasons mathematics traditionally constitutes a fundamental component of pedagogy that is of the socio-cultural indoctrination of children. Mathematics trains children towards "the correct" modes of thinking, "the correct" modes of deducting, "the correct" modes of decision making. A relation of this type between school mathematics and its subject matter may not in any case be considered as an essentially learning relation.

It is above all a relation of ideological indoctrination of children, which on the ground of a commonly valuable teaching subject matter habituates them in particular standpoints and impresses specific patterns of behaviour towards mathematics on them directing through mathematics towards the associated prevailing social values.

To sum up, the teaching of mathematics as well as the teaching of many school subjects incorporates a double relation with its subject matter. A scientific relation as it is a vehicle for theoretical knowledge of its subject matter in conjunction to an ideological relation as it is a vehicle for practical knowledge about this subject matter. A practical knowledge that essentially refers to patterns of behaviour towards the theoretical and the social function of the subject matter of teaching. In this sense the teaching of mathematics in schools is a vehicle for mathematical knowledge in itself in conjunction to a vehicle for an ideology concerning mathematical activity and its outcomes; that is for an ideology of dealing with mathematical knowledge which is based on a specific conception of the place and function of mathematical activity and its outcomes in the present-day particular social reality.

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