Proceedings of the 3rd Mediterranean Conference on Mathematical Education
3- 5 January 2003, Athens, Greece
p.p. 409 - 417

GREEK PRIMARY SCHOOLS TEACHERS' BELIEFS ABOUT MATHEMATICAL KNOWLEDGE

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ABSTRACT

The study of teachers' beliefs and values about mathematics, its learning and teaching, has been of continuing interest to researchers world-wide the late thirty years, as it became widely accepted that teachers' beliefs play a significant role in shaping their teaching practices. In this context, the present paper reports on findings from an exploration of the beliefs concerning mathematics held by Greek primary school teachers. Seventy teachers serving in schools in the area of Thessaloniki were interviewed, and their beliefs about mathematical knowledge as a discipline and as a school subject were probed. A narrow vision for mathematical knowledge limited essentially to the subject taught in school; an absolutist conception of mathematics as a discipline considered as a unified body of knowledge involving a set of facts, rules and procedures to be applied; and an individualistic approach to the learning of mathematics were found to be the main characteristics of teachers' beliefs.

INTRODUCTION

Thirty years ago the distinguished mathematician Rene Thom pointed out that '*all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics*' (Thom, 1973, p.204). And few years later Hersh underlined this position emphatically noting that ' *one's conceptions of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it. … The issue, then, is not, What is the best way to teach? but, What is mathematics really about? …Controversies about … teaching cannot be resolved without confronting problems about the nature of mathematics*' (Hersh, 1979, p.33).

Independent from these earlier philosophical positions, the systematic study of teachers' ideas, views, conceptions or beliefs about mathematics as a whole or about elements of mathematics, as well as its learning and teaching, has been of

continuing interest to researchers the years followed. As it became widely accepted that teachers' beliefs and values about mathematics play a significant role in shaping characteristic patterns of their teaching practices (Thompson, 1992).

For some, this relationship between teachers' beliefs and practices has been confirmed by substantial empirical evidence (Thompson, 1984; Frank, 1990; Ball, 1990; Foss and Kleinsasser, 1996), whilst for others is an enquiry on a rather philosophical realm with, inevitably, conjectural outcomes (Lerman, 1983, 1990; Steiner, 1987; Ernest, 1989a, 1989b, 1995).

In any case, the relevant research is important because "*an analysis of belief structures, attention to the intensity with which beliefs are held, and the nature of the evidence that supports beliefs can provide a forum by which our teacher education programs will be better able to address issues of reform*" (Cooney et al., 1998, p. 331). However, at the same time, considering that there exist many conditional or situational factors beyond the epistemological beliefs and conceptions that teachers hold about mathematics that exert influence on teachers' thoughts and practices within classrooms, many of which are neither visible nor easily identifiable. As, for instance, are the values, beliefs and traditions of the education system which are manifested in the adopted curricula and textbooks, in educational practices, in expectations of pupils, parents, colleague teachers and administrators.

In this account, teachers' beliefs about mathematics, its learning and teaching, have been explored in various national contexts around the world and a remarkable body of research has now been accumulated. For example, in Spain (Carillo and Contreras, 1994; Camacho et al., 1998), in France (Nimier, 1986), in England (Andrews and Hatch, 1999), in Germany (Toerner, 1996), in Finland (Pehkonen, 1999), in the United States (Bush et al., 1990; Cooney et al., 1998; Frank, 1990; Foss and Kleinsasser, 1996; Nisbet and Warren, 2000; Stipek et al., 2001; Thompson, 1984), in Canada (Dionne, 1988; Gattuso and Mailloux, 1994; Chapman, 1999; Mura, 1993; 1995), in New Zealand (Irwin and Britt, 1994), in Australia (Crawford et al., 1998; Perry et al. 1999). Comparative studies of the issues have also recently been presented (e.g., Andrews and Hatch, 2000; Pepin, 1999).

In the Greek context, however, the nature of teachers' beliefs about mathematics and its learning and teaching, as well as the influence of those beliefs on teachers' classroom practices, are relatively new areas of study and the relevant research evidence is quite limited. Against this background, the present paper reports on findings from an exploration of the beliefs concerning mathematical knowledge held by Greek serving primary school teachers.

