

# Evidence of non-extensivity and complexity in the seismicity observed during 2011-2012 at the Santorini volcanic complex, Greece.

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## Abstract

During the last months of 2011, an increase in the seismicity rates of the volcanic complex system of Santorini Island, Greece, was observed. In the present work, the temporal distribution of seismicity, as well as the magnitude distribution of earthquakes, has been studied. Using the concept of Non-Extensive Statistical Physics (NESP) along with the time evolution of Tsallis' information measure, we have investigated the inter-event time distribution between the successive earthquakes. Non-Extensive Statistical Physics, which is a generalization of Boltzmann-Gibbs statistical physics, seems a suitable framework for studying complex systems. The observed inter-event time distribution of Santorini seismicity can be described (fitted) with NESP models exceptionally well. This implies the inherent complexity of the Santorini volcanic seismicity, the applicability of NESP concepts to volcanic earthquake activity and the usefulness of NESP in investigating phenomena exhibiting multifractality and long-range coupling effects.

## 1. Introduction

Thera (Santorini) is the most active volcano of the Aegean volcanic arc (Bond and Sparks, 1976). Its volcanic activity began 600 thousand years ago (Perissoratis, 1995). The present day caldera was formed during the last main "Minoan" eruption, ~ 3600 years ago (Bond and Sparks, 1976). After the Minoan eruption, the volcanic activity concentrated in the central part of the caldera complex, producing lava domes, flows and pyroclasts forming Palaia and Nea Kameni islets between 197BC and 1950 AD (Stiros et al., 2010). Since then, the caldera has remained quiet. As far as it concerns the seismicity in recent years, the caldera is characterized by the almost complete absence of earthquakes and the main cluster of epicenters is located at the Coloumbo Reef, a submarine volcano about 10 km northeast of Santorini (Dimitriadis et al., 2009). During the last year, an increase in the seismicity rate of the volcanic complex system of Santorini was observed. The increasing earthquake activity in the caldera is accompanied with an important ground deformation, possibly the greatest since the 1950 eruption (Newman et al., 2012).

In the present work, the magnitude and temporal distribution of seismicity have been studied. Using the concept of Non-Extensive Statistical Physics (NESP; Tsallis, 2009) along with the evolution of the Tsallis' information measure we investigate the inter-event time distribution between the successive earthquakes. The analysis is based on the earthquake catalogue of the Geodynamic Institute of the National Observatory of Athens for the period 1/10/11-31/3/12 (<http://www.gein.noa.gr/>) for the seismicity of the Santorini volcanic complex area. According to Fig.1, the highest seismic rate is observed in December 2011 and in January 2012.

