On possible scaling laws between electric earthquake precursors (EEP) and earthquake magnitude

F. Vallianatos
Technological Educational Institute of Crete, Chania Branch, Crete, Greece.

A. Tzanis
Department of Geophysics and Geothermy, University of Athens, Athens, Greece.

Abstract. We assume, without reference to any particular electrification mechanism, that a pre-seismic, time dependent polarization appears in a number of spherical volumes distributed in some earthquake preparation zone embedded in a half space of constant resistivity. We estimate the resulting transient electric and magnetic fields in the quasi-static approximation. On assuming that the number of polarized spheres \( N \) is scaling with their radii \( r \) as \( N = r^{D} \), we show that at some distance \( r \) from the zone, the electric field and the magnitude of the earthquake are related as \( \log E = aM + c \), where \( a = (3-D)/2 \) and similarly for the magnetic field. Fragmentation experiments and theoretical simulations indicate that \( 2.2 \leq D \leq 2.6 \), yielding \( 0.4 > \alpha > 0.2 \). The lower fractal dimensions correspond to the case of dynamic crack propagation. Letting \( D=2.3 \), yields \( \alpha = 0.35 \) which is comparable to the experimental value of 0.35 given by Varotsos and Alexopoulos, (1984) on the basis of a few earthquake sequences in Greece. This indicates that electric and magnetic earthquake precursors may obey scaling laws that are direct consequence of the fractal distribution of their generators and also implies that transient precursors may result from microfracturing and fragmentation processes in the earthquake preparation zone.

1. Introduction

The possibility of Electric Precursors to Earthquakes has been subject of intensive research over the past few decades. Laboratory experiments of electric field generation in rocks have been encouraging (e.g. see Molchanov and Hayakawa, 1995; Hayakawa and Fujinawa, (eds), Electromagnetic Phenomena Related to Earthquake Prediction, pp 253-359, 1994), but the same is not true for field experiments involving long term observations and relying on statistics to associate "EEP signals" and earthquakes. Several examples of anomalous electric field variations prior to earthquakes have been reported, (e.g. Sobolev, 1975; Mizutani et al., 1976; Rikitake, 1987; Varotsos and Alexopoulos, 1984; Park et al., 1993), but in most cases, their generation mechanism and relationship to earthquakes has not been demonstrated. To date, there's no comprehensive theory to account for the generation and propagation of EEP and progress is slow, exploring concepts such as are the piezoelectric effect (Sornette and Sornette, 1990; Yoshida et al., 1997), the electrokinetic effect (e.g. Bernard, 1992), the motion of charged dislocations (e.g. SIlkfin, 1993; Vallianatos and Tzanis, 1998, 1999), contact electrification and piezostimulated currents (e.g. Varotsos and Alexopoulos 1986). All these mechanisms are ultimately related to stress and strain changes in the earthquake source.

One of the longest and most interesting experiments is undoubtedly the one carried out by the VAN group in Greece, continuously since the early 80's (e.g. Varotsos and Alexopoulos, 1984, 1986). However, signals and statistics similar to those reported by VAN have not been unambiguously observed elsewhere and the VAN method remains highly controversial (e.g. see Special Issue of GRL v23 N#11, 1996; Lighthill (ed), "A critical review of VAN", 1996). In one case however, (Varotsos and Alexopoulos, 1984), the group has reported a set of empirical laws for the behavior and propagation of the EEP. The most interesting of these, (henceforth referred to as the V-A scaling law), was constructed on the basis of data from a very few earthquake sequences in western Greece and associates signal amplitude and earthquake magnitude with a relationship of the form \( \log E = \alpha M + C \) where \( \alpha \) is a positive slope in the range 0.3–0.4 and \( C \) is different for different seismic regions. The authors attribute the almost universal slope to fundamental processes at the source, but cannot explain it.

An interpretation attempt was made by Sornette and Sornette (1990), on the basis of a self-organized critical system at the earthquake focus, long range correlation between the source and the observer and piezoelectricity as the fundamental electrification process. Recently Molchanov (1999) has reproduced the relationship on the basis of the electrokinetic effect, making the crucial assumption that the electric signal is a product of foreshock activity.

