ALGEBRAIC HYPERSTRUCTURES and APPLICATIONS

Proceedings of the
Fifth International Congress on
Algebraic Hyperstructures and Applications
Jasi, Rumania, July 4–10, 1993

edited by

M. Stefanescu

H P

HADRONIC PRESS, INC.

35246 US 19 North # 115, Palm Harbor, FL 34684, USA

Algebraic Hyperstructures and Applications
M. Stefanescu, Editor
Hadronic Press, Palm Harbor, FL 34682–1577, U.S.A.
ISBN 0–911767–76–2, Pages 251–265, 1994

AUTOMATA AND HYPERMODULOIDS

Gerasimos G. Massouros
54, Klious st., 155 61 Cholargos, Athens, Greece

(Dedicated to the memory of GEORGE GH. MASSOUROS)

Abstract

Here appear some of the results reached during the connection of the theory of Automata and Languages to the theory of hypercompositional structures. In the beginning, the notion of the linguistic hyperringoid is being introduced and its action as a set of operators, or hyperoperators, on a given set is being studied. Next the notion of the (s, F)-acceptable subset, along with the notion of the hypermoduloid and the supermoduloid are being introduced and connected to the notions of the deterministic and non deterministic automata. One of the results of this study is the proof of Kleene's theorem.

1. Introduction.

Let A be an alphabet and let A^* be the set of the words over A. As it has been proved [10], the consideration of bisets of words leads to the creation of a new hypercomposition structure $(A^*, +)$ which is a special kind of join hypergroup [5],[9] that we named *B-hypergroup*.

Definition 2.1. A hypergroup $(H, +)$ is called *B-hypergroup*, if for the hypercomposition "+" we have: $a + b = \{a, b\}$, for every $a, b \in H$.

Motivated from the notion of the empty set from the theory of languages, we introduced the *null word* and so we have the definition:

Definition 2.2. A *fortified join hypergroup* is a join hypergroup that satisfies the additional axioms:

FJ_1 There exists a unique neutral element in H, denoted by 0, such that: $x \in x + 0$ and $0 + 0 = 0$, for every $x \in H$.

FJ_2 For every $x \in H \setminus \{0\}$ there exists only one element $x' \in H \setminus \{0\}$, called *opposite* or *symmetric* of x, denoted by $-x$, such that $0 \in x + (-x)$.
Also $-0=0$

It has been proved [11] that if a B-hypergroup is enriched with an element 0 and if it is defined $0+0=0$ and $x+x=\{x,0\}$, if $x \neq 0$, then the deriving hypergroup is a special kind of fortified join hypergroup:

Definition 1.3. A *dilated B-hypergroup* is a fortified join hypegroup for which:

(i) $x + y = \{x, y\}$, if $x \neq y$ and $x, y \neq 0$,

(ii) $x + x = \{x, 0\}$, if $x \neq 0$ and ~~$x \neq 0$~~,

(iii) $0 + 0 = 0$.

Therefore the hypercomposition structure $\underset{\sim}{A}^* = A^* \cup \{0\}$ is a dilated B-hypergroup [11]. Also it is known, [7],[17], that the set A^* with operation of concatenation of the words is a non commutative (with the exception of A being a singleton) monoid, the unit element of which is the empty word. Similarly, the set $\underset{\sim}{A}^*$ is a monoid as well. We have proved, [10],[11] that the concatenation of the words is bilaterally distributive w.r.t. the hypercomposition defined in the set of the words and thus a new multiplicative-hyperadditive structure [10] appeared:

Definition 1.4. A non void set Y endowed with a composition "·" and a hypercomposition "+" is called a *hyperringoid* if:

(i) $(Y, +)$ is a hypergroup;

(ii) $(Y, \cdot)$ is a semigroup;

(iii) The composition is bilaterally distributive to the hypercomposition.