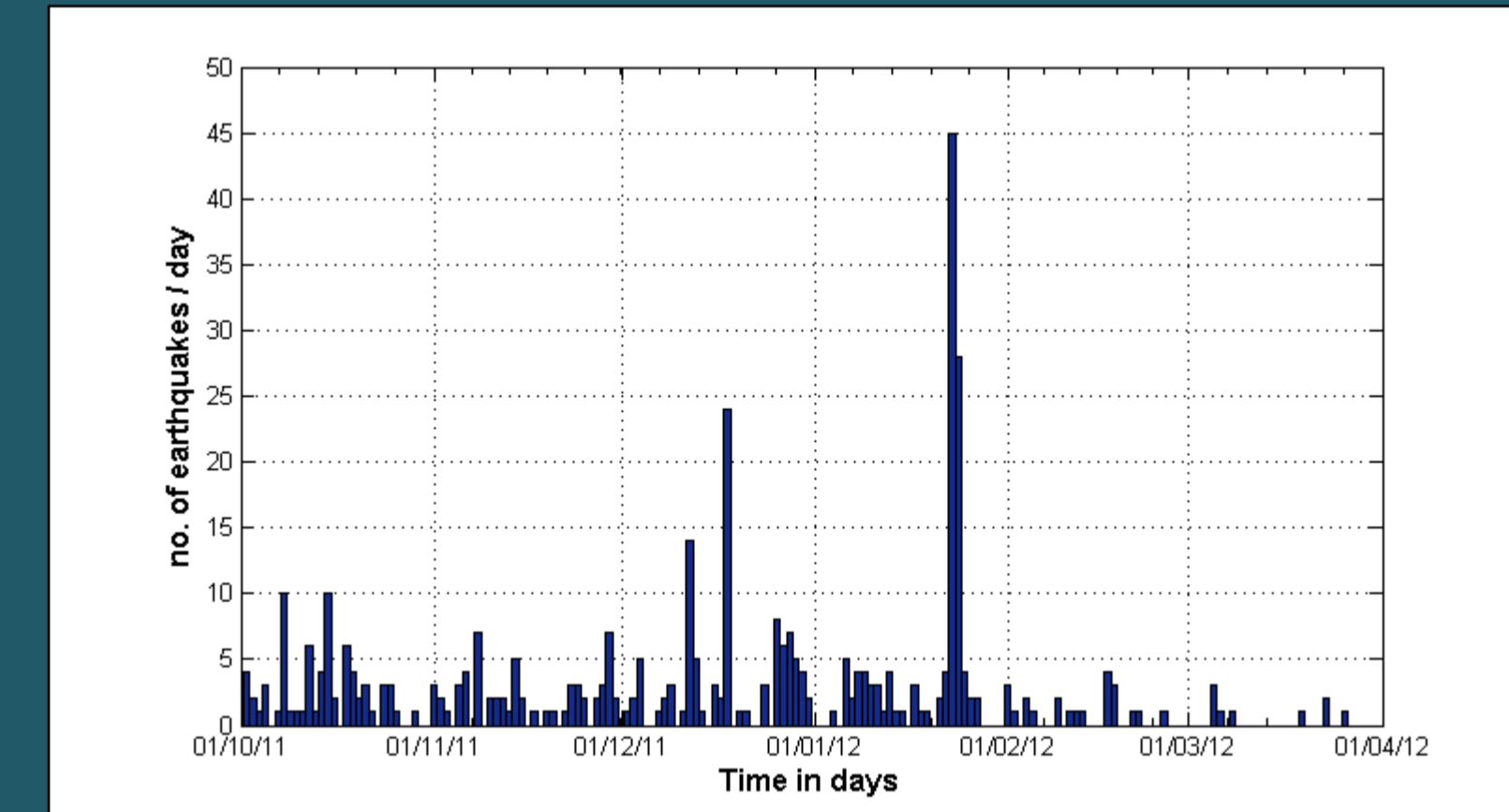


Figure 1: The seismicity rate in the Santorini volcanic complex area.

## 2. Frequency-Magnitude Distribution

The earthquake size distribution can be described by a power law relationship over a wide range of magnitudes which is commonly referred to as the Gutenberg-Richter law (Gutenberg and Richter 1944):

$$\log N(M) = a - bM \quad (1)$$

, where  $N$  is the cumulative number of earthquakes with magnitude  $\geq M$  and  $a$  and  $b$  constants. The so-called "b-value" is the slope of the frequency-magnitude distribution (FMD) and describes the relative size distribution of the earthquake events. While for tectonic earthquakes  $b$  has typical values below 1, higher b-values have been reported for volcano related earthquakes (McNutt 1996). High b-values associated with volcanic areas have been attributed to crustal heterogeneities, high thermal gradients and high pore-pressures at the vicinity of the magma chambers, all of which can cause a decrease of the effective normal stress (Wys 1973).

The FMD for the Santorini seismicity, referred to the period 1/10/11 – 31/3/12, has a bimodal character (Fig.2). The maximum likelihood solution (Aki 1965) of the Gutenberg – Richter law gives a b-value of 0.746 that holds for the magnitude range 1.2 – 2.6. According to the lowest magnitude for which this solution holds, we consider

