A FRAMING OF THE WORLD BY MATHEMATICS: A STUDY OF WORD PROBLEMS IN GREEK PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

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Assuming that mathematical concepts, beyond their mathematical meaning, acquire multiple referential meanings whenever applied to different real-world situations, it is claimed in this paper that the word problems included in mathematics textbooks convey specific social values and patterns of thought, assigning relevant contextual meanings to the mathematical concepts employed. Evidence to support this claim is briefly reported, drawn from an analysis of the word problems included in two sets of primary school mathematics textbooks which have been used in Greece in the last twenty-five years.

INTRODUCTION

Mathematics textbooks play an important role in mathematics education as they not only identify and organise the mathematical content of classroom teaching but also actually structure classroom lessons with examples, exercises, problems and activities. They, therefore, may be considered to provide a particular interpretation of mathematics to teachers, students and their parents.

In Greece, as possibly in many other countries, the teaching of mathematics in primary and secondary schools is almost exclusively based on the use of textbooks. Therefore, it may be claimed that textbooks define that is called "school mathematics" as well as determine what is known as the "learning path" for the majority of students.

A typical organisation of content found in most mathematics textbooks involves three parts: exposition, examples and exercises, the latter in many forms (Love & Pimm, 1996). The exposition part is intended to support students' learning of mathematics concepts and techniques which are taught by the teacher in the classroom. The examples either pave the way to mathematics concepts within a rationale of 'guided discovery' or follow the exposition part as prototypes to be copied by the students answering exercises and solving problems posed in the third part.

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The third part accommodates exercises and problem solving tasks, sometimes graded in order of difficulty. Exercises and problem solving tasks are quantitatively dominant in many mathematics textbooks (Pepin & Haggarty, 2001) and among them word problems represent a common way in which "real world" contexts are introduced into school mathematics. A word problem is defined as any "verbal description of problem situations wherein one or more questions are raised the answer to which can be obtained by application of mathematical operations to numerical data available in the problem statement. In their most typical form, problems take the form of brief texts describing the essentials of some situation wherein some quantities are explicitly given and others are not and wherein the solver - typically a student who is confronted with the problem in the context of a mathematics lesson or a mathematics test – is required to give a numerical answer to a specific question by making explicit and exclusive use of the quantities given in the test and mathematical relationships between those quantities inferred from the text." (Verschaffel et al, 2000, p. ix).

Word problems may also combine a written text with other kinds of information, e.g. a table, a picture, a drawing.

Word problems have been extensively problematised and their many dimensions have been analysed from a variety of perspectives, including mathematical (e.g. Verschaffel et al, 2000), linguistic (e.g. Gerofsky, 1993), psychological (e.g. Lave, 1992) and sociological (e.g. Cooper and Dunne, 2000). Considering word problems as the main vehicle for introducing real world contexts into school mathematics, then the type and kind of real life situations selected and used to frame the mathematical questions posed by these problems is of crucial importance, particularly as a topic for perceiving the ideological influences that mathematics teaching exerts on children conveying specific social values and patterns of thought. At the same time, however, mathematics is considered as a politically and ideologically neutral discipline.

As an example, it is claimed in this paper that the word problems included in primary school mathematics textbooks used in Greece assign value-laden meanings to the mathematical concepts employed; research evidence for supporting this claim is briefly reported.

CONCEPTUAL AND REFERENTIAL MEANING OF MATHEMATICS

As has been elsewhere analysed (Chassapis, 1997), each mathematical concept acquires its meaning by a particular mathematical theory in which it is embedded. This is its conceptual meaning, assigned by the propositions of a particular mathematical theory. For example, the concept of addition for natural numbers is defined recursively by the Peano axioms (i) $a + 0 = a$, and (ii) $a + Sb = S(a + b)$, where Sa is the successor of a. Accordingly in set theory addition is defined by the cardinality of the disjoint union and in any other kind of mathematical structure is defined in terms of its propositions. Mathematical concepts, however, are used to describe or are "applied" to the real world on the basis of a mapping between them and real world situations.