When the additive hypergroup is a join one then we have the *join hyperringoid*. A special join hyperringoid is the *B-hyperringoid*, where the hypergroup is a B-hypergroup. But when the hypergroup is a fortified join one we have the *fortified hyperringoid*, or *join hyperring* [10],[11] (terminology in accordance to [15],[12]). A detailed study of the fortified join hypergroups and of the hyperringoids appears in [11] and it is the subject of other papers of mine.

Now, we shall introduce some new hypercomposition structures, and after studying some of their properties we will show how all these can be connected to the Theory of Automata.

2. Linguistic hyperringoids.

From the following Proposition, the relation of the hyperringoids to the set of the words A^* over an alphabet A derives:

Proposition 2.1. *The set of the words A^* is a B-hyperringoid and the set $\underline{A}^* = A^* \cup \{0\}$ is a join B-hyperring.*

The hyperringoid mentioned in the above Proposition have the property: Every one of their elements (words) has a unique factorization into irreducible elements which are the elements of the alphabet (letters). So these hyperringoids have a finite *prime subset*, that is a finite set of *prime* or *initial* and *irreducible* elements, such that every one of their elements has a unique factorization with factors from their prime subset. In this sense they have a property similar to the one of the Gauss' rings.

Definition 2.1. A *linguistic hyperringoid* (resp. *dilated linguistic hyperringoid* is a unitary B-hyperringoid (resp. dilated B-hyperringoid) which has a finite prime subset P and which is non commutative for $|P| > 1$.

And so:

Proposition 2.2. *From every non commutative free monoid with finite base, a linguistic hyperringoid derives.*

Example 2.1. Consider all the 2×2 matrices deriving from products of matrices with elements $0, 1$. We observe that none of the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

can result as a product of the other two. Furthermore it can be verified that all the considered matrices can be written as products of the above three ones. So the B-hyperringoid deriving from the 2×2 matrices with elements 0 and 1 is a linguistic hyperringoid and its prime subset consists of the above three matrices. The empty word in this hyperringoid is the unitary matrix.

Example 2.2. As another example we may consider a system with internal memory and external inputs, a JK flip-flop for instance, a sort of the physical components that actually go into the brain of a say counter device. It is known that all the JK flip-floap are "clocked" and that their majority works on the Master-Slave principle. The set of states of the JK flip-flop is the biset $S = \{s_1, s_2\}$. From the set of the conditions leading from one state to another, which in the binary code is $C = \{00, 01, 10, 11\}$, we get the alphabet used by the flip-flop. The linguistic hyperringoid P^* of the JK flip-flop, has prime subset $P = \{01, 10, 11\}$ and unitary element for the multplication the element 00.

It must be mentioned though that every B-hyperringoid is not a linguistic hyperringoid.

Also sets that are important in the theory of Languages appear here as special subsets of a hyperringoid. So at the end of this paragraphe we give the definition of the notion of the rational subsets of a hyperringoid.

Definition 2.2 *Rational subsets* of a unitary hyperringoid Y are the:

(i) finite subsets of Y;

(ii) finite sums and finite products of rational subsets;

(iii) series of the form $\sum_{n=0}^{\infty} A^n$, where A is a rational subset.

Since, as we have previously mentioned, the set of the words has the structure of a special hyperringoid (linguistic hyperringoid), series of the form $\sum_{n=0}^{\infty} A^n$ will be denoted by A^*, generalizing thus in an arbitrary hyperringoid the notation that Kleene introduced in [17] in the Theory of Languages.

3. Hypermoduloids

The notions of the set of operators and hyperoperators from a hyperringoid Y over an arbitrary set M have been introduced in [11] in order to describe the action of the state transition function δ and the extended state transition function δ^*. So

Definition 3.1. Y is a *set of operators* over M, if there exists an external operation from $M \times Y$ to M, with $(s\alpha)\beta = s(\alpha\beta)$, where $s \in M$ and $\alpha, \beta \in Y$. If there exists an external hyperoperation from $M \times Y$ to $P(M)$ which satisfies the above axiom, then Y is a *set of hyperoperators*.