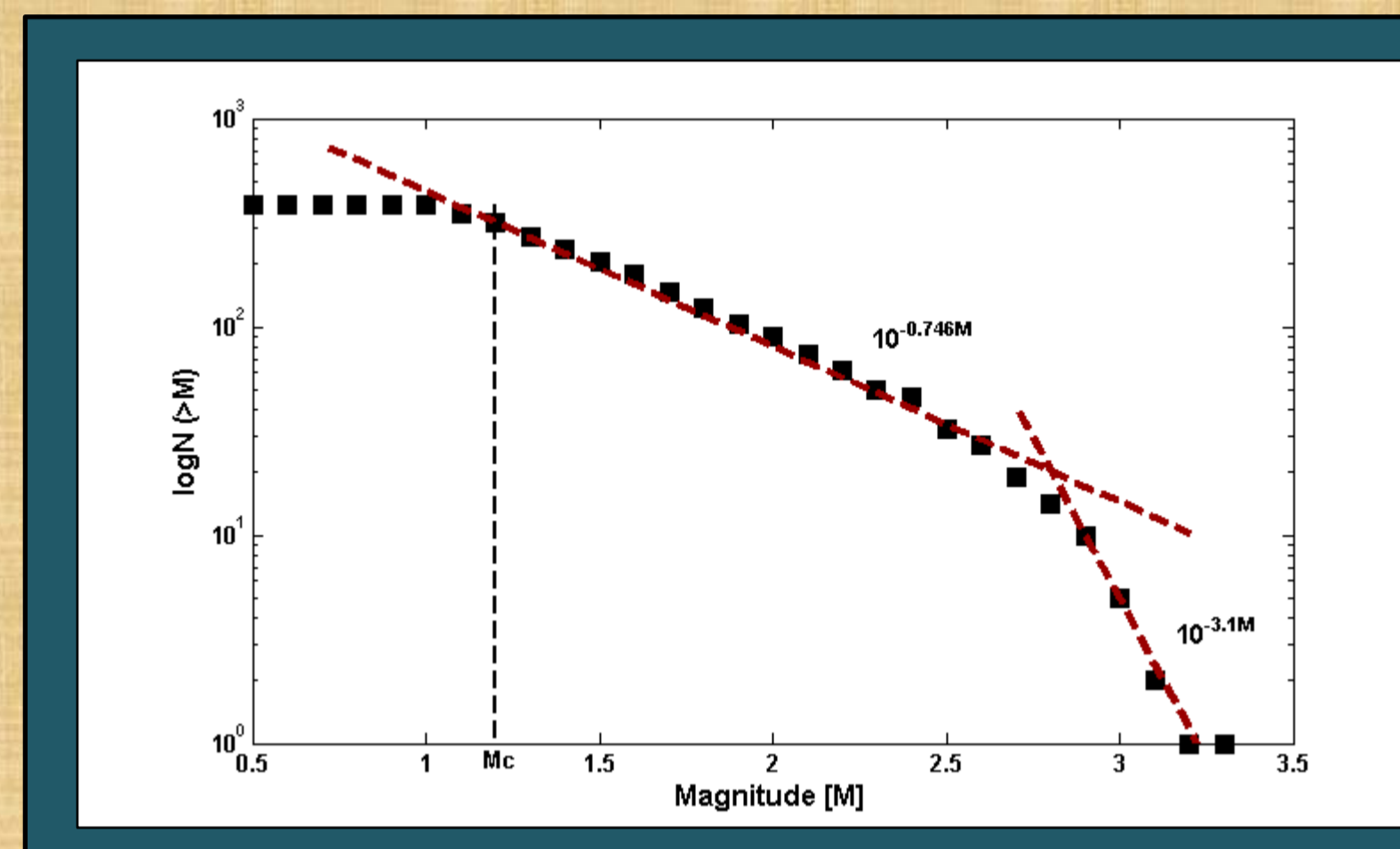


Figure 2: The cumulative (squares) size distribution on logarithmic scale for the Santorini seismicity. The distribution has a bimodal character with different b-values. We consider the magnitude of completeness ( $M_c$ ) as 1.2.

that our dataset is complete for magnitudes equal or above 1.2 ( $M_c$ ). As we can see from Fig.2, the slope of the FMD is getting higher with magnitude and takes the value of 3.1 for the higher observed magnitudes. Bimodal distributions in the FMD for volcanic areas have been attributed to earthquake swarms of similar sizes related to magma movements (Wiemer and Wyss 2002). For Santorini 2011-2012 seismic activity, the low b-value of 0.746 implies that seismicity has a strong tectonic component and that the related magma and fluid movements occur mainly through the pre-existing fractured surfaces and cracks. This is strongly supported by the SW-NE hypocenter distribution of the earthquakes during this period (Newman et al., 2012), along an almost vertical fault beneath the Kameni islets that is the site of the past intra-caldera eruptions over the 20<sup>th</sup> century. The departure from the power-law distribution at higher magnitudes implies that small earthquakes dominate and possibly indicates that the source size is restricted.