It is now well accepted that earthquakes are self-organized critical processes and that faults and fractures obey fractal distribution laws. All possible electrification mechanisms are related to stress/strain changes which presumably occur as part of such systems and processes. Therefore, and given that the V-A law is also strongly suggestive of a self-similar system, it is interesting to investigate the properties of the electric signal generated in such a system. Independently of any underlying generation mechanism and without requiring any long range correlations, we consider a set of electric field emitters in the earthquake preparation volume, distributed according to a self-similar fractal law. Then, we attempt to theoretically derive an Amplitude - Magnitude scaling law, compare it to V-A law and investigate whether it may be a result of the geometrical distribution the emitters in the preparation zone.
2. Construction of the Amplitude – Magnitude scaling law

For simplicity, we consider spherical earthquake sources which include spherical electric emitters but as will become apparent, the theory may be directly generalized to sources with different shapes. Consider a sphere of radius $L$ and volume $V$, embedded in a conductive medium of constant resistivity $r$. Next, assume that, the spherical volume acquires time dependent induced polarization $P(t)$ and becomes a source of electric and magnetic fields. For the moment, suppose that the polarization vector has only a vertical component, $P(t) = Pf(z,t)$, and that at $t=0$, there’s a step change in polarization from zero to some finite value. Immediately after $t=0$, the potential $\psi_e$ at an external point to the sphere will be given by (e.g. Griffiths, 1996)

$$\psi_e = \frac{P V \cos \theta}{4 \pi \sigma^2}$$  \hspace{1cm} (1)

Since the sphere is embedded in a conducting medium, currents flow to reduce the surface charge $q_1 = P \cos \theta$ and just outside the surface of the sphere, the normal current density $J_n$ is

$$J_n = \frac{2q_1}{3 \sigma} = \frac{2q_1}{3 \sigma} \text{ and } J_n = \frac{d q_1}{dt}$$

Hence, we conclude that

$$\frac{dq_1}{dt} + \frac{2q_1}{3 \sigma} = 0$$  \hspace{1cm} (2)

which shows that the surface charge of a spherical object embedded in a conducting full-space will decay with a time constant equal to $3 \sigma/2$. Introducing the frequency response indicated by (2) into (1) we obtain

$$\psi_e(r) = \frac{P \omega V \cos \theta}{4 \pi \sigma^2} \left( \frac{io}{io + 2/3 \sigma} \right)$$  \hspace{1cm} (3)

Herein, we consider fields in the quasi-static approximation and may neglect the feedback from the magnetic field. Moreover, our result can be readily generalized for an arbitrary orientation of the polarization vector, in which case the electrostatic field becomes:

$$E(r) = -V \psi_e = \frac{V}{4 \pi} \frac{io}{io + 2/3 \sigma} \sqrt{\frac{P(\omega) \cdot r}{r^3}}$$  \hspace{1cm} (4)

Equation (4) has a corner frequency at $\omega_c = 2/3 \sigma$. For $\sigma < 10^3 \Omega m$, $\omega_c > 120$ kHz, well above the frequency range under consideration. By taking the low frequency asymptote,

$$E(\omega) = \frac{3V \omega}{8 \pi} \left( \frac{io}{io + 2/3 \sigma} \right)$$  \hspace{1cm} (5)

and on transforming back to the time domain,

$$E(t) = \frac{3V \omega}{8 \pi} \sqrt{\frac{P \cdot r}{r^3}} \text{ and } P = \frac{d P(t)}{dt}$$  \hspace{1cm} (6)

