ON THE EXISTENCE OF SOLUTIONS TO THE GENERAL (THREE-DIMENSIONAL) INVERSE PROBLEM OF PASSIVE ELECTROMAGNETIC SOUNDING

Andreas Tzanis

Department of Geophysics and Geothermy, University of Athens, Panepistimiopoli, Zografou 157 84, Greece; e-mail: atzanis@cc.uoa.gr

A very important question arising in the interpretation of geophysical data, is that of the existence of solutions to the inverse problem, based on measured (incomplete and inconsistent) data. This is a particularly important problem in the case of passive electromagnetic sounding methods, (MagnetoTelluric and Geomagnetic Deep Sounding), because the weak natural fields are notoriously susceptible to, and can often be swamped and distorted by noise of various denominations. The problem of existence of MT and GDS data may have a solution. On the assumption of causality, it is shown that in the case of general (three-dimensional) conductivity distributions integrable over any cuboid region of the Earth, the MT and GDS response functions possess very restrictive analytic properties: Their singularities are all simple zeros, confined on the positive imaginary axis of the complex frequency plane. This means that MT and GDS responses are meromorphic functions, expressible in terms of their zeros, in the form of tensor Mittag-Leffler expansions (finite conductivity) or tensor Stieltjes-Lebesgue integrals (diverging conductivity). The results are completely analogous to those derived for one-dimensional Earth structures by Weidelt (Z. Geophys., 38, 257-289, 1972; J. Geopys., 59, 171-176, 1986) and Parker (J. Geophys. Res., 85, 4421-4428, 1980). In fact, they may be thought of, as a direct generalization of their pioneering work. In the case of measured, incomplete (finite bandwidth) and inconsistent (associated with finite uncertainty error) responses, there exist finite versions of the tensor Mittag-Leffler representations, which can be used for the development of practical algorithms of testing the existence of physically valid data. One such set of tests assumes the form of simple inequality constraints that comprise necessary and sufficient conditions, similar to those of Weidelt (1986) for the one-dimensional response. A much more effective algorithm involves the solution of the analytic expansions for the zeros of the response functions, which is then used for the reconstruction of theoretical tensor sounding curves that can be compared with the measured data (errors taken into consideration). This approach has the advantage of facilitating the reconstruction of moderately distorted response functions by analytic interpolation and is conceptually similar to Parker and WhalerŠs (J. Geophys. Res., 86, 9574-9584, 1981) quadratic programming solution of the finite Cauer representation of one-dimensional

MT responses. Examples of these procedures are given, as well as a discussion of their applicability to other methods of geophysical investigation.