## 3. Analysis of Inter-event Time Distributions

In this section we analyze the temporal properties of the Santorini seismicity using the inter-event time distribution between successive earthquakes, which can provide useful insights in the physical mechanism of the earthquake process. For the analysis we use the Non-Extensive Statistical Physics concept (NESP) that has been frequently used to characterize complex dynamical systems that exhibit scale-invariance, (multi)fractality and long-range interactions. This concept refers to the non-additive entropy  $S_q$  (Tsallis 2009), which is a generalization of Boltzmann-Gibbs entropy  $S_{BG}$ . The non-additive entropy  $S_q$  reads as:

$$S_q = k_B \frac{1 - \sum_{i=1}^W p_i^q}{q-1}, q \in \mathbb{R} \quad (2)$$

, where  $k_B$  is Boltzmann's constant,  $p_i^q$  is a set of probabilities,  $W$  is the total number of microscopic configurations and  $q$  the entropic index. For the particular case where  $q=1$  we obtain the Boltzmann-Gibbs (BG) entropy. Using this concept, Abe and Suzuki (2005) investigated the temporal properties of the seismicity in California and Japan and more recently Vallianatos et al. (2012) investigated the spatiotemporal properties of the 1996 Aigion earthquake aftershock sequence. In these studies, the inter-event time distributions between successive earthquakes are nicely fitted by a q-exponential distribution and its inverse, the q-logarithmic distribution, implying the complexity and the existence of long range correlations in the temporal evolution of seismicity.

The inter-event time  $\Delta t$  is the time between two successive earthquakes defined as  $\Delta t = t(i+1) - t(i)$ . If  $P(>\Delta t)$  is the cumulative distribution of the probability finding an event greater than the inter-event time  $\Delta t$ , then the q-logarithmic distribution is  $\ln_q(P(>\Delta t))$ , where

$$\ln_q(x) = \frac{1}{1-q} (x^{1-q} - 1) \quad (3)$$

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After the estimation of the appropriate  $q$  that describes the observed distribution  $P(>\Delta t)$ , the q-logarithmic distribution  $\ln_q(P(>\Delta t))$  is linear with  $\Delta t$  (Vallianatos 2011 and references therein).