CONCEPTUAL FRAMEWORK

BELIEFS AND THEIR ROLE

The concept of beliefs has many definitions in the relevant literature. Here we take beliefs to be one's subjective knowledge (which also includes affective loadings) of a certain object or concern for which indisputable grounds may not necessarily be found in objective considerations. Beliefs cover personal convictions mixed with facts and external knowledge; thus the beliefs' subjective certainty ranges from truth-like facts to vague assumptions.

As personally held mental constructs, beliefs, unlike knowledge, do not require community consensus or agreement to establish their validity (Nespor, 1987). Knowledge is taken to be built up through intellectual activity: experimentation, debate and reasoning, and is stored in the form of propositions that are open to further evaluation and change. Beliefs on the other hand are not developed through rational thought, but are mental summaries of significant past episodes. An individual's beliefs may fail to exhibit logical consistency, conflicting with each other and with knowledge or the observed world. The power of beliefs to filter new information and colour one's comprehension of events means that once established they are not likely to change even in the presence of contradictory evidence (Nespor, 1987).

Very often, beliefs take precedence over knowledge, shaping the interpretation of presently held knowledge and selectively admitting or rejecting new knowledge claims playing in this way a major role in the organization and interpretation of one's knowledge (Abelson, 1979). Thus it is possible for two persons, possessing similar knowledge, to present distinctly different interpretations of content through their activities, in expressing different beliefs. Thus, despite their looser structure and potential inconsistencies, beliefs are much more influential than knowledge in defining individual behaviour.

The process of how and for what reasons a belief is adopted and defined by the individual is not well understood. The adoption of a belief may be based on some generally known facts as well as beliefs, and on logical conclusions drawn from them. But, in each case, the individual makes their own choice of the facts and beliefs to be used as reasons and their own evaluation of the acceptability of the belief in question. Often for the individual there seems to be no objective distinction between facts and beliefs. The individual compares their beliefs with new experiences and the beliefs of other individuals, and therefore these beliefs are subjected to continuous evaluation and hence undergo change. When an individual adopts a new belief, it will automatically form a part of the larger structure of their personal knowledge and of their belief system, since beliefs never fully develop independently (Green, 1971).

As mentioned above, beliefs always come in sets or groups, which are linked in a belief system, a network of belief clusters, never in complete independence of one another, each focused on a particular situation or facet of life (Pajares, 1992).

A belief system about mathematics contains at least four main, and interrelated, components: (1) beliefs about the nature of mathematical knowledge; (2) beliefs about oneself as a learner and user of mathematics; (3) beliefs about the process of learning mathematics; and (4) beliefs about the nature of teaching mathematics (Ernest, 1989a; Pehkonen, 1994).

These main categories of beliefs, in turn, can be split into smaller units. The category of ‘beliefs about the nature of mathematical knowledge’, which is the focus of this study, comprises a wide spectrum of beliefs, which includes, among others, the following components: (1) beliefs about the origins of mathematical knowledge; (2) beliefs about the nature of mathematical knowledge as a discipline; (3) beliefs about the nature of mathematical knowledge as a subject taught in schools; (4) beliefs about the nature of mathematical problems and tasks; (5) beliefs about the relationships between mathematical knowledge and empirical reality, and in particular about the applicability and utility of mathematical knowledge (Törner, 1996).

BELIEFS ABOUT MATHEMATICAL KNOWLEDGE

Ernest (1989a), arguing from a philosophical perspective, has distinguished three 'psychological belief systems' or conceptions of mathematics:

First of all, there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus, mathematics is a process of enquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view). Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus, mathematics is monolith, a static immutable product. Mathematics is discovered, not created (the Platonist view). Thirdly, there is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external end. Thus, mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalist view). (p.250)

Closely associated with each conception of mathematics is a mental model of learning and teaching mathematics, which Ernest has described in detail using various discriminating constructs (Ernest (1991). In a brief and simplified wording, the problem solving view of mathematics is associated with the learning as the active construction of understanding, possibly even as autonomous problem posing and solving, and the teacher as facilitator; the Platonist view of mathematics is associated with the individual learning as the passive reception of knowledge, and the teacher as explainer; the instrumentalist view of mathematics is associated with the learning as compliant mastery of skills, and the teacher as instructor.