The set of the operators can endow M with a hypercomposition and thus, if Y is a unitary hyperringoid, M becomes a hypergroup. Before proceeding though with this, let's give the definition:

Definition 3.2. The element s_2 of M will be named *connected* to the element s_1 if there exists $\omega \in Y$ such that $s_2 = s_1\omega$ in the case of external operation or $s_2 \in s_1\omega$ in the case of external hyperoperation.

It is worth mentioning that the above definition does not imply that s_2 is connected to s_1, if s_1 is connected to s_2. Let us suppose now that Y is the unitary hyperringoid of the operators [11],[16]. Then we introduce in M a hypercomposition "+" defined as follows:

$$(3.1) \qquad s_1 + s_2 = \begin{cases} \{s \in M \mid & s = s_1\omega \text{ and } s_2 = s\psi, \text{ with } \omega, \psi \in Y\}, \\ & \text{if } s_2 \text{ is connected to } s_1, \\ \{s_1, s_2\}, & \text{if } s_2 \text{ is not connected to } s_1. \end{cases}$$

We remark that the result of the hypercomposition always contains the two participating elements, since Y is unitary and thus $s+M = M$ for every $s \in M$. Moreover, for this hypercomposition the associativy holds. Thus:

Proposition 3.1 *If the set of operators Y is a unitary hyperringoid, then M endowed with the hyperoperation* (3.1) *is a hypergroup.*

We remark that this hypercomposition is not commutative and that the join axiom is not valid. So if we consider the set M to be the set S of the states of an automaton (A, S, s_0, δ, F), then

Proposition 3.2 *The set of the states of an automaton, endowed with the hypercomposition* (3.1), *is a hypergroup.*

It is worth mentioning that a similar to the above proposition can be deduced for the vertices of the directed graphs, that is:

Proposition 3.3. *The set of vertices of a directed graph is endowed with the structure of the hypergroup, if the hypercomposition of two vetices v_i and v_j is defined to be the set of the vertices which appear in all the possible ways that connect v_i to v_j and if there are not such, the biset $\{v_i, v_j\}$.*

Consequently, with Proposition 3.2, we have attached a hypergroup in the set of states of an automaton, namend *the attached hypergroup of the paths.*

But the hypercomposition $s_i + s_j$ defined in 3.1 is directly related to the hyperringoid Y, since it defines its subset:

$$\Gamma[s_i + s_j] = \{\omega \in Y \mid s_i\omega = s_j\}.$$

If Y is not commutative and if it has a prime subset, then all the elements of $\Gamma[s_i + s_j]$ have the property:

$$\omega \in \Gamma[s_i + s_j] \Rightarrow s_i z \in s_i + s_j, \text{ for every } z \in \text{Prefix}(\omega),$$

where $\text{Prefix}(\omega) = \{\psi \in Y \mid \psi\chi = \omega, \text{ for some } \chi \in Y\}$ [17].

Next let's suppose that the set of operators of M is a linguistic hyperringoid with prime subset A. If $P \subseteq M$, then we shall prove that the set:

$$\Gamma[(s_1 + s_2) \cup P] = \{\omega \in Y \mid s_1 z \in (s_1 + s_2) \cap P, \text{ for every } z \in \text{Prefix}(\omega)\}$$

is a rational subset of Y, for every finite subset P of M.

Consider the set $B(s_1 + s_2) = \Gamma[s_1 + s_2] \cap A$. This set is a finite subset of Y and therefore rational. Moreover, $(s_1 + s_2) \cap P$ is a singleton when $s_1 = s_2$ and $P = \{s_1\}$. In this case we have:

$$\Gamma[(s_1 + s_2) \cap \{s_1\}] = [B(s_1 + s_2)]^*$$

which is a rational set.

Now if $P = \{s_1, s_2, s\}$ then

$$\Gamma[(s_1 + s_2) \cap P] = B(s_1 + s)[B(s + s)]^* B(s + s_2)$$

which again is a rational set.

