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The Curvelet Transform in the analysis of 2-D GPR data: Signal enhancement and extraction of orientation-and-scale-dependent information

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The Ground Probing Radar (GPR) has become a valuable means of exploring thin and shallow structures for geological, geotechnical, engineering, environmental, archaeological and other work. GPR images usually contain geometric (orientation/dip-dependent) information from point scatterers (e.g. diffraction hyperbolae), dipping reflectors (geological bedding, structural interfaces, cracks, fractures and joints) and other conceivable structural configurations. In geological, geotechnical and engineering applications, one of the most significant objectives is the detection of fractures, inclined interfaces and empty or filled cavities frequently associated with jointing/faulting. These types of target, especially fractures, are usually not good reflectors and are spatially localized. Their scale is therefore a factor significantly affecting their detectability. At the same time, the GPR method is notoriously susceptible to noise. Quite frequently, extraneous (natural or anthropogenic) interference and systemic noise swamp the data with unusable information that obscures, or even conceals the reflections from such targets. In many cases, the noise has definite directional characteristics (e.g. clutter). Raw GPR data require post-acquisition processing, as they usually provide only approximate target shapes and distances (depths).

The purpose of this paper is to investigate the Curvelet Transform (CT) as a means of S/N enhancement and information retrieval from 2-D GPR sections (B-scans), with particular emphasis placed on the problem of recovering features associated with specific temporal or spatial scales and geometry (orientation/dip).

The CT is a multiscale and multidirectional expansion that formulates a sparse representation of the input data set (Candès and Donoho, 2003a, 2003b, 2004; Candés et al., 2006). A signal representation is sparse when it describes the signal as a superposition of a small number of components. What makes the CT appropriate for processing GPR data is its capability to describe wavefronts. The roots of the CT are traced to the field of Harmonic Analysis, where curvelets were introduced as expansions for asymptotic solutions of wave equations (Smith, 1998; Candès, 1999). In consequence, curvelets can be viewed as primitive and prototype waveforms – they are local in both space and spatial frequency and correspond to a partitioning of the 2D Fourier plane by highly anisotropic elements (for the high frequencies) that obey the parabolic scaling principle, that their width is proportional to the square of their length (Smith, 1998).

The GPR data essentially comprise recordings of the amplitudes of transient waves generated and recorded by source and receiver antennae, with each source/receiver pair generating a data trace that is a function of time. An ensemble of traces collected sequentially along a scan line, i.e. a GPR section or B-scan, provides a spatio-temporal sampling of the wavefield which contains different arrivals that correspond to different interactions with wave scatterers (inhomogeneities) in the subsurface. All these arrivals represent wavefronts that are relatively smooth in their longitudinal direction and oscillatory in their transverse direction. The connection between Harmonic Analysis and curvelets has resulted in important nonlinear approximations of functions with intermittent regularity (Candès and Donoho, 2004). Such functions are assumed to be piecewise smooth with singularities, i.e. regions where the derivative diverges. In the subsurface, these singularities correspond to geological inhomogeneities, at the boundaries of which waves reflect. In GPR data, these singularities correspond to wavefronts. Owing to their anisotropic shape, curvelets are well adapted to detect wavefronts at different angles and scales because aligned curvelets of a given scale, locally correlate with wavefronts of the same scale.
The CT can also be viewed as a higher dimensional extension of the wavelet transform: whereas discrete wavelets are designed to provide sparse representations of functions with point singularities, curvelets are designed to provide sparse representations of functions with singularities on curves.

This work investigates the utility of the CT in processing noisy GPR data from geotechnical and archaeometric surveys. The analysis has been performed with the Fast Discrete CT (FDCT – Candès et al., 2006) available from http://www.curvelet.org/ and adapted for use by the matGPR software (Tzanis, 2010). The adaptation comprises a set of driver functions that compute and display the curvelet decomposition of the input GPR section and then allow for the interactive exclusion/inclusion of data (wavefront) components at different scales and angles by cancelation/restoration of the corresponding curvelet coefficients. In this way it is possible to selectively reconstruct the data so as to abstract/retain information of given scales and orientations.

It is demonstrated that the CT can be used to: (a) Enhance the S/N ratio by cancelling directional noise wavefronts of any angle of emergence, with particular reference to clutter. (b) Extract geometric information for further scrutiny, e.g. distinguish signals from small and large aperture fractures, faults, bedding etc. (c) Investigate the characteristics of signal propagation (hence material properties), albeit indirectly. This is possible because signal attenuation and temporal localization are closely associated, so that scale and spatio-temporal localization are also closely related. Thus, interfaces embedded in low attenuation domains will tend to produce sharp reflections rich in high frequencies and fine-scale localization. Conversely, interfaces in high attenuation domains will tend to produce dull reflections rich in low frequencies and broad localization.