Recall that (6) is valid only for a polarized sphere in a conducting full-space. In order to estimate the field at the surface of the Earth (i.e. the top of a conductive half-space), we use image theory and the boundary condition $E_z(z=0) = 0$. We define a spherical co-ordinate system with $\hat{z}$ a unit vector in the vertical direction, and $\hat{e}_1$, a unit vector perpendicular to $\hat{z}$, on the plane defined by the vectors $\hat{z}$ and $r$ (figure 1). Then,

$$P = P_1 \hat{z} + P_1 \hat{e}_1 + P_2 \hat{e}_2$$  \hspace{1cm} (7)

where $P_1$ is the vertical component of the polarization and $P_{1,2}$ are the horizontal components in the source-receiver direction and perpendicular to it respectively. Then, using the fields of the real and image sources we compute the horizontal electric field $E_h$ at the surface of the Earth:

$$E_h(t) = \frac{3V \omega}{4 \pi \sigma r^3} \left( \frac{\partial P_1(t)}{dt} \cos \theta \sin \theta + \frac{\partial P_2(t)}{dt} \sin^3 \theta \right) \hat{e}_h$$  \hspace{1cm} (8)

with $\hat{e}_h$ a unit vector in the horizontal direction, lying in the plane defined by $\hat{z}$ and $r$. An equivalent expression for $K_{sr}$ is

$$K_{sr}(\theta, r) = \frac{3 \sigma \omega^3}{4 \pi \sigma r^3}$$  \hspace{1cm} (9)

meaning that at distances far enough from the source such that $\sigma \ll r$, it becomes

$$K_{sr}(\theta, r) = \frac{3 \sigma \omega^3}{4 \pi \sigma r^3}$$  \hspace{1cm} (10)

which indicates that the field produced by the horizontal components will predominate as the field of the vertical term decreases inversely with distance.

A similar approach can be used for estimating the magnetic field. From Maxwell’s equations we have, directly,

$$\mathbf{v} \times \mathbf{B} = \mu \left( J + \frac{\partial \mathbf{E}}{\partial t} \right) = \mu \left( \frac{E}{\rho} + \frac{\partial \mathbf{E}}{\partial t} \right)$$  \hspace{1cm} (11)

and on using Stokes’ theorem and transforming the result in the frequency domain,

$$\int \mathbf{B} \cdot d \ell = \mu \left[ \mathbf{v} \times \mathbf{E} = \mathbf{J} \right]$$  \hspace{1cm} (12)

An appropriate contour along which to compute the line integral is the perimeter of the surface $S$. The problem has spherical symmetry and the magnetic field lines form circles centered around the axis of symmetry and on a plane perpendicular to it. Thus, we can choose such a circle for the contour of integration, in which case $S$ is the enclosed disk. After some algebra, $B$ is given by

$$B = \frac{\mu}{4 \pi} \left[ \mathbf{v} \times \mathbf{E} = \mathbf{J} \right]$$  \hspace{1cm} (13)

By taking the low frequency asymptote, (i.e. $\omega_c = 2/3 \sigma$),

$$B = \frac{3 \mu \omega^3}{8 \pi \sigma}$$  \hspace{1cm} (14)

which is easily transformed to time domain.

Figure 1. Spherical co-ordinate system for computation of the electric field due to a polarised sphere.
To estimate the magnetic field at the surface we apply the same image theory approach and we find

$$B = \frac{3\mu_0\nu}{4\pi r^2} = \frac{\mathbf{P}}{r^2}.$$  

(15)

This implies that the magnetic field should be mainly vertical and observable only if the seismogenic process generates a source with polarization rate perpendicular to the plane \((x, y)\).

Next, consider an earthquake source volume \(\mathcal{V}\), (again assumed spherical for simplicity), with radius \(L_r\). Next, consider a set of distributed spherical sub-volumes \(v_i\) with radius \(l_i\) in \(\mathcal{V}\), which develop coherent, time-dependent electrical polarization. This brings us, (and will attempt to justify later on), that the direction of the polarization vectors is consis-

$$V = \sum_{i=1}^{n} \frac{4\pi A D}{3} l_i^{3-D} (S_{i})^{D},$$  

(18)

where \(l_{max}\) and \(l_{min}\) are the uppermost and lowermost radii sizes in the set \(v_i\) and \(S_{i} = l_i^{3-D} / l_{max}^{3-D}\) a scaling range factor \((0 < S_{i} < 1)\). It is expected that the upper limit \(l_{max}\) is a fraction \(\alpha\) of \(L_r\), the excited domain. Thus, we may assume that \(l_{max} = \alpha L_r\), \(0 < \alpha < 1\). Hence,

$$V = \sum_{i} \frac{4\pi A D}{3} \left(\frac{\alpha L_r}{3} \right)^{3-D} (S_{i})^{D} S_{i}.$$  

