POME

PHILOSOPHY OF MATHEMATICS EDUCATION JOURNAL

11 (1999)

ON CONCEPTUAL KNOWLEDGE AND THE OBJECTS OF MATHEMATICS

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The adoption of a social constructivist standpoint about mathematical knowledge and its objects in contrast to the ontological assumptions of Platonism and mathematical realism, a standpoint like that elaborated by Ernest for instance, beyond the various particular issues it raises, it makes, in principle, the fundamental status of mathematics as a scientific discipline ostensibly problematic. In the sense that, mathematics as any scientific discipline is a sector in the quest for truth about experienced reality and, If that conception is given up or radically modified, then mathematics seems to run a risk to be reduced to a kind of esoteric art form---to an intellectual play.

If it is accepted, that "the ontology of mathematics is given by the discursive realm of mathematics, which is populated by cultural objects, which have real existence in that domain", and "mathematical signifiers and signifieds are mutually interacting and constituting, so the discourse of mathematics which seems to name objects outside of itself is in fact the agent of their creation, maintenance and elaboration, through its use" (Ernest, 1997), if, in other words, it is accepted that the "mathematical discourse as a living cultural entity, creates the ontology of mathematics" (Ernest, 1997), then it is seemed that mathematical discourse, if not actually self-referential, constitutes a closed system, and mathematics as a discipline seems to not provide a theoretical knowledge of any realm of social or material reality, which is existing independently and outside of itself. That is, mathematics by itself seems to not deal directly and precisely with any realm of experienced reality, providing the necessary grounds for confirming or refuting its theoretical propositions. Conceptual knowledge of mathematics and the objects it appropriates seems to coincide and any reference of mathematics to an independently and outside of itself existing reality, it seems to be possible only through its mediation by other scientific disciplines, which are utilising mathematics. If so, the constitution of the particular scientific methods which mathematics employs to correlate theoretical knowledge to its objects and simultaneously to test the efficacy of that knowledge for the conceptual appropriation of its objects is also problematised in many ways.

As a first step in a course of elaborating convincing answers to the above mentioned and many relevant questions, within a framework ensuring the fundamental scientific status of mathematics, it may be -- adopting a thesis expressed by Raymond (1978) -- to consider any mathematical text (in the usual sense since mathematics is always written or in the broad sense of a text according to Derrida) as being composed, in a way constituent for itself, by two distinct parts: the “mathematical” part and the “mathematicised” part of the text. In any particular mathematics text and exclusively in relation to that text, its “mathematicised” part constitutes the conceptually appropriated object, while its “mathematical” part constitutes the conceptual knowledge which accomplishes this appropriation. In other words, the “mathematical” part of the text is considered to play the role of theoretical knowledge and the “mathematicised” part is assigned the role of the conceptually appropriated reality. The “mathematicised” in particular, is always an already theorised field, hence variable, which has been constituted as such, either outside any branch of mathematics, or inside someone of them, however devoid of any relations to another branch of mathematics. The “mathematical” on the other hand, may be a mathematical conceptual system already having been constituted as such, which play the role of theoretical knowledge against an assigned as object of its study. The “text” of analytic geometry, as an example, constitutes the conceptual appropriation of the “mathematicised” of the Euclidean geometry by the “mathematical” of algebra.

These two parts of a mathematical text are inextricably interweaved and their definite segregation, in the sense that any particular paragraph or proposition of the text falls exclusively within one or the other of the two parts of the text, is essentially impossible. In addition, the partition of a particular mathematical text in two parts, the one designated as constituting the “mathematical”, i.e., the conceptual knowledge and the other the “mathematicised” part, i.e., the conceptually appropriated object , has to do with that and only that particular text. Neither the particular partition nor the corresponding designations are necessarily preserved in any other mathematical text. According to the case, the part which constitutes the “mathematicised” in a particular mathematical text may constitute the “mathematical” in another mathematical text and conversely. In other words, the designation of a part as the “mathematical” or the “mathematicised” part of a mathematical text refers to a function, which that particular part of the text performs and consequently, it does not reflect any self-existent and definable per se property of the involved mathematical conceptual knowledge or its objects of study.

Let us consider as an example, the concept of circle, as it is actually utilised in the domain of a particular social practice, land-surveying or navigation for instance, with the practical rules which dictate its permissible uses in that particular practice. The concept of circle is being mathematicised after its relative conceptual independence from every aspect of that one, as well as any other particular practice. Such a conceptual independence is equivalent to its transformation in some kind of “object”, which possesses pertinent “properties” and which generates “facts” or “events”, as for instance is the “fact” that, ‘the angle at the centre of a circle is twice the angle at its circumference." Just as, and to the extent that, the concept of circle attains its conceptual autonomy from any particular practice, it becomes susceptible to a pertinent conceptual elaboration. This means that, all the “facts” or “events” which this “object” generates require their conceptual appropriation by a consistent and rigorous conceptual system. A such conceptual system is the above mentioned “mathematical” part of a mathematics “text”, which attains the conceptual appropriation of all “facts” or “events” generated by a mathematical “object”.