Now let's suppose that, for $|P| < n$, the $\Gamma[(s_1 + s_2) \cap P]$ is rational and let's consider a subset P of M with $|P| = n$. We have the cases:

i) Let $s_1, s_2 \in P$. Also let ω be an element from $\Gamma[(s_1 + s_2) \cap P]$. Then

$$\omega = \chi_1 \chi_2, \ldots, \chi_k \omega' \text{ with } k \geq 0$$

and

$$\chi_1, \chi_2, \ldots, \chi_k \in \Gamma[(s_1 + s_1) \cap (P \setminus \{s_1\})].$$

So, if $s_1 = s_2$, then $\omega' = 1$ and therefore

$$\Gamma[(s_1 + s_1) \cap P] = (\Gamma[(s_1 + s_1) \cap (P \setminus \{s_1\}])^*$$

which has been considered rational from the hypothesis of the induction. Now if $s_1 \neq s_2$ then we consider the word ψ, with the minimum length, for which $s_1 \psi = s_2$ and $\omega' = \psi z$. Then

$$\psi \in \Gamma[(s_1 + s_2) \cap (P \setminus \{s_1, s_2\})],$$

$$\omega = \chi_1 \chi_2 \ldots \chi_k \psi z, \text{ with } z \in \Gamma[(s_2 + s_2) \cap (P \setminus \{s_1\})].$$

Thus we have

$$\Gamma[(s_1 + s_2) \cap P] =$$

$$= (\Gamma[(s_1 + s_1) \cap (P \setminus \{s_1\})]])^* \Gamma[(s_1 + s_2) \cap (P \setminus \{s_1, s_2\})] \Gamma[(s_2 + s_2) \cap (P \setminus \{s_1\})]$$

and so $\Gamma[(s_1 + s_2) \cap P]$ is rational.

ii) Let $s_1, s_2 \notin P$. If $\omega \in \Gamma[(s_1 + s_2) \cap P]$ then $\omega = \psi \omega' z$ with $\psi, z \in B$ where $s_1 \psi = s_3$ with $s_3 \neq s_1$, $s_4 z = s_2$ with $s_4 \neq s_2$ and $s_3 \omega' = s_4$. Thus, $\omega' \in \Gamma[(s_3 + s_4) \cap P]$ and therefore

$$\Gamma[(s_1 + s_2) \cap P] = \sum_{s_3, s_4 \in P} B(s_1 + s_3) \Gamma[(s_3 + s_4) \cap P] B(s_4 + s_2).$$

But, because of (i), $\Gamma[(s_3 + s_4) \cap P]$ is rational and so $\Gamma[(s_1 + s_2) \cap P]$ is rational. Following the same reasoning for the other cases, we have the Proposition:

Proposition 3.4. *The subset* $\Gamma[(s_1 + s_2) \cap P]$ *of* Y *is always rational.*

Thus, according to Proposition 3.4, if M is the set of states of an automaton $\mathcal{A}$ with start state s_0 and s_τ one of its final states, then $\Gamma[s_0 + s_\tau]$ is a rational subset of A^*. So if F is the set of the final states of $\mathcal{A}$, then

$$L = \sum_{\tau \in F} \Gamma[s_0 + s_\tau]$$

is also a rational subset of A^*. But L is the language of the automaton $\mathcal{A}$, therefore

Proposition 3.5. *The language of an automaton is a rational subset of A^*.*

Since the rational subsets of A^* derive from regular expressions [17] and since every regular expression defines a rational subset of A^*, it is obvious that from the above Proposition the direct part of Kleene's theorem [6] derives.

Next let M be an arbitrary set with operators or hyperoperators from a hyperringoid Y and let s be an element of M. Every element α of Y is being associated to $s\alpha$. If Y is a set of operators, then this mapping, which will be denoted by φ_s, is a function from Y to M, while if Y is a set of hyperoperators, then φ_s is a function from Y to $P(M)$. Hence we introduce the definition:

Definition 3.3. A subset L of Y will be named (s, F)-*acceptable* from M, or simply *acceptable*, when there is no danger of confusion, if there exists $s \in M$ and $F \subseteq M$, in the case of external operation, or $F \subseteq P(M)$, in the case of external hyperoperation, such that: $\varphi_s^{-1}(F) = L$. So, according to the definition, for different s and F, we shall generally have acceptable from M subsets of Y. Therefore we have the question: for a given $F \subseteq M$ can all the acceptable from M subsets of Y be found? With regard to the above we prove the Proposition:

