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Crossing the Bridges of Koningsberg in a Primary Mathematics Classroom

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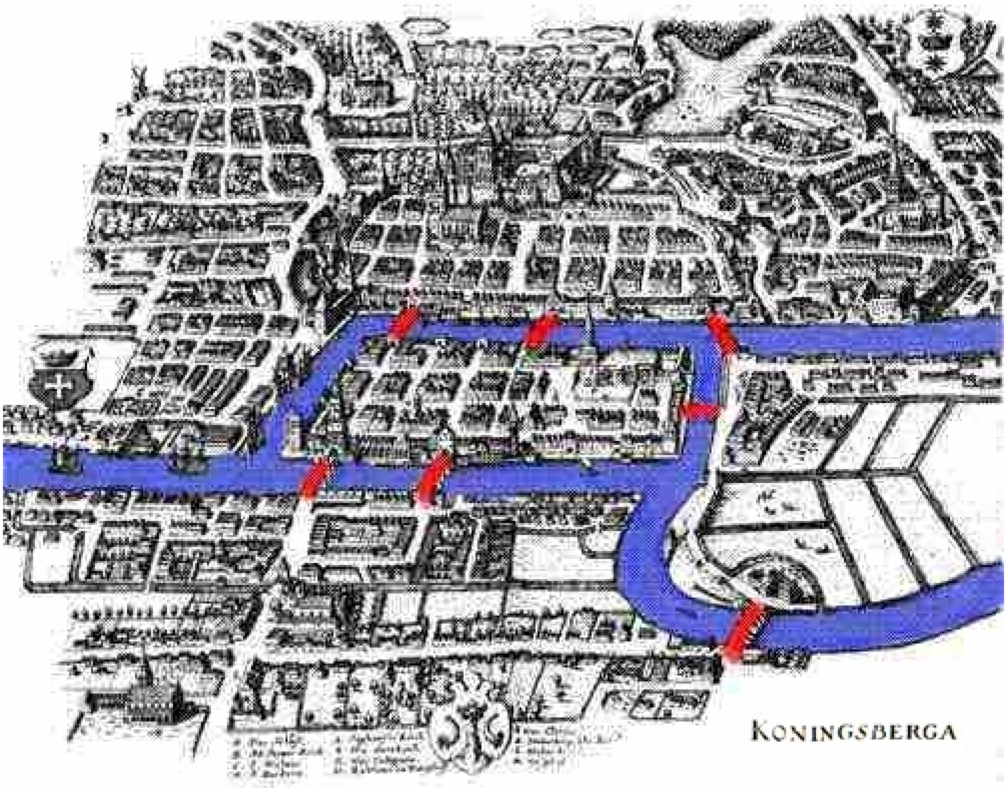
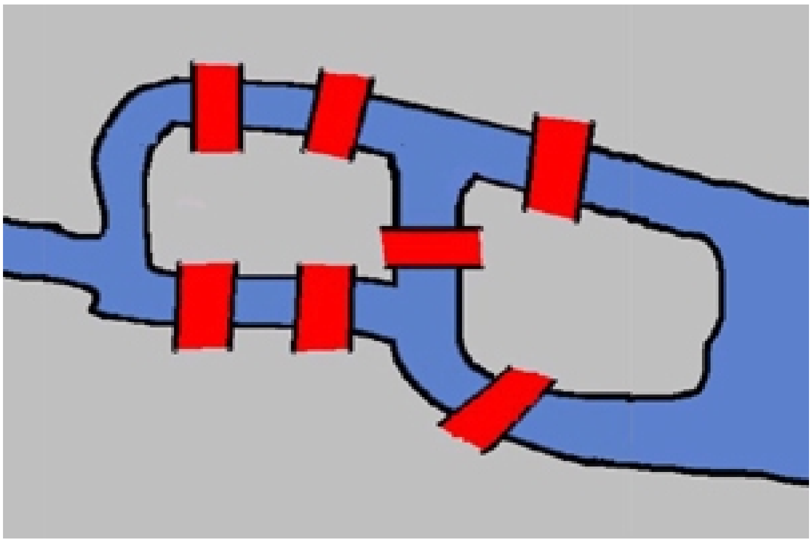


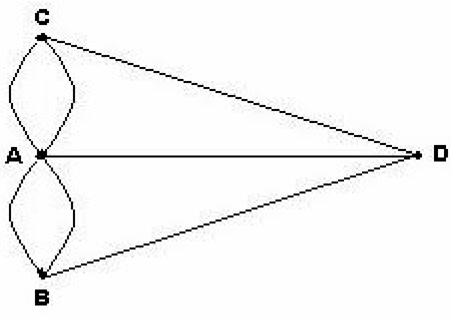
The bridges of Koningsberg

The mathematics classroom activity reported in the following article has been developed in a 6th grade class in a primary school at Thessaloniki, Greece, around a problem from the history of mathematics known as "the bridges of Koningsberg problem". Koningsberg, a small city of old East Prussia (nowadays Kaliningrad in Russia), was built on both banks of the river

Pregel as well as on two islands in the river. Seven bridges connected the four parts of the city to each other.