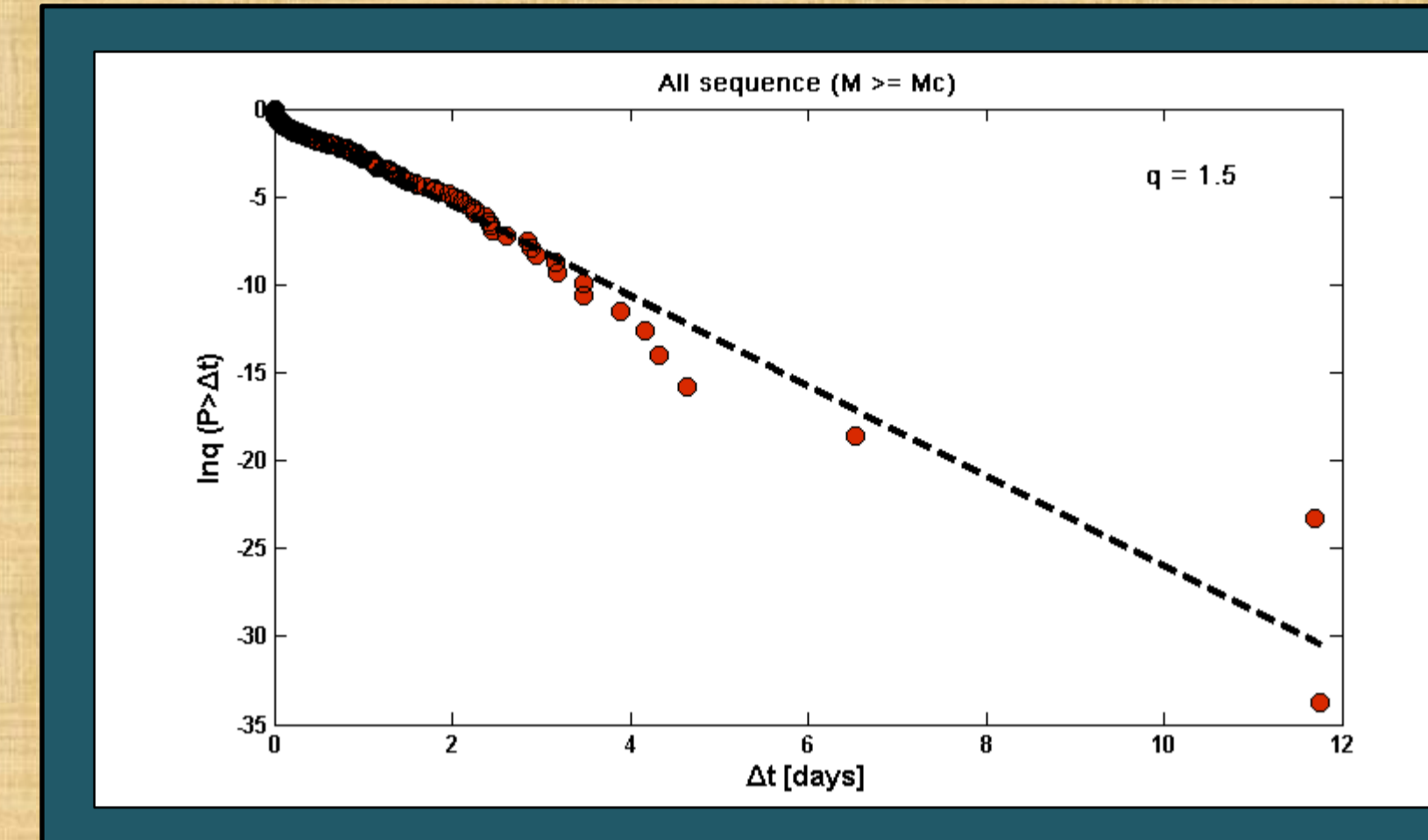


Figure 3: The semi- $q$ -log plot of the inter-event times cumulative distribution for the entire dataset. The dashed line represents the  $q$ -logarithmic function. The value of  $q$  for the best fit regression is  $q = 1.5$  with a correlation coefficient  $\rho = -0.9844$ .

Applying the NESP concept to the inter-event time distribution of the Santorini seismicity for the magnitudes  $\geq M_c$ , we can observe that for the value of the entropic index  $q=1.5$ , the inter-event time distribution  $P(>\Delta t)$  is nicely described by the q-logarithmic function (3) (Fig.3). We have also performed this analysis for different time periods of the earthquake evolution in order to detect possible alterations in the dynamics of the complex volcanic system (Fig.4 and Fig.5). We chose two periods, one from the beginning of this sequence at 1/10/11 until 22/1/12 and the other for the 23/1/12-24/1/12 sudden seismic crisis that is possibly related with a sudden magma or related fluids movement. The inter-event time distribution of the 23/1/12-24/1/12 seismic crisis is characterized by a higher q-value of  $q=1.73$  than for the period before, where  $q=1.3$ . This alteration implies that different time clustering and correlations exist at the temporal evolution of the Santorini seismicity, indicating the complexity that characterizes the dynamics of the system.