From this point of view, the Euclidean geometry may be considered as the first in history "mathematical" which succeeded in the conceptual appropriation of all the "facts" or "events" generated by geometrical concepts, under its corresponding historical conditions.

Many examples may be added to: the "mathematicised" of Newton's fluxions and Leibniz's differentials have been conceptually appropriated by the "mathematical" of differential and integral calculus of Cauchy and his successors, the "mathematicised" of probabilities arisen from games of chance has been conceptually appropriated by the "mathematical" of measure theory, and so on.

The "mathematicised" is not, of course, limited to, or constituted by, the initial and possibly non integrated stages of a mathematicisation. Since the "mathematicised" is not referred to an entity but to a function, which is performed in accordance to a particular mathematical text, any part of mathematics may be functioning as such in relation to another part of mathematics. The earlier mentioned example of analytic geometry is a such indicative case.

On this account, the development of mathematics, as it is well known, is not only owed to an "internal" development of the various mathematical conceptual systems, but equally to an interweaving of these mathematical systems or their parts, as it is derived by their alternation between the functions of the "mathematicised" and the "mathematical" in corresponding mathematical "texts".

Every effort to the conceptual appropriation of any "fact" or "event" generated by a mathematical "object" - identified in the mathematical practice as "proof" - is carried out within a mathematical "text", employing the conceptual system defined by the related "mathematical" part of that "text". The attainment of any usual mathematical "proof" implies the following distinct results (Baltas, 1978).

First, the existence of a "proof" by its own removes the status of hypothesis from a stated proposition and transforms it to an actual mathematical "fact" or "event". Second, the "proof" itself constitutes the conceptual appropriation of the mathematical "fact" or "event" ensured by its own existence within the relevant mathematical conceptual system. Third, the "proof" validates retrospectively the capacity of that mathematical conceptual system to accomplish successfully the conceptual appropriation of the particular "fact" or "event". Finally, the deductive chain composing the "proof" records the actual process, which validates the mentioned capacity of the relevant mathematical system.

In brief, the existence of a "proof" validates by itself the capacity of the relevant mathematical system to appropriate conceptually the specific "fact" or "event" conjectured by the hypothesis, while it is the "proof" itself which constitutes that validation. In this sense, the "proof" of a hypothesised mathematical "fact" or "event" and the validation of the capacity of the pertinent mathematical system to ensure the conceptual appropriation of that "fact" or "event" are coincident.

This is not the case in other disciplines, as for instance is physics or chemistry. In these disciplines, the procedures of any proof are exclusively subjected to the relevant conceptual system, in contrast to the procedures of validation, which are completely subjected to the scientific methods, experimental as a rule, employed to associate the conceptual system with its objects of study. Although, the scientific methods employed by any discipline are, in the last analysis and in some extent, shaped by, and simultaneously shaping both the relevant conceptual system and its objects of study, for that reason being simultaneously material and mental, concrete and abstract.

In mathematics, the conceptual systems and their objects constitute, as commented earlier, just functions of a mathematical "text", alternating their playing roles in the various particular mathematical "texts". Thus, a mathematical conceptual system and its objects share the same fundamental characteristics, which implies that the proof methods of mathematics share also the same fundamental characteristics. Therefore, proofs and validations inevitably coincide in mathematical practice, being in fact, likewise the mathematical conceptual systems and their objects, functions of a mathematical "text". For that reason, whatever constitutes a proof of a mathematical "fact" or "event" in a particular mathematical "text", it may constitute a validating procedure of a mathematical conceptual system in another mathematical "text" and vice versa.

Conceiving the objects of mathematics as being determined by the mathematical discourse, when functioning as "mathematicised" in relation to a "mathematical", then any foundational project of mathematics is in vain, since mathematics are founded on the already "mathematicised", not concerned with sources and the processes which it has acquired from its initial functional status. What have been denoted in the history of mathematics as "foundations", may be considered as simply broad conceptual systems aiming at the conceptual appropriation of all the classes of the registered at the moment and the prospective to be defined mathematical "objects".

For the reasons outlined above, and possibly many more others, it actually can be maintained in agreement with Ernest, that the ontology of mathematics is created by the mathematical discourse. As a consequence, mathematics as a scientific discipline maintains an autonomy and self-sufficiency, being depended on its own pertinent means and procedures for the conceptual appropriation of the objects, which mathematics itself determines as such, as well as for the evaluation of its own theories, means, procedures and products. For the same reasons however, mathematics appeals to, and is a privileged scientific discipline for, Platonism.

References

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