Lerman (1990) identified two alternative conceptions of the nature of mathematics, which he named 'absolutist' and fallibilist', and which, in his view, correspond to two competing schools of thought in the philosophy of mathematics: Euclidean and Quasi-empirical (Lakatos 1978). From the absolutist perspective, mathematics is an immutable body of knowledge based on universal and 'true' foundations, and as such is 'the paradigm of knowledge, certain, absolute, value-free, and abstract'. From the fallibilist perspective, mathematics develops through conjectures, proofs and refutations, and uncertainty is inherent in the discipline. Accordingly, the absolutist perspective of mathematics is associated with the pedagogic model, which is based on the transmission of mathematical knowledge by the teacher in well-defined and replicable ways, in contrast to the model, which is based on the presentation of mathematical knowledge as a social construction learnt through an engagement in problem solving, associated with the fallibilist perspective of mathematics.

Between Ernest's Platonic and problem-solving views and Lerman's absolutist and fallibilist perspectives of mathematics there are obvious parallels.

On the other hand, various researchers have proposed similar distinctions, each from a different viewpoint, in a different framework and, of course, for a different purpose. For instance, Dionne (1984) has described three perspectives of mathematics: Mathematics as a set of skills *(traditional* perspective). Doing mathematics is doing calculations, using rules, procedures and formulae; Mathematics as logic and rigour *(formalist* perspective). Doing mathematics is writing rigorous proofs, using precise and rigorous language and using unifying concepts; Mathematics as a constructive process *(constructivist* perspective). Doing mathematics is developing thought processes, building rules and formulae from experience and reality and finding relationships between different notions. Or,

along similar lines, Skemp (1978) has proposed two conceptions of mathematics accounting for sharp differences in classroom practices: 'instrumental' and 'relational' conception of mathematics. The 'instrumental' conception considers mathematics as a set of 'fixed' plans for performing mathematical tasks involving step-by step procedures, whereas 'relational' conception considers mathematics as a conceptual structure that enables individuals to construct several plans for performing a given task.

Based primarily on the taxonomies introduced by Ernest (1989a, 1989b, 1991) and Lerman (1990), a bipolar conceptual framework for recording and analysing teachers' beliefs about mathematical knowledge has been developed and employed in the study reported here, which is based on the following dichotomies:

(1) Mathematical knowledge considered as: a unified body of knowledge involving a set of facts, rules and procedures that are applied to yield unique, fixed and objectively right answers vs. a tool for problem solving and a set of cultural understandings that arise out of human activity.

(2) Mathematical knowledge considered as: static, fixed, absolute, certain and predictable vs. dynamic, potentially flawed and continually undergoing change and revision.

(3) Mathematical knowledge taught in primary schools considered as: a medium for training children's minds with its logic vs. a valuable knowledge per se.

(4) Mathematical knowledge taught in primary schools considered as: having a primacy over any other knowledge included in the curriculum vs. being equal in rank to any other school subject.

(5) Learning mathematics is considered: an individual issue depended on the inherent faculties of a person vs. an educational issue depended on the school and the adopted teaching methods.

It seems obvious, that the above discriminating constructs constitute a least, indispensable, set of criteria for the categorisation of belies about mathematical knowledge as a discipline in connection with mathematical knowledge as a subject taught in primary schools, and from this standpoint the conceptual framework based on it fulfilled principally a heuristic function in our study.

RESEARCH FINDINGS

Seventy (70) teachers serving in primary schools in the area of Thessaloniki were the subjects of the investigation, which forms the basis of the empirical work reported in this paper. The distribution between males and females were almost equal (33 with 37), their total teaching experience ranged from 5 to 25 years, whilst all the teachers had been graduated from the Pedagogical Academies (higher teachers' training colleges, offering three-years courses, which was operating in Greece up to 1985, when they replaced by university departments).

Data gathered through semi-structured interviews using open questions derived from the aforementioned conceptual framework, so as to elicit as far as possible the teachers' spontaneous thoughts rather than to have them respond to predetermined points of interest. The teachers participated in the study, were approached and interviewed individually at a time suited them. All the interviews were audio-taped and transcribed. Here it must be noted that in these interviews not all teachers answered the questions addressed to them and talked about the mathematical knowledge in an explicit way. Some explained their views indirectly, using examples and personal instances, which in turn gave the researcher indications of their beliefs and conceptions concerning the issues in question.

From the analysis of the evidence collected in this study, four conclusions were generated, which are outlined in the following.