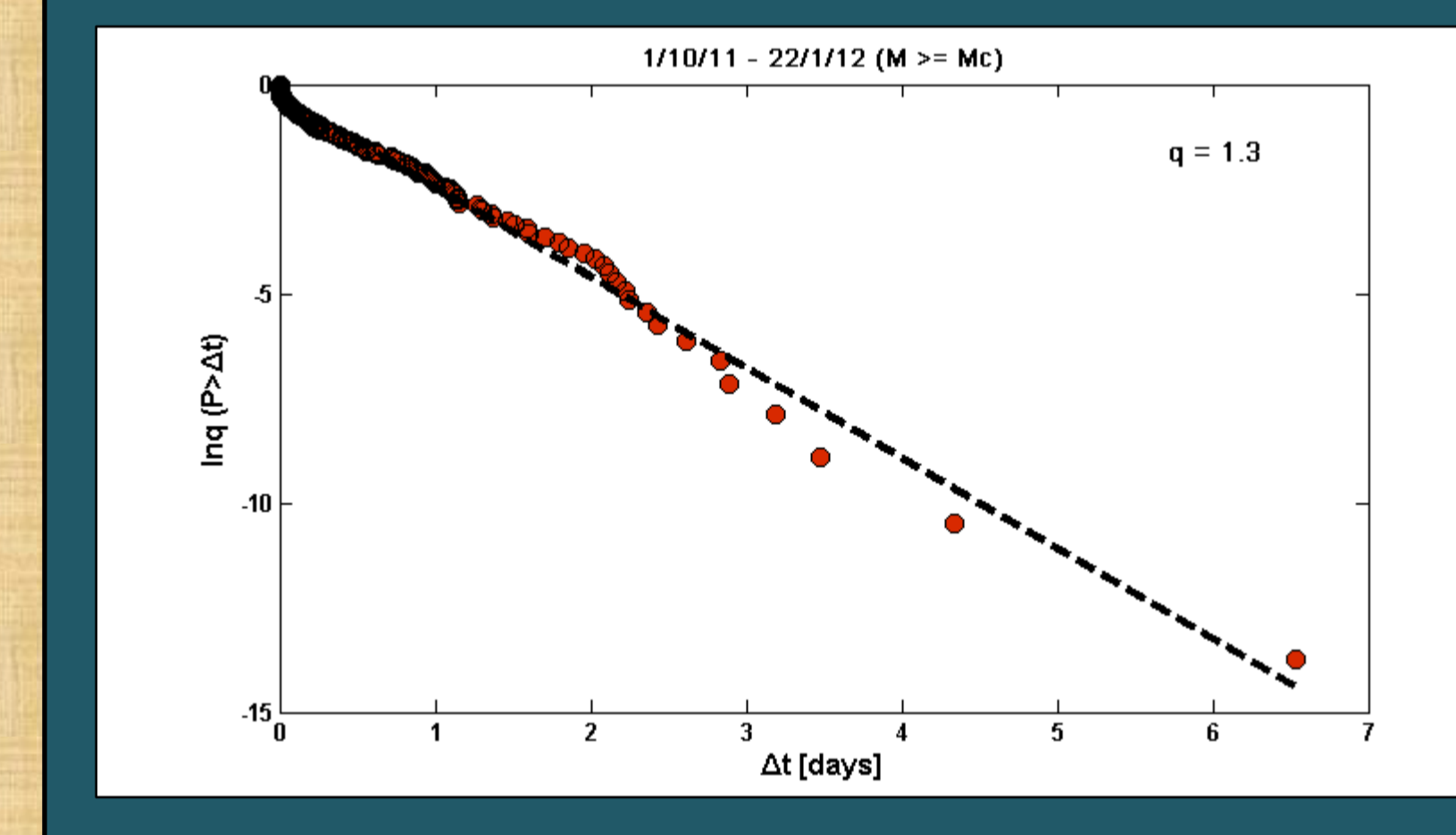


Figure 4: The semi- $q$ -log plot of the inter-event times cumulative distribution for the period 1/10/11-22/1/12. The dashed line represents the  $q$ -logarithmic function. The value of  $q$  for the best fit regression is  $q = 1.3$  with a correlation coefficient  $\rho = -0.9938$ .

