The mediation of hand-held instruments vs. dynamic geometry software in the formation of geometric concepts of perpendicular and parallel lines by children in two different cultures

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Proceedings of the International Scientific Conference eRA-7, Athens GR, 2012 (in press)

Abstract

The reported study adopts a theoretical perspective on human thinking developed by Vygotskian socio-cultural school and focuses on the process by which children appropriate formal geometry concepts of perpendicular and parallel lines within the context of a teaching experiment carried out in two culturally and educationally different environments. Four groups of children attending the 4th class of primary school and the 1st class of lower secondary school in Greece and four groups of children attending classes at corresponding school levels in Jordan (a total of approximately 130 students in both countries) were posed a series of geometry tasks asking them to identify, define and construct parallel, perpendicular and intersecting at various angles lines on a plane in two different situations. Half of the students in each group were posed and carried out the tasks on a squared paper having at their disposal the traditional hand-held geometric instruments of ruler, protractor and right-angled triangle as well as a pencil whilst the other half of them encounter the same tasks on a computer using the dynamic geometry software of GeoGebra4 after having introduced to it. Our study paid attention on the qualitative differences induced by the two types of tools adopted in the teaching and learning of parallel and perpendicular lines and to the extent to which these different types of tools contributed to the appropriation of geometric concepts. The data collected and analyzed adopting an analytic framework based on Van Hiele levels of geometric thinking. The main findings of this analysis indicate that the use of geometric tools either material or computerized have an overall positive influence on the formation of analytical concepts of parallel and perpendicular lines on a plane by children in both cultural and educational environments of Greek and Jordanian schools. On the other hand, the use of computer and dynamic geometry software, despite its tools offered and their functional capacities, they seem not to structure in a radically different fashion than traditional hand-held instruments the activities of indentifying and defining parallel and perpendicular lines on a plane bringing into play qualitative different thinking and acting processes concerning the appropriation of the fundamental geometry concepts that were investigated in this study.

Key-words: teaching-learning geometry, dynamic geometry software, Geogebra

1. Theoretical background

1.1 Drawing instruments in geometry

Geometry has been founded on the use of instruments with particular emphasis in compass used for drawing circles and arcs and straightedge used as a guide for the pencil when drawing straight lines. Euclid's "Elements", the founding text of geometry, implicitly defines and clearly theorizes the use of drawing instruments both for defining geometry concepts and for solving problems by proper geometric constructions, although the instruments are never directly quoted.

Compass and straightedge as well as their rules of use correspond to axioms and theorems of Euclidean geometry and for any given geometric construction there is a theorem stating the relationships between the elements of the geometrical figure represented by the drawing produced [1]. The radical transformation of geometry from the classical static constructions restricted by the use of compass and straightedge to geometric investigations resulting from mechanical motions, which took place during the 17th century, raised in the novel context the issue of drawing instruments and devices. During the followed 18th and 19th centuries drawing instruments of many types were designed and adopted in geometric investigations. The development of computer technology and dynamic geometry software renew the interest for the tools used in geometry, although from a qualitatively new viewpoint.

The crucial role of using drawing and measuring instruments in learning geometry is supported by the Vygotskian perspective of tool mediation [2]. A central claim of this perspective is that children's mental functioning and development can be accounted for in terms of their engagement in culturally organised practices in which technical and symbolic tools play a crucial role. Such tools have been developed in a culture over extended periods of time and have become an integral part of human activity. By acting as mediators, technical and symbolic tools, structure human activity and bring into play differentiated mental processes which in turn regulate and qualitatively transform that activity. Mediatory means, thinking processes, and human activities become functionally intertwined in their development, shaping each other in a dialectical interdependence.

In this account it is assumed that the use of different types of mediatory means structures practical activities in different ways and hence has a differentiated impact on thinking and, consequently, on the genesis of concept-appropriating processes [3].

1.2 Concepts of perpendicular and parallel straight lines

Children are taught in school geometry both in Greece and Jordan, that two straight lines are perpendicular to each other if they form congruent adjacent angles, i.e. if they are at right angles (90°) to each other. Therefore, it may be easily verified if two straight lines are perpendicular or not by using a right-angled triangle (a modern version of gnomon) or a protractor.

On the other hand, school geometry includes three equivalent but different definitions of parallel lines on a plane. First, two straight lines on the same plane are parallel if they are the same distance apart at any given point. Second, two straight lines are parallel if they do not intersect even assuming that they extend to infinity in either direction. And third, two straight lines on a plane are parallel if they share a transversal line through a point that intersect them at right angles, i.e. if they are both perpendicular to a third straight line, or more generally, if they form equal the corresponding angles of intersection with a transversal line.

Therefore, it may be claimed that two straight lines on a plane are parallel or not by measuring their perpendicular distance at any two points using a ruler or by inspecting if they are perpendicular or not to a third line or if they form equal the corresponding angles of intersection or not with a transversal line using a right-angled triangle or a protractor.