(19)

Under the condition \(l_{min} \ll l_{max}\) (which will be justified later on), it is straightforward to assume that \(S_{i} = 1\). The total horizontal electric field observed at a distance \(r\) from the emitters can be computed on the basis of the superposition principle, by substituting (19) into (8):

$$E_h = \frac{3\mu_0}{4\pi} \sum_{i} \left(\frac{\alpha L_r}{3} \right)^{3-D} (S_{i})^{D} S_{i}.$$  

(20)

If the electrification mechanism has approximately similar geometry for different events located in the same seismogenic zone, (thus producing similar geometry of the polarization rates), it is evident from equation (20) that the polarization of the received signal depends only on the azimuthal parameters included in \(S_{i}\), which is constant for a given observation point. Taking the logarithm of (20),

$$\log(E_h) = (3-D)\log(L_r) + \log \left(\frac{3\mu_0}{4\pi} \frac{\alpha L_r}{3} \frac{3-D}{(S_{i})^{D}} \right)$$  

(21)

which, by virtue of the well known scaling relationship \(\log(L) = 0.5M + \text{Constant}\), (e.g. Scholz, 1990), reduces to

$$\log(E_h) = \frac{3-D}{2} M + C^B,$$  

(22)

where \(C^B\) includes the second and third terms in the right hand side of (21).

A number of fragmentation experiments indicate that \(2.2 < D < 2.8\) (e.g. see Table 3.2 of Turcotte, 1997), although deviations from this range have also been observed. However, the condition \(D > 2\) is necessary to constrain the total area of the fragments to a finite value. Observations of fault networks indicate that the two-dimensional fractal dimension \(D_2 = 1.6\), (e.g. Turcotte, 1997 pp 67-76 and references therein). However, unfragmented blocks are bounded by micro- and macro-fractures and faults, so that a fractal distribution of block sizes in three dimensions can be related to the fractal distribution of fractures and faults in two-dimensions. This relationship is demonstrated by Turcotte (1997, pp71-72) on the basis of the comminution model of fragmentation, so that \(D_2 = 1.27\) for the surface topography, which can be generalized as above to \(D_2 = 2.27\) and is consistent with the experimental results quoted therein. Hirata et al. (1987) produced explicit experimental results in granites showing that \(D_2 = 2.75\) for transient creep, \(D_2 = 2.66\) for steady creep and \(D_2 = 2.25\) for acceleration creep. The latter corresponds to the phase of dynamic crack propagation (microfracturing) and clustering: as the creep progresses, the 3-D crack network becomes increasingly clustered and the fractal dimension decreases. Thus, we may assume that \(D\) varies in the range (2.25-2.6), taking the lower values during dynamic crack propagation. Accordingly, the constant slope \(\alpha = (3-D)/2\) varies in the range (0.375-0.2). In the presumed case of microfracturing, \((D=2.3),\)

$$\log(E_h) = 0.35 M + C^B,$$  

(23)

in which the slope \(\alpha = 0.35\) is very comparable to the experimental slope of the \(V-A\) law. Applying the same procedure for the magnetic field we get

$$B_s = \left(3-D\right) \log(L_r) + \log \left(\frac{\mu_0 \alpha L_r}{3-D} \frac{3-D}{(S_{i})^{D}} \right)$$  

(24)

and introducing the scaling expression between \(L_r\) and \(M\),

$$\log(B) = \alpha M + C^B, \quad \alpha = \frac{(3-D)}{2}.$$  

This indicates that in case we observe a preseismic vertical magnetic field, this will be scaled with the magnitude according to the same law and the same universal slope as the preseismic electric field.

3. Discussion

We have derived a scaling law between EEP amplitude and the associated earthquake magnitude, on the assumption of a fractal distribution of multiple sources within the earthquake preparation zone. This is the linear relationship (22), with a slope \(\alpha\) which is exclusively controlled by the fractal exponent \(D\), i.e. by the geometric distribution of the electric field emitters. Furthermore we explore the conditions under which a vertical preseismic magnetic field may be observed and construct a similar scaling law with identical slope.