Every such mapping is indispensably mediated by a class of non-mathematical concepts that circumscribes the real world situations to which the mathematical concept is applied as well as by a set of pertinent linguistic or more generally symbolic expressions that signify these concepts. These non-mathematical concepts and their symbolic expressions assign another non-mathematical meaning and simultaneously specify a reference of the mathematical concept to a particular description or application. This meaning is the contextual or the referential meaning of a mathematical concept. Number addition, for example, may be used to describe or may be applied to a class of real world situations circumscribed by concepts of change, combine or compare. Particular instances of these concepts (e.g., the growth of a quantity as an instance of change, the union of two quantities as an instance of combination or the difference of two quantities as an instance of comparison) together with their signifying linguistic expressions attribute to the mathematical concept of addition various referential meanings, according to the case, which are not in their every aspect identical.

The referential meaning of a mathematical concept beyond its practical sources is related to the values associated by the people employing it in their everyday activities and/or by various communities using mathematics applied to their practices. This second aspect is socially and historically determined, since one mathematical concept can be valued in one context and de-valued in another, while its value in the same context can change over time due to social changes. Adopting the view that real world situations acquire their meanings by the implicated human activities which are always meaningful since intentional, it may be claimed that any real world situation and its representations bear meanings that are never value-free nor ideologically neutral. One step further, it may be claimed that the selected real world situations and consequently the associated references of the mathematical concepts to specific aspects of the real world used as examples, applications, questions or problems-to-be-solved in the teaching of school mathematics are never value-free and bear - in any feasible case - a more or less definite, even if not clear, ideological orientation. They thus assign to the mathematical concepts analogous, ideologically oriented, referential meanings. The ideological orientations of the referential meanings assigned to number addition, for instance, when applied to, and interpreted as describing, a growth process of profit in a situation of commercial dealings or a growth process of nuclear waste in a situation of environmental pollution are not identical. The two situations highlight different aspects of human activity, and implicitly emphasise different attitudes and patterns of thinking towards human activities, support different life values and ultimately transmit different social ideologies.

From this point of view, school mathematics, just as many other school subjects, may not be considered as an ideologically and hence socially neutral subject of knowledge, derived from a similarly neutral scientific mathematical activity. It has to be conceived as a school subject composed of selected mathematical topics bearing ideologically oriented referential meanings assigned to mathematical concepts and tools by their selected mappings in selected applications to selected real world situations.

AN ANALYSIS OF EXAMPLES AND WORD PROBLEMS INCLUDED IN GREEK PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

Two sets of textbooks used in Greek primary schools for teaching mathematics were analysed. The first set was adopted and used from the school year 1982-83 until 2005-6 (hereinafter referred to as set A) and the second from the school year 2006-7 to the present day (hereinafter referred to as set B). Each set includes one textbook for each of the six primary school grades being the unique textbook for every primary school in Greece, distributed free of charge to the students according to relevant legal provisions. These textbooks follow the mathematics curriculum which is centrally prescribed in detail by the Ministry of Education and includes the study of natural numbers and fractions, their fundamental operations and associated algorithms, basics of measurement, basic geometric relations and shapes as well as handling and presenting numerical data. Exercises and problems are emphasised for each topic while references to everyday activities are dominant in both sets of mathematics textbooks. Every mathematics textbook is accompanied by a teacher's manual dictating in detail every teaching unit, its content, teaching method and learning tools. Since the curriculum, the teaching materials and methods are centrally designated and controlled, mathematics teaching in Greek primary schools may be considered as uniform in any of its main aspects.

The related question encountered in this paper is twofold. First, which are the prominent characteristics of the real world situations prevailing in the Greek primary mathematics textbooks as references of mathematical concepts to real world and second, which is the social ideology, if any, advocated by the meanings of these references? In other words, which aspects of the real world are selected and in which patterns are they structured and nominated by the Greek primary mathematics textbooks as the prominent real world objects of mathematical activity.