Proposition 3.6. *Let M be a finite set with $|M| = n$ and with operators in a B-hyperringoid Y. Then, for a given F, the acceptable from M subsets of Y form the solution of an $n \times n$ system.*

Proof. Let $M = \{s_i \mid i = 1, \ldots, n\}$ and $\underline{Y}$ be the dilated B-hyperringoid which derives from Y. With every $s_i \in M$ we associate the set

$$X_i^{\prime} = \{\omega \in Y \mid s_i\omega \in F\}$$

which obviously is the (s_i, F)-acceptable from M subset of Y. If $X_i^{\prime} = \emptyset$ then we define $X_i \subseteq \underline{Y}$ to be the singleton $\{0\}$, (0, by usual conventions) while in every other case we put X_i equal to $\hat{X_i}$. Next we define the sets $\hat{A_{ij}}$ as follows:

$$A_{ij}\hat{} = \{\omega \in Y \mid s_i\omega = s_j \text{ and } \omega \text{ irreducible}\}.$$

We consider again the subsets A_{ij} and B_i of Y defined as follows:

$$A_{ij} = \begin{cases} \mathrm{A}_{ij}\hat{}, & \text{if } \mathrm{A}_{ij}\hat{} \neq \emptyset \\ 0, & \text{if } \mathrm{A}_{ij}\hat{} = \emptyset \end{cases} \qquad B_i = \begin{cases} 0, & \text{if } s_i \notin F \\ 1, & \text{if } s_i \in F \end{cases}$$

and next we form the system:

$$\begin{aligned} X_1 &= A_{11}X_1 + A_{12}X_2 + \ldots + A_{1n}X_n + B_1 \\ X_2 &= A_{21}X_1 + A_{22}X_2 + \ldots + A_{2n}X_n + B_2 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_n &= A_{n1}X_1 + A_{n2}X_2 + \ldots + A_{nn}X_n + B_n \end{aligned}$$

Solving this system according to the theory developed in [11], we get $X_1, \ldots, X_n$ which are always rational subsets of Y [11].

Remark 3.1. It must be noted that the (s, F)-acceptable subsets of a hyperringoid Y are not necessarily countable. Indeed let us consider the set $\mathbb{R}$ of the real numbers, endowed with the usual multiplication and with the hypercompositional structure of the B-hypergroup. Then $\mathbb{R}$ is a B-hyperringoid (more precisely, join hyperfield) and becomes a set of operators over $\mathbb{R}^2$ if the external operation is defined in the following way: $(a, b)\lambda = (a\lambda, b\lambda)$. $\mathbb{R}^2$ endowed with the structure of the B-hypergroup is a B-hypermoduloid over $\mathbb{R}$). Let $s = (1, 1)$ and $F = \{(\chi, \chi) \in \mathbb{R}^2 \mid \chi \geq \alpha\}$. Then the (s, F)-acceptable subset of $\mathbb{R}$ is the interval $[\alpha, +\infty)$, which is a non countable set.

At this point we should also note that Proposition 3.5 can derive as a direct consequence of the above Proposition 3.6, if we consider the set M to be the set of states S of the automaton. Thus, if s_1 is the start state of the automaton, the solution for X_1 is the language L of the automaton, which, consequently, is a rational subset.

A number of Propositions follows, which deal with subjects relevant to the (s, F)-acceptable subsets of the hyperringoid of operators. In order to avoid the further extention of this paper, we skip the proofs, since they can be found in [11].