At a single scale and with respect to points (a) and (b) above, the results of the CT processor are comparable to those of the tuneable directional wavelet filtering scheme proposed by Tzanis (2013). With respect to point (c), the tuneable directional filtering appears to be more suitable in isolating and extracting information at the lower frequency – broader scale range.

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THE CURVELET TRANSFORM IN THE ANALYSIS OF 2-D GPR DATA:
SIGNAL ENHANCEMENT AND EXTRACTION OF ORIENTATION-AND-SCALE-DEPENDENT INFORMATION

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MOTIVATION

GPR is an invaluable tool for civil and geotechnical engineering applications. One of the most significant objectives of such applications is the detection of fractures, inclined interfaces, empty or filled cavities frequently associated with jointing/faulting and a host of other oriented features. These types of target, especially fractures, are usually not good reflectors and are spatially localized. Their scale is therefore a factor significantly affecting their detectability. Quite frequently, systemic or extraneous noise, or other significant structural characteristics swamp the data with information which blurs, or even masks reflections from such targets, rendering their recognition difficult.

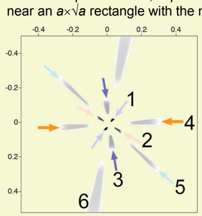
This presentation investigates the utility of the *CURVELET TRANSFORM (CT)* in processing noisy GPR data from geotechnical and archaeometric surveys and extracting information geometrical information (oriented and scale-dependent structural characteristics).

THE 2G CURVELET TRANSFORM

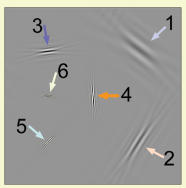
- The CT is a *multiscale* and *multidirectional* expansion that formulates a *sparse representation* of the input signal (Candès and Donoho, 2003, *Appl. Comput. Harmon. Anal.*, 19, 162-222.
- Curvelets are obtained by parabolic dilations rotations and translations of a specifically shaped function φ and are indexed by a scale parameter a such that $0<a<1$), a location b and an orientation θ and have the approximate form:

$$\varphi_{a,b,\theta}(x) = a^{-3/4}\varphi\left(\mathbf{D}_a\mathbf{R}_\theta(x-b)\right), \quad \mathbf{D}_a = \begin{bmatrix} 1/a & 0 \\ 0 & 1/\sqrt{a} \end{bmatrix}$$

$\mathbf{D}_a$ is the parabolic scaling matrix, $\mathbf{R}_\theta$ is the rotation by θ radians and $\varphi(x_1, x_2)$, $x_1, x_2 \in \mathbb{R}^2$ is an admissible profile. Thus, if φ is supported near the unit square, the envelope of $\varphi_{a,b,\theta}$ is supported near an $a\times\sqrt{a}$ rectangle with the minor axis pointing in the direction of θ.



Frequency (left) and *spatial* (right) representation of six curvelets at different scales, rotations and translations. Numbering is in order of *decreasing* scale. In the dyadic partitioning of the frequency domain, each wedge represents the compact frequency support of the corresponding curvelet, i.e. each pair of opposing wedges represents a real curvelet. Image adapted from Herrmann and Hennenfent, 2008, *GJI*, doi:10.1111/j.1365-46X.2007.03698.x).



Curvelets obey the principle of harmonic analysis: It is possible to decompose and reconstruct an arbitrary function $f(x_1, x_2)$ as a superposition of curvelets.

If the scale, rotation and location are discretized as:

$$a_j = 2^{-j}, \qquad j = 0,1,2,\ldots$$

$$\theta_{j,l} = 2\pi l \cdot 2^{-\lfloor j/2 \rfloor}, \qquad l = 0,1,\ldots,2^{\lfloor j/2 \rfloor}-1 \qquad \text{so that} \qquad \varphi_{j,k,l} = \varphi_{a,b_k^{(j,l)},\theta_{j,l}}$$

$$b_k^{(j,l)} = \mathbf{R}_{\theta_{j,l}}\begin{bmatrix} k_1 2^{-j} \\ k_2 2^{-j/2} \end{bmatrix}, \quad k_1,k_2 \in \mathbb{Z}$$

the function f can be expressed in terms of the curvelet family ($\varphi_{j,k,l}$) as:

$$f = \sum_{j,k,l}\left\langle f,\varphi_{j,k,l}\right\rangle \varphi_{j,k,l}\,,$$

(tight frame).

$$\|f\|_2^2 = \sum_{j,k,l}\left|\left\langle f,\varphi_{j,k,l}\right\rangle\right|^2, \qquad \forall f \in L^2(\mathbb{R}^2)$$