For the analysis of textbooks a technique of textual analysis was employed, comprising two steps. First, all worked examples and problems included in mathematics textbooks were sorted out and classified into categories according to their contexts and the activities described or referred to. The outcome of this first step of quantitative analysis is outlined in Table I.
Second, the subjects referred to as actors in word problems and examples as well as the material objects manipulated by these subjects or simply involved in calculations were spotted and counted. Due to space limitations in this paper only the most significant data from this analysis will be reported.

The findings of these analyses suggest that in the word problems included in mathematics textbooks three classes of situations are privileged as references of mathematical concepts to the real world, in almost equal share in both sets of textbooks:

Table I: Context of examples and problems by textbook set and school grade

Context	Textbook set	School grade					
		1st	2nd	3rd	4th	5th	6th
Counting -Measuring	A	22 73%	38 53%	72 40%	38 15%	87 25%	27 7%
	B	48 63%	47 27%	31 27%	60 30%	58 31%	101 31%
Buying - Selling	A	6 20%	22 31%	58 33%	92 36%	37 10%	110 30%
	B	14 18%	45 25%	28 25%	53 27%	45 24%	83 26%
Production-Consumption	A				13 5%	30 8%	39 10%
	B			5 4%	18 9%	2 1%	15 5%
Income-Expenses-Salaries-Taxes-Deposits-Insurance	A				8 3%	1 1%	7 2%
	B						
Sharing	A		9 12%	7 4%		12 3%	
	B	5 7%	10 6%				
Craft - Manufacturing-Construction- Household-calculations	A		3 4%	35 20%	51 20%	84 24%	92 25%
	B		9 5%	25 22%	31 16%	36 19%	79 24%
Trip & travel expenses	A				32 13%	31 9%	21 6%
	B		7 4%	9 8%	11 6%	30 16%	38 12&
Game scores	A			6 3%	19 8%	11 3%	7 2%
	B	5 7%	15 8%	15 13%	14 7%	18 10%	9 3%
Drawing-Graphing-Geometry	A	2 7%				60 17%	68 18%
	B	4 5%	44 25%		12 6%		
Total of problems & examples	A	30	72	178	253	353	371
	B	76	177	113	199	189	325
Total of problems & examples out of any context	A	24 80%	38 53%	72 40%	38 15%	147 42%	95 26%
	B	52 68%	91 51%	31 27%	72 36%	58 31%	101 31%

Total of problems & examples framed in a real world context	A	6 20%	34 47%	106 60%	215 85%	206 58%	276 74%
	B	24 32%	86 49%	82 73%	127 64%	131 69%	224 69%

(1) First is a class of financial and, especially, commercial situations devoid of any pertinent social relationships. Buying and selling of commodities, money returns and payments, business profit accounts, individual incomes and expenses, consumption bills, funds debit and credit, tax payments, insurance and related transactions compose the prevailing references of the mathematical concepts.
For example:

"You have 20 Euro. How many will be left over if you buy a T-shirt costing 15 Euro?" (1st grade, textbook set A)

"Mark wants to buy balloons for his birthday costing 3 Euro each. He has 25 Euro. How many balloons may he buy? Will he have any money left over?" (3rd grade, textbook set B)

"The nearby stationary shop sells notebooks for 30c each. George paid 90c and John 60c for buying some of them. How many notebooks did each one buy?" (3rd grade textbook set A)

"One chair constructed by a cabinet-maker cost 35.22 Euro and it sold with a profit of 8.8 Euro. How much was it sold for?" (6th grade textbook set A)

Even calculations of interest on capital deposits and loans are included as a distinct teaching unit for the 6th grade in the set A of mathematics textbooks, containing problems as the following:

"A farmer took a loan from a bank which is payable in 10 months at a rate of 10%. What amount ought he to pay back to the bank?" (6th grade, textbook set A)

"Mrs George has deposited today at a bank an amount of 293.5 Euro at a rate of 22% for 18 months. The interest is added to the capital at the end of every six-month period. How much will she have after 18 months and will it suffice to buy an appliance which is on sale for 836.4 Euros?" (6th grade, textbook set A)