A frequent topic of conversation in the city of the eighteenth-century was whether or not it was possible for a person to take a journey that starts any place in the city and crosses all seven bridges without having to cross any bridge more than once. Nobody was able to find one, but people kept on trying. Until, around 1735, Leonhard Euler dealt with this problem and showed conclusively that such a journey was impossible, not only for the Konigsberg bridges, but for any network of bridges anywhere. His work on this problem led to the foundation of a whole new branch of mathematics, known as graph theory or topology of networks.



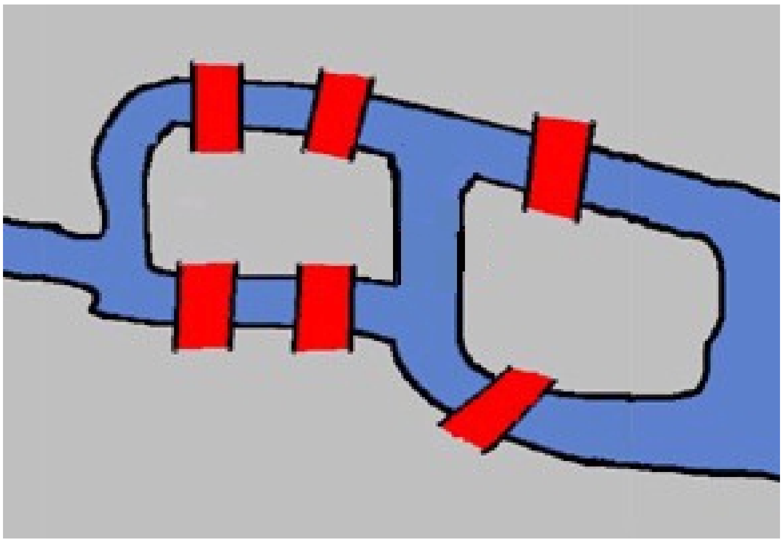
Euler reasoned that, for such a journey to be possible, each part of the city should have an even number of bridges connected to it, so that half the number of them could be used for entering and the other half for leaving that part of the city. Further, if the journey would begin at one part of the city and end at another, then exactly those two parts of the city could have an odd number of connecting bridges while all other parts must have an even number of connecting bridges. All the parts of Koningsberg, however, have an odd number of connecting bridges and a journey across all bridges without crossing any bridge more than once proves to be impossible.

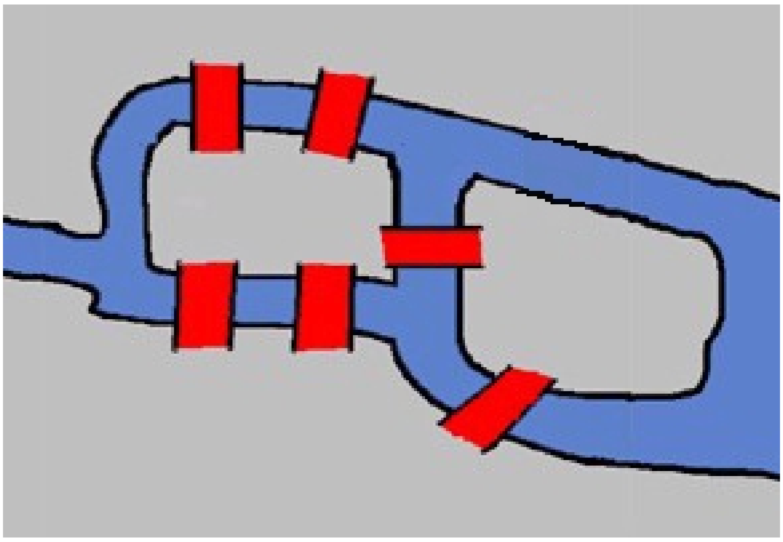
Crossing the bridges of Koningsberg in a mathematics classroom activity