Proposition 3.7. *Let M_1 and M_2 be sets with operators from a hyperringoid Y. If L_1 and L_2 are subsets of Y acceptable from M_1 and M_2 respectively, then there exists a set M with operators from Y such that $L_1 \cup L_2$ is also acceptable from M.*

Corollary 3.1. *If M_1 and M_2 are sets with operators from a B-hyperringoid Y, and L_1, L_2 are subsets of Y acceptable from M_1 and M_2 respectively, then there is a set M with operators from Y, such that the $L_1 + L_2$ is acceptable from M.*

Proposition 3.8. *Let M be a set and let Y be a hyperringoid of hyperoperators with hyperoperation "$\circ$". If L is a subset of Y acceptable from M, then there exists a set N over which Y is a set of operators with operation "$\cdot$" and such that L is acceptable from N.*

Corollary 3.2. *For every non deterministic automaton there exists a deterministic automaton which accepts the same language with the first one.*

Proposition 3.9. *Let M_1 and M_2 be sets with operators from a hyperringoid Y. If L_1 and L_2 are subsets of Y acceptable from M_1 and M_2 respectively, then there exists a set M with operators from Y such that L_1L_2 is acceptable from M.*

Proposition 3.10. *If L is a subset of Y acceptable from a set M, then $L \setminus \{1\}$ is also acceptable from the set $M \times \{0,1\}$.*

Proposition 3.11. *Let Y be a hyperringoid produced from a set B and every element of which has a unique factorization from elements of B. If L is an (s, F)-acceptable from a set M subset of Y, not containing the 1, then the $L^* \setminus \{1\}$ is also an acceptable subset of Y.*

Proposition 3.12. *If L is an acceptable from a set M subset of a B-hyperringoid Y, then L^* is also an acceptable subset of Y.*

From the proofs of the above propositions it follows that, if the subsets of Y mentioned in the suppositions of these propositions are acceptable from finite sets, then the same thing happens for the subsets of Y mentioned in their conclusions. Also it is known from the Theory of Automata that every finite set of words defines a language of an automaton. So:

Proposition 3.13. *Every rational subset of A^* is acceptable from an automaton.*

From this Proposition the converse of the Theorem of Kleene [6] is obtained and therefore, from Propositions 3.5 and 3.13, we have:

Theorem of Kleene. *A subset of A^* is acceptable from an automaton A, if and only if it is defined by a regular expression.*

Now let's consider that the hyperringoid of the operators Y acts on a set M endowed with the structure of the hypergroup. Then, in correspondence with the classical theory and the theory of the hypercompositional structures [13],[14],[8],[4],[3] we have:

Definition 3.4. If M is a hypergroup and Y a hyperringoid of operators over M such that for every $k, \lambda \in Y$ and $s, t \in M$, the axioms:

$$\text{(i)}\ \ (s+t)\lambda = s\lambda + t\lambda, \qquad \text{(ii)}\ \ s(\lambda + k) \subseteq s\lambda + sk$$

hold, then M is called *hypermoduloid* over Y. If Y is a set of hyperoperators, then M is called *supermoduloid.*

Relevant structures have been defined in [11] (also see [16]). We remark that if the second of the above axioms holds as an equality, then we have the *strongly distributive* hypermoduloid.

Example 3.1. Let's suppose that M is a B-hypergroup and Y is a B-hyperringoid, then, depending on whether Y is a set of operators or hyperoperators, M becomes a hypermoduloid or a supermoduloid.

In a similar way to what we have defined up to this point, M will be called *B-hypermoduloid* or *B-supermoduloid* respectively.

Proposition 3.14. *Every B-hyperringoid is B-hypermoduloid on itself.*

Proposition 3.15. *The B-hypermoduloids and the B-supermoduloids are strongly distributive.*

In the following we will assume that all the hyperringoids we refer to, are unitary.

Proposition 3.16. *If M_1, M_2 are two B-hypermoduloids, then $M = M_1 \times M_2$ becomes an Y-hypermoduloid, defining the hypercomposition $(s_1, t_1) + (s_2, t_2) = \{(s,t) \mid s \in s_1 + s_2,\ t \in t_1 + t_2\}$ and an external operation from $M \times Y$ to M, $(s,t)\lambda = (s\lambda, t\lambda)$. M is not strongly distributive, even when M_1 and M_2 are strongly distributive.*

As we have mentioned in the second paragraph (motivated from $\underset{\sim}{A}^*$) if the hypergroup of a linguistic hyperringoid is a fortified join one, then we get a fortified hyperringoid of a special form, the dilated linguistic hyperringoid. The dilated linguistic hyperringoid appears directly in the construction of the automaton. We start with the Proposition.