2. The teaching experiments

This study focused on differences in children's thinking impacted by the use of hand-held instruments vs. dynamic geometry software in a series of tasks asking to identify, define and draw parallel, perpendicular and intersecting lines on a plane. The inquiry was undertaken within the context of a sequence of experimental situations in two culturally and educationally different environments during the school-year 2011-12. Four groups of children attending the 4th class of primary school and the 1st class of lower secondary school in Greece and four groups of children attending classes at corresponding school levels in Jordan (a total of approximately 130 students in both countries) participated in the study.

These students, who had been taught before our experiments in their regular math classes the concepts of parallel and perpendicular lines on a plane, were posed a series of tasks asking to identify, define and construct parallel, perpendicular and intersecting lines on a plane in two different situations.

Half of the students in each group were posed and carried out the tasks on a squared paper having at their disposal the traditional hand-held geometric instruments of ruler, protractor and right-angled triangle as well as a pencil whilst the other half of them in each group encounter the same tasks on a computer using the dynamic geometry software of GeoGebra4, after having introduced to it by the researcher.

GeoGebra4 was selected from the pool of available software packages for mathematics teaching and learning for many reasons, the main being that it is available both in Greek and Arabic language and so it could be used in the two cultural and educational environments of our study, it could be utilized in the grade levels of primary and secondary schools selected for our teaching experiments and its users can use the software intuitively without having advanced computer skills [4].

In the first stage of our study reported here, the experimental tasks required the identification, definition and reasoning on the relationships of straight lines on a plane with particular focus on parallel and perpendicular lines. In these tasks the students were given figures of two lines drawn on a squared grid and asked to identify their relationship (perpendicular, parallel or intersecting) and to justify their responses. In the justification part of each activity the students were asked to provide evidence for their claims so as their conceptions of perpendicular and parallel lines to be deduced. Attention paid on the qualitative differences induced by the two types of tools used in identifying and defining parallel, perpendicular and intersecting straight lines and to the extent to which these different types of geometry tools contributed to the appropriation of these geometric concepts.

The data of the study are children’s responses to the tasks and their interviews which registered their voices as responses to the researcher’s questions as well as the researcher’s notes. The data were analyzed adopting an analytic framework based on Van Hieles’ levels of geometric thinking enhanced by observations concerning the utilization of geometric instruments by children in offering and supporting their claims [5].

3. Key findings

The key findings of our analysis indicate that the use of geometric tools had an overall positive influence on the formation of analytical concepts of parallel and perpendicular lines by children in both cultural and educational environments of Greek and Jordanian schools. The percentages of children who based their arguments about the relationships of two straight lines on

the use of a suitable geometric tool increased significantly after our teaching experiments, which involved discussions between children and researcher on the characteristic properties of perpendicular and parallel lines. On the other hand, the use of Geogebra4, despite its dynamic computerized tools and their functional capacities they seem not to structure the activities of indentifying and defining parallel or perpendicular lines in a radically different fashion than traditional hand-held instruments. According to our evidence, the radically different type of dynamic geometry software tools they did not brought into play qualitative different thinking and acting processes at the level of appropriating the fundamental geometry concepts investigated in this study.
The following extracts of children's responses to a task are illustrative. Children having at their disposal a ruler, a protractor and a right-angled triangle as well as a pencil were asked to identify the relationship of two lines and to justify their responses (fig. 1).
The majority of the children both in Greek and Jordanian schools claimed that the two lines are intersecting based exclusively on a visual appreciation. Asked by the researcher to justify their claims most of the children picked up the ruler and - ignoring its equally spaced markings along its length for measuring a distance - use it as a straightedge extending the two lines so as to be clearly visible that they intersect at a point on the paper sheet.

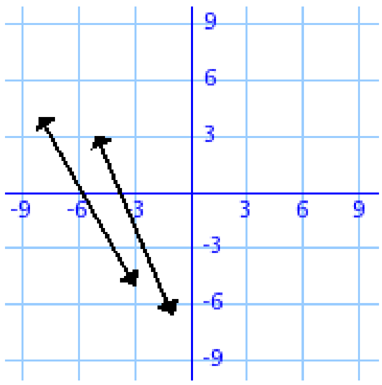

Fig. 1: The two lines are…. parallel, perpendicular or intersecting at an angle?

Very few children in both educational levels and environments of our experiments used the ruler and measured the distance between the two lines in order to verify their claims that the two lines are intersecting.

The same task was posed on a computer to the children who asked to respond using the tools offered by the dynamic geometry software of GeoGebra4 (fig. 2). In previous sessions of this experimental activity the children had been introduced to the tools and functions of Geogebra4 by the first of the researchers.