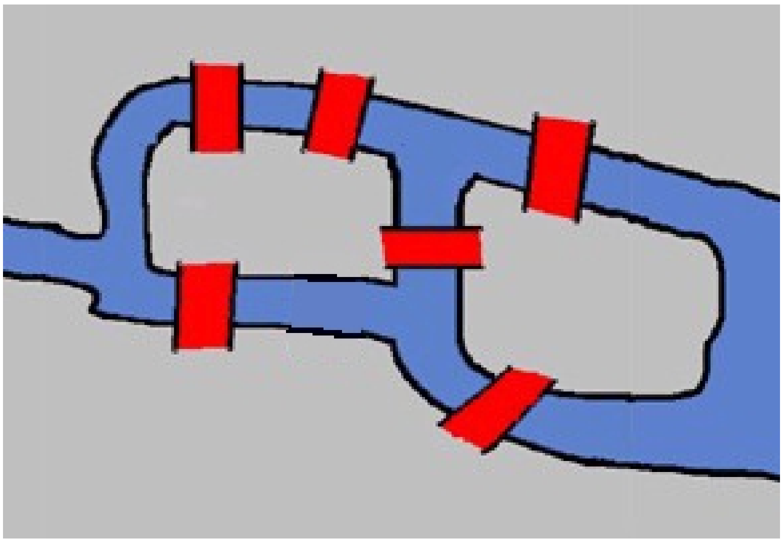
The mathematics classroom activity of crossing the bridges of Koningsberg developed in three stages, each one involving the reformulation of the Koningsberg bridges problem in different terms and consequently the consideration of varying queries, which required from the children a re-orientation of their problem-solving approaches. During each stage of the activity, the children had at their disposal copies of sketch maps depicting the network of the Koningsberg bridges in various versions according to the case. An account of the problem was given to the children by their teacher as the story of a puzzle which was put to the notable Swiss mathematician Leonhard Euler by a group of curious strollers he accidentally met when he visited the city of Koningsberg, years ago. After that, the relevant classroom activity started by inviting the children to participate in the story, investigating on their own the question posed to Leonhard Euler and conceiving an answer.

The first stage of the activity asked for an initial investigation of the question posed by the original Koningsberg bridges problem: can a route be found a journey across all seven bridges of Koningsberg without having to cross any bridge more than once? This question was, after a short probing, clarified by a supplementary query: does the selection of the starting point matter when trying to find such a journey? Question: can the required journey be found if it starts from a particular bank of the river or from a particular island?" The children had at their disposal a sketch map depicting the network of the seven bridges of Koningsberg and tried to trace a journey such as the required one using various starting points. All their efforts ended up in failure since, in every case, either one of the bridges had to be left out of their traced journey or one of the bridges had to be crossed twice. Thus, before long, the children concluded that such a journey could not be found regardless of its starting point and somehow the number of bridges had something to do with this fact.

The second stage of the activity developed from this puzzling situation by further inquiring into the number of bridges: is the number of bridges really the reason for not being able to find a journey as required? That is, if there were not seven bridges in Koningsberg, but one fewer or one more bridge, would it be possible to find a journey across all the bridges without having to cross any bridge more than once? The children were allocated random various sketch maps depicting networks of six bridges connecting the four parts of Koningsberg in different arrangements between them and shortly everyone had traced on his or her map a journey across all the six bridges without having to cross any bridge more than once.







After that, every child was asked to describe the journey which he or she had traced and this process revealed, step by step, the diversity of the solutions found regarding in particular the starting and ending points of the journeys. The same task was repeated for networks of eight bridges connecting the four parts of Koningsberg, again in different arrangements between them. As a result of the former outcome an additional, more clarifying, question emerged which was verbalised by the teacher recapitulating the children’s vague and implicit queries: does the existence of a journey such as the required one across all the bridges of Koningsberg depend on the total number of existing bridges? Or does it depend on their arrangement and therefore on the particular numbers of bridges connecting the four parts of the city?

The third, and final, stage of the activity focused on discussing this question and, under the tactful guidance of the teacher, an inkling of a solution to the original question of the Koningsberg bridges problem emerged. A journey across all the bridges of any network of bridges connecting any number of pieces of land, without having to cross any bridge more than once, is possible in only two cases. Either (a) each piece of land has an even number of bridges connected to it or (b) only two pieces of land, where the starting and ending points of the journey occur, have an odd number of connecting bridges. Neither of these two cases applies to the Koningsberg bridges and thus the required journey is impossible. In the words of one of the children participating in this activity, "… we have here many starting points as well as many ending points and thus we cannot choose one to start and another one to end our journey…..". The solution to the problem was narrated to the children as the final answer, given by Euler, to the puzzle put to him by a group of curious strollers he met, when he had visited the city of Koningsberg, years ago.

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