Proposition 3.17. *The set of the states of an automaton can be endowed with the structure of the dilated B-hypergroup with zero the start state (and selfopposite elements).*

Also the next Proposition holds:

Proposition 3.18. *The set of the states of an automaton endowed with the structure of the dilated B-hypergroup becomes a hypermoduloid over the dilated linguistic hyperringoid* $(\underline{A}^*, +, \cdot)$ *which is defined by the alphabet A of the automaton (B-hypermoduloid) [and where the external operation* $S \times \underline{A}^* \to S$ *is obviously being defined from the function* δ^**].*

So when the logical circuit, which is the electronical realization of an automaton, is being constructed, we find it convenient to use a special symbol, $< EOS >$ (End of String), in order to determine the end of a word. $< EOS >$ has the property $\delta^*(s, < EOS >) = s$ for every state s of the automaton. Thus we see that $< EOS >$ corresponds to the unitary element of the hyperringoid, since in the hypermoduloid of the automaton we have: $s \cdot 1 = s$, for every $s \in S$, that is $\delta^*(s, < EOS >) = \delta^*(s, \lambda) = s\lambda = s$. Another symbol, useful to the electronical realization of the automaton is the symbol $< SOS >$ (Start of String). This symbol acts as an annihilator, since it leads from every state to the start state. It is therefore the corresponding symbol to the zero element of the dilated linguistic hyperringoid, since in the hypermoduloid of the automaton we have: $s \cdot 0 = s_0$, for every $s \in S$, and so $\delta^*(s, < SOS >) = \delta^*(s, 0) = s \cdot 0 = s_0$.

Let's consider the automaton of figure 1 as an example. When we introduce the elements $< EOS >$ and $< SOS >$, the automaton of figure 1, is being transformed to the one of figure 2.

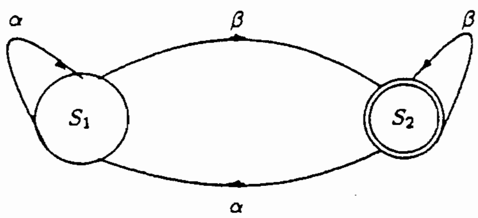


Figure 1.

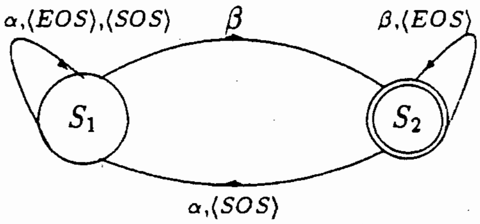


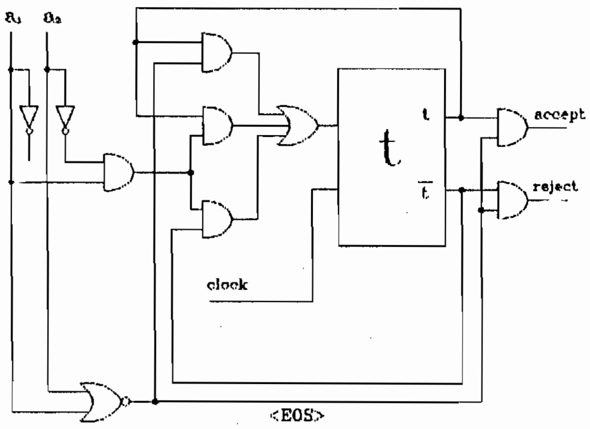
Figure 2.