Many of the financial transactions described in mathematics textbooks derive from a socially abstract material production and distribution, which is introduced in the word problems by statements of the type "a factory constructs", "a farm produces", "a store sells", or "bottles are packed", "apples are sold", "drinks are canned" etc. All these appear as of being on their own, in the absence of any human agent, out of any spatial, temporal or social structure. On the other hand, whenever any person is participating in such a situation, it is indicated by an occupational identity (e.g. a bookseller, a grocer, a confectioner, a craftsman), by a position in a division of labour (e.g., an employer, an employee, a manager, a producer) or is presented as a socially indefinable agent (e.g., a man, a woman, a buyer, a seller). In any other case, the student is assigned the role of an actor in financial transactions being addressed by the personal pronoun "you", a speech act inciting participation both in mathematical and even in imaginary financial activities.

Furthermore, an interesting feature of many problems of this class of situations is the confusing use of economic terms, especially those concerning "values", such as "cost" and "price" of goods or services, which are always used interchangeably, thus obscuring fundamental aspects of commercial reality. For instance:

"You have 20 Euro and you buy a t-shirt that <u>costs</u> 15 Euro. How much money will you have left?" (1st grade, textbook set A) and *"Apples <u>were sold</u> for 78c per kg last year and this year their price increased by 12c per Kg. How much are they sold per kg?"* (3rd grade, textbook set A)

(2) Second, mathematical concepts are referred to a class of situations, similar in its main characteristics to the previous class, however requiring calculations concerning various construction, manufacturing or household activities. Coins constitute an indispensable element of this as well as the previous class of situations and they are indirectly ascribed an almost natural existence. Their manipulation is introduced as a prominent reference of numerical activities from the first teaching units of the primary school mathematics onwards, as may be concluded from the data shown in Table II.

Table II: Problems including manipulation of coins

textbook set	School grade					
A	1 4%	14 19%	70 33%	108 38%	16 6%	115 30%
B	9 13%	41 26%	30 31%	44 27%	46 29%	75 32%

Coins are used even in quite unrealistic cases as in the following problem posed as an application of decimal fractions.:

"You had 8/10 of a ten-drachma coin and spent 5/10 of it. How many tenths of the ten-drachma coin have you got left?" (4th grade, textbook set A).

A marked division of activities by sex is also clear in the problems classified in this type of situations. Men are crafting, constructing, building, while women are cooking, sewing, knitting, typing and sometimes calculating expenses either within or outside their home. For instance

"The 15 girls of the 5th grade of a school bought 9.75m of fabric in order to make embroideries of the same size. How much fabric did each girl use?" (5th grade, textbook set A).

(3) Third, word problems refer to socially indefinite or artificial situations coming out of an ostensibly abstract, and thus neutral, world. A world made up of material objects, (playthings, clothing, pieces of furniture or everyday objects), plants (flowers, vegetables or trees), fruits (apples, oranges or bananas) and animals

(birds, dogs or cats), which being as a rule out of any meaningful context are unmediated objects of mathematical manipulations, usually incited by the question "How many?" or "How much?". For instance, the problems included in textbooks for the 1^{st} grade refer mainly to toys (43% of total problems in set A and 33% in set B) and foods and fruits (26% and 28% respectively).

The persons, if any, involved in this class of situations, required to count, measure, compare or share objects are referred to by personal names (e.g., George, Helen), family relations (e.g., mother, brother, sister), sex identity (e.g., a girl, a boy), membership of a group (e.g., students, passengers, spectator) or personal pronouns (me, you, we). All these activities are, as mentioned, detached from any social setting and their exclusive aim of exercising makes their context obviously superfluous. For example

A window had 8 panes but 3 of them broke. How many panes were left? (1^{st} grade textbook set A)

Summing up, it may be claimed that the commercial market is directly endorsed by the Greek primary school mathematics textbooks as the prominent field of mathematical activity and indirectly as a dominant aspect of the social world. In addition commercial transactions are nominated as the dominant mode of human activity while commodity production and distribution are abstractly presented as impersonal and socially neutral, therefore as essentially technical, activities. Such options are not directly imposed by any learning requirement of primary mathematics per se and therefore these may not be considered as ideologically neutral options. Intentionally or not, the authors of both sets of the analysed textbooks, using the aforementioned real world applications of mathematics, reinforce and promote a particular image of worthy modes of human activity and thus a particular conception of society. The following problem, albeit ridiculous, is an exemplary culmination of this point of view:

"Bill Gates, the owner of Microsoft Company, had during the year 2003 an income of 5c for every second of the time. How much money did he make in one minute, one hour, one day, one month and one year?" (5th grade, textbook set B).