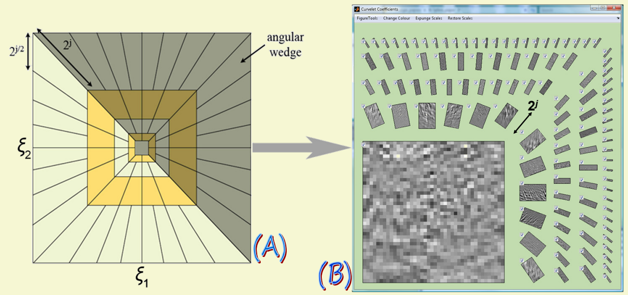
- The curvelet transform is organized in such a way, that most of the energy is localized in only a few coefficients $\langle f, \varphi_{j,k,l}\rangle$.
- Curvelets are ideally adapted to represent functions with *curve-punctuated smoothness* (or *intermittent regularity*) which are piecewise smooth with discontinuities (singularities) along a curve of bounded curvature.
  1. A curvelet intersecting a discontinuity parallel to its longitudinal support will have coefficients of significant amplitude.
  2. A curvelet intersecting a discontinuity at an arbitrary angle will have small coefficients.
  3. A curvelet not intersecting a discontinuity will have zero coefficients.
- There is no basis in which coefficients of an object with an arbitrary singularity curve can decay faster than in a curvelet frame (*optimally sparse representation*). In comparison:
  - Discontinuities destroy the sparsity of a Fourier series (Gibbs effect) – a large number of terms in required to reconstruct a discontinuity.
  - Wavelets, are localized/multi-scale and perform much better but because their frames are isotropic they do not represent higher-dimensional singularities (curves) effectively.

- 2-D GPR data (B-scans) contain wavefronts that correspond to reflections from structural inhomogeneities; these are generally curved, relatively smooth in their longitudinal direction and oscillatory in their transverse direction.
- Wavefronts are functions with intermittent regularity and their singularities correspond to geological inhomogeneities at which waves reflect.
- Owing to their anisotropic shape, curvelets are well adapted to detect wavefronts at different angles and scales because aligned curvelets of a given scale, locally correlate with wavefronts of the same scale.

IMPLEMENTATION

The analysis has been performed with the *Wrapping* Fast Discrete Curvelet Transform (WFDCT) algorithm (Candès et al., 2006, *Multiscale Modelling and Simulation*, 5, 861-899) available from http://www.curvelet.org/ and adapted for use by the matGPR software (Tzanis, 2010, *FastTimes*, 15, 17-43).

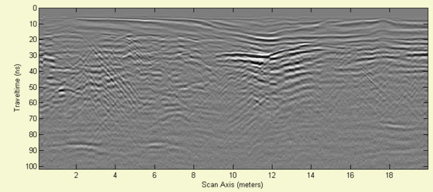
- The DCT partitions the Frequency-Wavenumber plane into a parabolically scaled series of second dyadic rectangular coronae and further sub-partitions each corona into angular wedges.
- The variable j represents the scale – each scale comprises a number of angular wedges that doubles at every second scale.
- Each angular wedge provides the support of a specific curvelet at scale 2^j and angle θ.



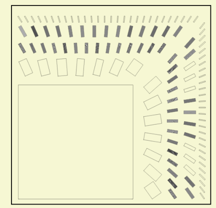
In the implementation presented herein:

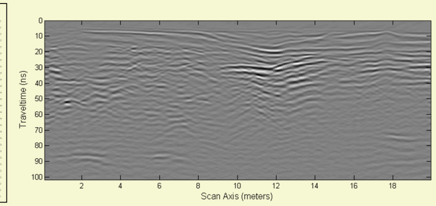
- The number of scales and number of angles at the second coarser scale is chosen.
- The WFDCT is computed and the upper right half of the f-k plane (dark shaded region in **A** above) is mapped as in **B** above.
- The user may select curvelet coefficients to include/exclude from the reconstruction of the data,
  - Either individually by un-checking/ checking the check-box associated with a particular set of coefficients, or,
  - Collectively (entire scales or angular subsets of a scale) using the drop-down menus

EXAMPLE 1

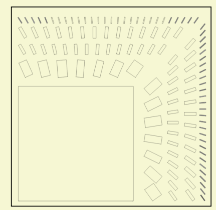


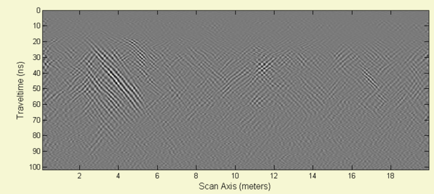
- Section distributed with package of Lucius and Powers (2002). Data measured with a GSSI SIR-2000 system and 500MHz antenna; size 1024×1024; sampling rates 0.099s/ 0.0194m. The data suffers from ***crossing clutter*** characteristic of multiple small targets or rough reflective surfaces.
- The noise can be locally strong, but it does not completely overshadow the data: the performance of the analysis can thus be easily evaluated.
- The curvelet decomposition comprised 7 scales; the 2nd coarser scale comprised 24 angular wedges