In order now to proceed to the electronical design of this automaton, we must firstly describe with proper number of bits [2] its states and the letters of its alphabet. So we have $t = 0$ for the state s_0 and $t = 1$ for the state s_1. Moreover, since the alphabet consists only of two letters, α and β and the symbols $< EOS >= 1 = \lambda$ and $< SOS >= 0$, we use the following combinations of bits for its description (we note that by convention $< EOS >$ is represented by the "lower" binary code and $< SOS >$ is represented by the "highest" binary code).

$$00 =< EOS >, \quad 01 = \alpha, \quad 10 = \beta, \quad 11 =< SOS > .$$

This reasoning leads to the complete truth table of the next state t':

t	a_1	a_2	t'
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



Among the other electronical parts it is necessary to use a D flip-flop [1],[2] since the outcome does not depend only on the input of the letter, but it also depends on the previous state of the circuit. Indeed, the input of the letter β, for instance, can either lead from state s_0 to s_1 or cause no change of states in the automaton of our example. Therefore the circuit should "remember" the previous state and for this reason we need a "memory cell". The electronical circuit which is the realization of the automaton of our example is the one of figure 3.

In this automaton the hypermoduloid consists of two elements, s_0 and s_1, while the dilated linguistic hyperringoid has the unitary element $< EOS >$, the zero element $< SOS >$, and the prime elements α and β.

Acknowledgement. I would like to extend my deep appreciation and sincere thanks to Professor J.Mittas for our fruitful discussions.

REFERENCES

1. BOOTH,T., *Introduction to Computer Engineering Hardware and Software Design.* Third edition. Wiley, 1984.

2. CLEMENTS,A., *The Principles of Computer Hardware.* Oxford Science Publications, 1985.

3. CORSINI,P., *Hypergroupes reguliers et hypermodules.* Ann. Univ. Ferrara, Sc. Math., 1975.

4. CORSINI,P., *Prolegomena of hypergroup theory.* Aviani Editore, 1993.

5. JANTOSCIAK,J., *Classical geometries as hypergroups.* Atti del Convegno su Ipergruppi, altre Strutture Multivoche e loro Applicazioni, Udine, 1985.

6. KLEENE,S.C., *Representation of events in nerve nets and finite automata.* Automata Studies, Princeton Univ. Press, Princeton N.J., 3-42, 1956.

7. KOHANI,Z., *Switching and Finite Automata Theory.* Second Edition, McGraw Hill.

8. MASSOUROS,C.G., *Free and cyclic hypermodules.* Annali di mat. pura ed appl., vol.CL, 153-166, 1988.

9. MASSOUROS,C.G., *Hypergroups and Convexity.* Riv. di mat. pura ed appl., vol.4, 7-26, 1989.

10. MASSOUROS,G.G., MITTAS,J., *Languages-Automata and hypercompositional structures.* Proceedings of the 4^{th} Internat. Cong. in Algebraic Hyperstructures and Applications, 137-147, Xanthi 1990. World Scientific.

11. MASSOUROS,G.G., *Automata-Languages and hypercompositional structures.* Doctoral Thesis. Department of Electrical Engineering and Computer Engineering of the National Technical University of Athens (1993).

12. MITTAS,J. - *Sur les hyperanneaux et les hypercorps.* Math. Balk. 3, 1973.

13. MITTAS,J. - *Sur certaines classes de structures hypercompositionnelles.* Πρακτικα της Ακαδημιας Αθηνών, 48, σελ. 298-318, Αθήναι 1973.

14. MITTAS,J., *Espaces vectoriels sur un hypercorps. Introduction des hyperspaces affines et Euclidiens.* Mathematica Balkanica 5, 199-211, 1975.

15. MITTAS,J., *Sur les structures hypercompositionnelles.* Proceedings of the 4th Internat. Cong. in Algebraic Hyperstructures and Applications, 137-147, Xanthi 1990. World Scientific.

16. MITTAS,J. - *Sur certaines classes de structures hypercompositionnelles.* Proceedings of the 5th Internat. Cong. in Algebraic Hyperstructures and Applications, Iaşi 1993.

17. SIPPU,S., SOISALM-SOININEN,E., *Parsing Theory.* Springer Verlag, 1988.