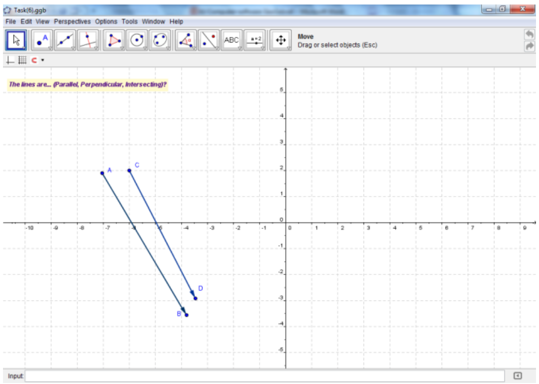

Fig. 2: The two lines are…. parallel, perpendicular or intersecting at an angle?

Most of the children in Greece and Jordan in both educational levels - although using the tools offered by Geogebra4 and their functions - responded to the task in a rather similar way to the children who used the hand-held geometry instruments and a pencil on a sheet of paper.

They initially claimed that the two lines are parallel on the basis of their visual appreciation. After that asked by the researcher to justify their claims they used the “Line tools” of Geogebra4 (option “Line through two points”) to extend the two lines in both directions and the “General tools” (option “Move graphics view”) to change the zoom of the graphics view so as to become visible that the two lines intersect in contrast to their visually based presumption.

That is, the children in this experiment simply replaced the material hand-held ruler by the line drawing functions offered by Geogebra4 without structuring their activities of indentifying and defining the relationship of the two lines in a radically different fashion than that of using the traditional ruler and pencil on a paper. The only difference was the use by the children of zoom facility offered by Geogebra4 in order to verify without any doubt, their claims that the two lines are intersecting or not. Verification visually appreciated by children and mathematically affirmed by the software utilities. It is interesting that few children used the “Measurement tools” of

Geogebra4 to measure the angle formed by the two intersecting lines in order to offer an additional justification of their claims and fewer used the same tool to measure the perpendicular distance between the two lines at any two points.
In conclusion, on a first level children used the drawing facilities offered by Geogebra4 to facilitate the material aspects of the geometry tasks, i.e. the extension of lines or the measurements of distances and angles, while they did not change the tasks conceptually and on second level they utilize Geogebra4 as a visual amplifier in identifying the relationships of two lines on a plane, since it is easier and more reliable to observe that two lines intersect or not in one point by zooming the diagram using the graphics view tool of the software than in a static paper-and-pencil diagram. In any case, children's responses to the tasks seem not to be affected by the dynamic geometry environment.
Summing up, it may be claimed that the children responding to tasks have not exhibited significantly different thinking processes and acting behaviors both in the two qualitative different experimental situations and in the two educationally and culturally different environments.

4. Conclusions

Two conclusions may be drawn from an analysis of the above reported findings. First, it seems that the instrumental approach suggested by Verillon and Rabardel, which distinguish artifacts (technical and conceptual tools) from instruments is endorsed by the findings of the present study. According to Verillon and Rabardel, an artifact is a material or abstract object, already produced by human activity, which aims at supporting new human activity in carrying out a type of task (e.g. a ruler or an algorithm for solving equations). An artifact is given to a human subject while an instrument is built by the subject from the artifact. An instrument may be psychological or material, but above all, is subjective, linked to a subject's activity and developed by the subject for responding to a given task. The transformation of an artifact to an instrument, a complex process so-called instrumental genesis, is linked to characteristics of the artifact and to the subject's activity, her/his knowledge and former working methods. The process of instrumental genesis has two components, the first one directed toward the artifact which is shaped by the users' activity and the second one directed toward the subject, whose activity is shaped by the artifact. In this process, a subject in order to perform a task constructs an instrument, which is composed of both artifact and subject's utilization schemes which allow her/him to perform the task and control her/his activity [6].

According to the evidence of this research, children have not transformed conceptually the available material and computerized geometric tools to proper geometric instruments for responding to tasks concerning the relationships of two lines on a plane. An explanation for this deficit may be sought to the practices of incorporating geometric tools in the teaching of geometry in schools both in Greece and Jordan.

Our second conclusion concerns the complexity of geometrical thinking and the corresponding cognitive demands required by children. Duval suggests that three types of cognitive processes are involved in geometrical reasoning: visualization processes supporting the visual representation of a geometrical statement, construction processes related to the use of geometrical tools and reasoning processes making possible the extension of knowledge, explanation, argumentation and proof [7]. As Duval points out, these different processes can be performed separately, however, are closely connected and their synergy is cognitively necessary for proficiency in geometry. The findings of our research indicate that children's geometric reasoning is based primarily on visualization processes, which are not connected to construction and reasoning processes related to the identification and definition of perpendicular and parallel lines on a plane.

Given that the synergy of visualization, construction and reasoning processes is necessary for appropriating geometry concepts and theorems, an important issue is their integration to the incorporation of tools – material and computerized- in the teaching and learning of geometry.

In summary, whatever the tools, their integration in geometry learning and teaching necessitates considerations of children's instrumental genesis in terms of the cognitive processes which are involved in geometrical reasoning, and especially in their synergy.

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