In addition, if we assume that the EEP generator is somehow associated with fracturing and crack propagation, whereupon \(2.2 < D < 2.6\), we obtain \(0.4 < \alpha < 0.2\). This also justifies our assumption that \(l_{min} << l_{max}\), because \(l_{min}\) would be the smallest fracture/crack size and \(l_{max}\) is very likely comparable to the size of the fault (also see Turcotte, 1997).

Our theoretical prediction for \(\alpha\) is consistent with the only existing experimental result, which associates earthquake magnitude and EEP amplitude, constructed on the basis of a handful of earthquake sequences in western Greece (Varotsos...
and Alexopoulos, 1984). The methods and procedures of these authors are subject to serious controversy, but providing that the data used by Varotsos and Alexopoulos (1984) were indeed genuine precursors, this remarkable coincidence may be of some consequence. It suggests that in a few cases at least, the EEP signal was indeed generated in a self-similar system of emitters which were undergoing fracturing and crack propagation. This kind of geometry and conditions however, are native to the terminal stages of the earthquake preparation, as at least predicted by the volume dilatancy models (stages of fracturing and crack propagation, e.g. see Scholz, 1990).

Electric field is indeed produced during microfracturing, as has been demonstrated in a large number of laboratory experiments (for example, see Molchanov and Hayakawa, 1995, and Hayakawa and Fujinawa, (eds), Electromagnetic Phenomena Related to Earthquake Prediction, pp 253-359, 1994). When cracks begin to propagate, they do so in unison, responding to the same stress field. The resulting electric field at any location, will be the superposition of the fields emitted by each individual crack and will have similar characteristics. There are several arguments pointing towards this field being dipole in nature (e.g. Slifkin, 1993; Molchanov and Hayakawa, 1994, 1995; Vallianatos and Tzanis, 1998, 1999). Thus, the possibility that some EEP signals are generated during crack propagation may also justify our assumption, that the direction of the electrical polarization vectors is consistent over the set of emitting sub-volumes.

There exists an alternative hypothesis, based on the assumption that the entire earthquake source volume becomes polarized and emits uniformly. In this case, \( E_2 = 4 \pi L_2^2 / 3 \), from which we derive \( \log(E_2) = 3 \log(M^2) + C_2 \). This would result in unlikely high EEP amplitudes. Moreover, it is inconsistent with the mechanics of fracturing and the associated fractal geometry of cracks and fractures, as well as with the existing experimental evidence.

We point out that the factors \( C_2 \) and \( C_3 \) are strongly dependent on the source and source-receiver path (second term in equations 21 and 24 respectively), but only weakly on the source-receiver separation (third term, due to logarithm). For instance, the product \( 3 \log(r) \) varies from 14 to 15 for \( r = 50 \) and 100 km respectively. Considering that the bulk transmission properties for a given source and propagation path, can hardly change over decakilometric ranges and time scales of a few months to some years, the scaling laws should be unique, if the geoelectric structure and noise environment in the neighborhood of the receiver do not change during the period of observations; if they do change however, this should only affect the constants \( C_i \) but not the slope. On the other hand, it is apparent that due to the strong dependence on the particular source properties and propagation path, any empirical realization of (22) for a given location and seismic region, cannot be used as a standard for predicting the magnitude of the impending earthquake from an electrical precursory signal recorded at a different seismic region.

References


Slifkin, L., Seismic electric signals from displacement of charged dislocations. Tectonophysics, 224, 149-152, 1993.

F. Vallianatos, Technological Educational Institute of Heraklion, Chania Branch, 3 Romanou St., Chania 73133, Crete, Greece.
(e-mail: avallian@chania.tei. Her.gr).
A. Tzanis, Department of Geophysics and Geothermy, University of Athens, Panepistimiopolis, Zografou 15784, Greece.
(e-mail: atzanis@atlas.cc.uoa.gr).

(Received November 16, 1998; revised March 16, 1999; accepted March 23, 1999)