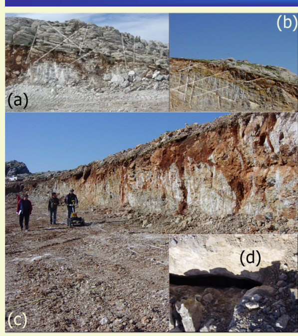
- Representation of noise-free data reconstructed from curvelets of scales 2^3 and 2^4 only; wavefront components with dip angles 110°–120° and 200°-300° also represent high angle noise and have been excluded.
- Empty boxes represent excluded curvelet coefficients





- Representation of the clutter reconstructed from curvelets of scale 2^5 while discarding low-angle wavefront components that comprise high-frequency (small scale) components of the data.
- Empty boxes represent excluded curvelet coefficients

EXAMPLE 2



The geological setting of the Ktenias ridge (altitude 1600m, NE Peloponnesus, Greece.

(a) Damaged thin-plated limestone with fractures and joints.

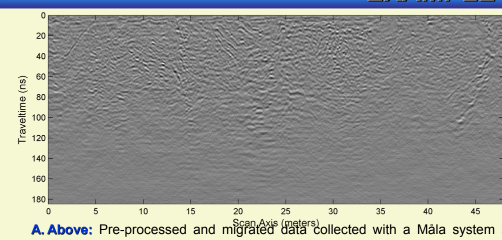
(b) Heavily fragmented and damaged limestone block overlying healthier bedrock of the same composition.

(c) Fragmented thin plated limestone with fractures filled with lateritic material; the bedding is also apparent.

(d) Gaping fracture in fragmented limestone block; such structures are abundant in the subsurface.

- In (a) and (d) the location of some significant faults, joints and interfaces is indicated with white lines.

All photographs are courtesy of Mr P. Sotiropoulos, Terra-Marine Ltd.



A. Above: Pre-processed and migrated data collected with a Måla system and 250MHz antenna on Mt Ktenias.

- The curvelet decomposition comprised 7 scales; the 2nd coarser scale comprised 24 angular wedges

B. Top Right: Signatures of steeply down-dipping reflections **(Type A)**, reconstructed from scale 2^6 with wavefront components oriented at 110°-180°.

- Type A reflections exhibit consistent angular relationship and dip at ±50° to ±60°: they correspond to joints and to synthetic and antithetic fractures.

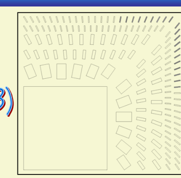
C. Middle-Right: Signatures of gently down-dipping reflections **(Type B)**, reconstructed from scale 2^5 with wavefront components oriented at 10° - 170°.

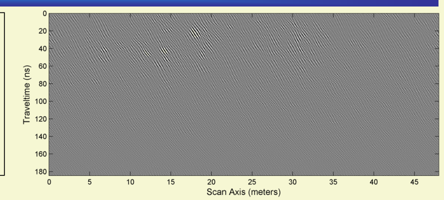
- The dip of Type B reflections is 10° – 20° and is consistent with the dip of the thin-plated limestone bedding and the limestone strata in general, (Figure 11a, b and c), which is observable at places because the aperture of the interfaces has been widened by damage and weathering and has been filled with lateritic material

D. Bottom-Right: Partial data reconstruction based on scales 2^2 and 2^3 and discarding wavefront components with dip angles 160 ° – 210° (noise).

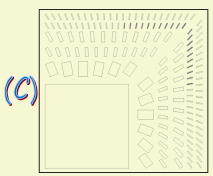
- The reconstruction comprises dull reflections from high attenuation interfaces forming anastomosing horizons or clustering in relatively narrow/ vertically extended complexes.
- These former are interfaces between limestone fragments filled with high attenuation lateritic material with the larger and internally less damaged blocks appearing in the form of lenses (hence the anastomoses).
- The latter (clusters) are signatures of cavities filled with lateritic material.

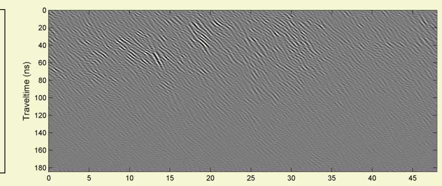
(B)





(C)





(D)