CONCLUDING COMMENTS: FRAMING THE WORLD THROUGH MATHEMATICS

School mathematics word problems framed in real world contexts play a mediating role between mathematics per se and real world situations, suggesting and in most cases creating templates for "reading" mathematically the objects and events of the world. In such a context the role and function of mathematics word problems may be understood from the viewpoint of what Goffman (1986) calls the "frame" of a (social) situation. Frame is primarily a psychological concept that refers to the cognitive process wherein people bring to bear background knowledge to interpret an event or circumstance and to locate it in a large system of meaning (Oliver & Johnston, 2000).

In Goffman's perspective, the concept "frame" implies that there is a definition of a situation which the participants share and most of them take for granted. A frame can be seen as the participants' shared response to the question "what is going on here" (Goffman, 1986, p. 18), which means that they have construed events, actions or utterances in line with the frame which they perceive as relevant.
Frames are basic, individual, cognitive structures, which guide the perception and representation of reality or, put in other words, frames structure which parts of reality become noticed. Frames select and organise information drawn from real experiences and about people and objects and which are actually in the world, therefore they orient and guide interpretation of individual experience, that is "enable individuals to locate, perceive, identify and label occurrences" (Snow et al. 1986, p. 464). A distinction between the concepts "frame" and "framing" is rather helpful. "Frame" is a mental structure. "Framing" is a behaviour by which people make sense of both daily life and the grievances that confront them. Frame theory, therefore, as developed after Goffman's founding contribution, embraces both cognitive structures whose contents can be elicited, inferred, and plotted in a rough approximation of the algorithms by which people come to decisions about how to act and what to say and the interactive processes of talk, persuasion, arguing, contestation, interpersonal influence, subtle rhetorical posturing, outright marketing that modify—indeed, continually modify—the contents of interpretative frames (Oliver & Johnston, 2000).

Conveying frames for reading the real world mathematically, textbook word problems infuse in children a practical relationship with mathematical knowledge, a relationship of usage. Such a relationship of usage without doubt contributes to the construction of mathematics knowledge. This knowledge however is primarily a practical knowledge of using mathematics and alongside it is a scientific knowledge of the mathematics subject matter. This fact may, in my view, be considered as the essential meaning of the concept of "mathematical enculturation"; a mathematical knowledge invested in a practical knowledge of its usage. For this reason among others, mathematics traditionally constitutes a fundamental component of the socio-cultural indoctrination of children. Mathematics trains children towards "the correct" modes of thinking, "the correct" modes of deducing, "the correct" modes of decision making. A relation of this type between school mathematics and its subject matter may not in any case be considered as an exclusively learning relation. Rather, it is a relation of ideological indoctrination of children, which, by using mathematics – a subject matter commonly agreed to be valuable - habituates them in particular standpoints and specific patterns of behaviour and directs them through mathematics towards the prevailing social values (Althusser, 1970).

In conclusion, school mathematics - as well as the teaching of many other school subjects - incorporates a double relation with its subject matter: a scientific relation as a means for theoretical knowledge of mathematics concepts and tools and an ideological relation as a vehicle for practical knowledge about the use of mathematics.

This practical knowledge concerns particular patterns of behaviour towards the theoretical and social function of mathematics. In this sense, the teaching of mathematics aims both at learning mathematics concepts and tools and at appropriating an ideology for mathematical activities and their outcomes, that is, an ideology concerning mathematics, based on a specific conception of place and function of mathematical activity, its outcomes and applications in the present-day dominant social reality.

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