(1) A narrow vision for mathematical knowledge, limited essentially to the subject taught in school, was found to characterise most of the teachers' beliefs.

In fact, very few of the interviewed teachers differentiated between mathematics as a discipline and mathematics as a school subject defined by curricula and textbooks, and talked about the characteristics of each area explicitly, showing a clear understanding of their differences. This confusion traced to permeate the answers and comments, which most of the teachers offered during the interviews of this study. It seems obvious from the relevant data analysis that the teachers had not actually engaged in any debate about the nature of mathematics as a discipline either as part of their pedagogical training or as part of any other in-service course. Thus they are unlikely to have explicitly studied ideas about what mathematics is, hence they are not used to conceive mathematics as a discipline as a field distinguished from the school subject. This confusion is, in my view, a source of many problems concerning mutual understanding between teachers and researchers or policy makers in mathematics education.

(2) An absolutist conception of mathematics as a discipline considered as a unified body of knowledge involving a set of facts, rules and procedures to be applied, was found to dominate most of the teachers' beliefs.

Most teachers' quotes about the nature of mathematical knowledge expressed implicitly a conception, which is close to that characterised in the literature as absolutist. Mathematics is a fixed, predictable, absolute, certain, value-free, culture-free and applicable body of knowledge involving a set of facts, rules and procedures to be used in the pursuance of some external end. Completely compatible with the absolutist perspective is the view that mathematical knowledge is timeless, and the discovery of new theories and truths are just added to the existing ones, also evident in the beliefs of most teachers. In addition, aligned with these beliefs is an image of mathematics as difficult, cold, abstract, theoretical, ultra-rational, but important knowledge, which has been traced in many teachers' quotes. It must be stressed, however, that besides the prevailing conception of mathematics identified as "absolutist", a variety of beliefs about the nature of mathematical knowledge, in most cases contradictory from an epistemological viewpoint, traced within the repertoire of each individual teacher. This is a quite conceivable finding, which has been recorded by most inquiries of teachers' beliefs (Thompson, 1992).

(3) A view that mathematics taught in primary schools is a privileged medium for training children's minds with its logic and for that reason mathematics have de facto a primacy over any other school subject found to be adopted by most of the teachers.

From what teachers mentioned answering questions concerning mathematics as a subject taught by themselves in their schools, it was clear that most of them acknowledged the more transcendent nature of it: mathematics above all trains the mind and develops rational reasoning through logical ways of thinking, as part of its nature. In accordance with this belief was their view that teaching mathematics had a high priority over any other school subject, also expressed by most of the interviewed teachers.

(4) An individualistic approach to the learning of mathematics was found to prevail in most of the teachers' beliefs.

Learning mathematics is considered by most of the teachers interviewed an individual, above all, issue depended essentially on the inherent faculties and the innate abilities, as well on the individual work of each person. A relatively considerable number of teachers supported this view claiming literally that "many students are just not able to learn mathematics' or on the contrary that "many students have a kind of "mathematical mind" and invoking examples from their teaching career. On the other hand, a great number of teachers assigned to the school context (mathematics curricula, textbooks, classroom organisation, teaching methods, system of assessment etc.) an important, although secondary role in the individual learning of mathematics, as well as to the associated social context (social valuation of mathematics, expectations of others including fellow students, parents, peers and teachers, etc.). For this group of teachers the relationships between individual and social factors in learning mathematics seems to be quite vague.

A CONCLUDING COMMENT

Associated to mathematics, regarded as an extremely complex world of experience and action, there is an equally complex and diverse world of beliefs about mathematics. A system of beliefs which, subject to the constraints and contingencies of the school context, regulates and shapes the mathematics teaching practices. Thus, it may be conjectured that the cluster of beliefs concerning mathematical knowledge outlined above, which prevail among the Greek primary school teachers' participated in this study, could potentially determine, in multiple ways and in various degrees, the outcomes of any attempted innovations in the mathematics classroom. Therefore, the appropriate change of individual teachers' beliefs about mathematical knowledge is a prerequisite of any relevant reform. However, any changes in the teacher's system of beliefs concerning the nature of mathematics, as well as the teaching and learning of mathematics, is neither an easy nor a simple venture. Moreover, as Ernest (1989a, p. 249) put it "*these changes in beliefs are associated with increased reflection and autonomy on the part of the mathematics teacher*".

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