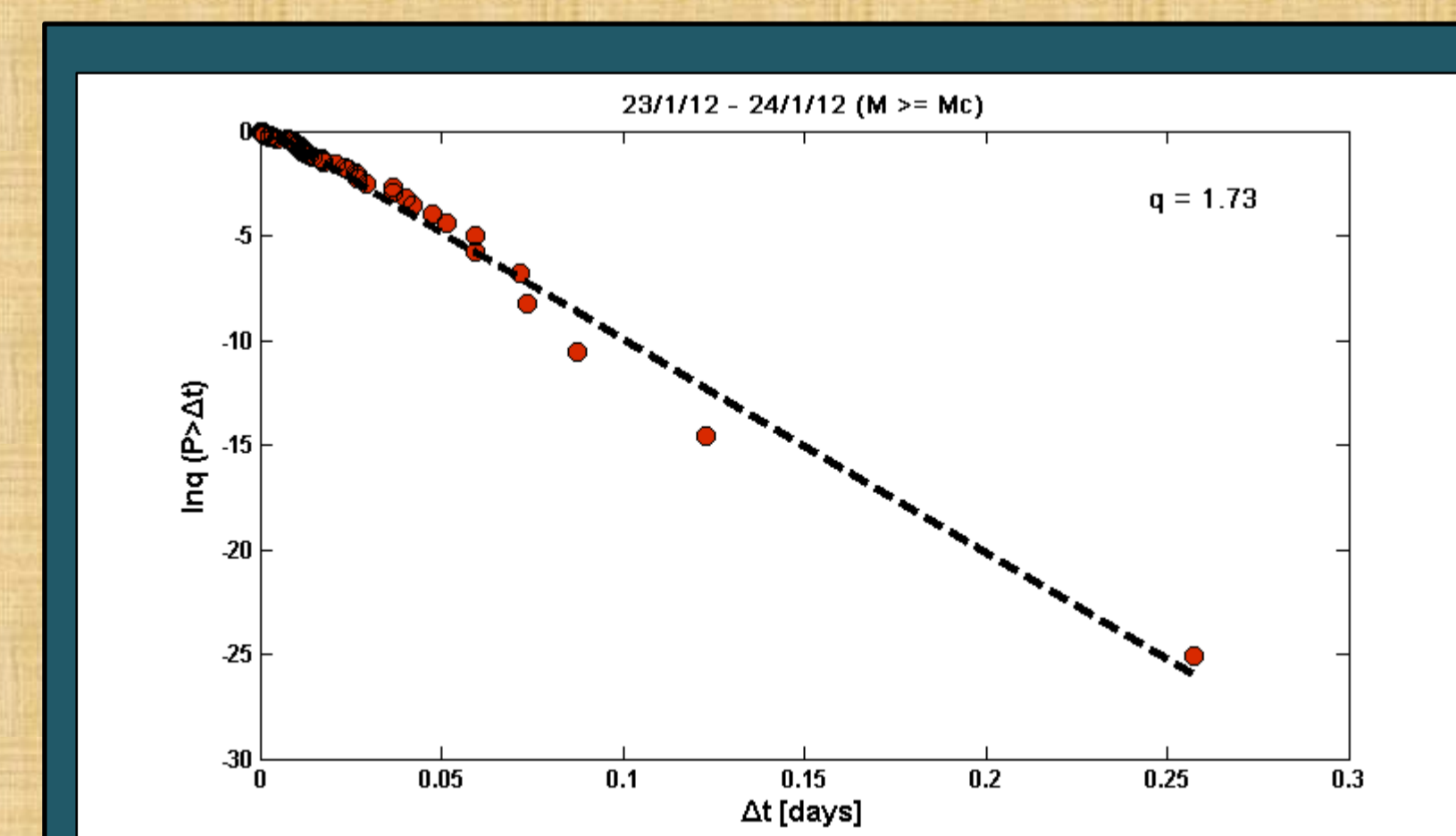


Figure 5: The semi- $q$ -log plot of the inter-event times cumulative distribution for the period 23/1/12-24/1/12. The dashed line represents the  $q$ -logarithmic function. The value of  $q$  for the best fit regression is  $q = 1.73$  with a correlation coefficient  $\rho = -0.9917$ .

## 4. Tsallis' Information Measure

In the previous section we have seen that the temporal evolution of Santorini seismicity, described by the inter-event time distribution, exhibits different patterns with time. This result has motivated us to study the statistical complexity that characterizes the inter-event time distributions through the time evolution of seismicity. Complexity measures

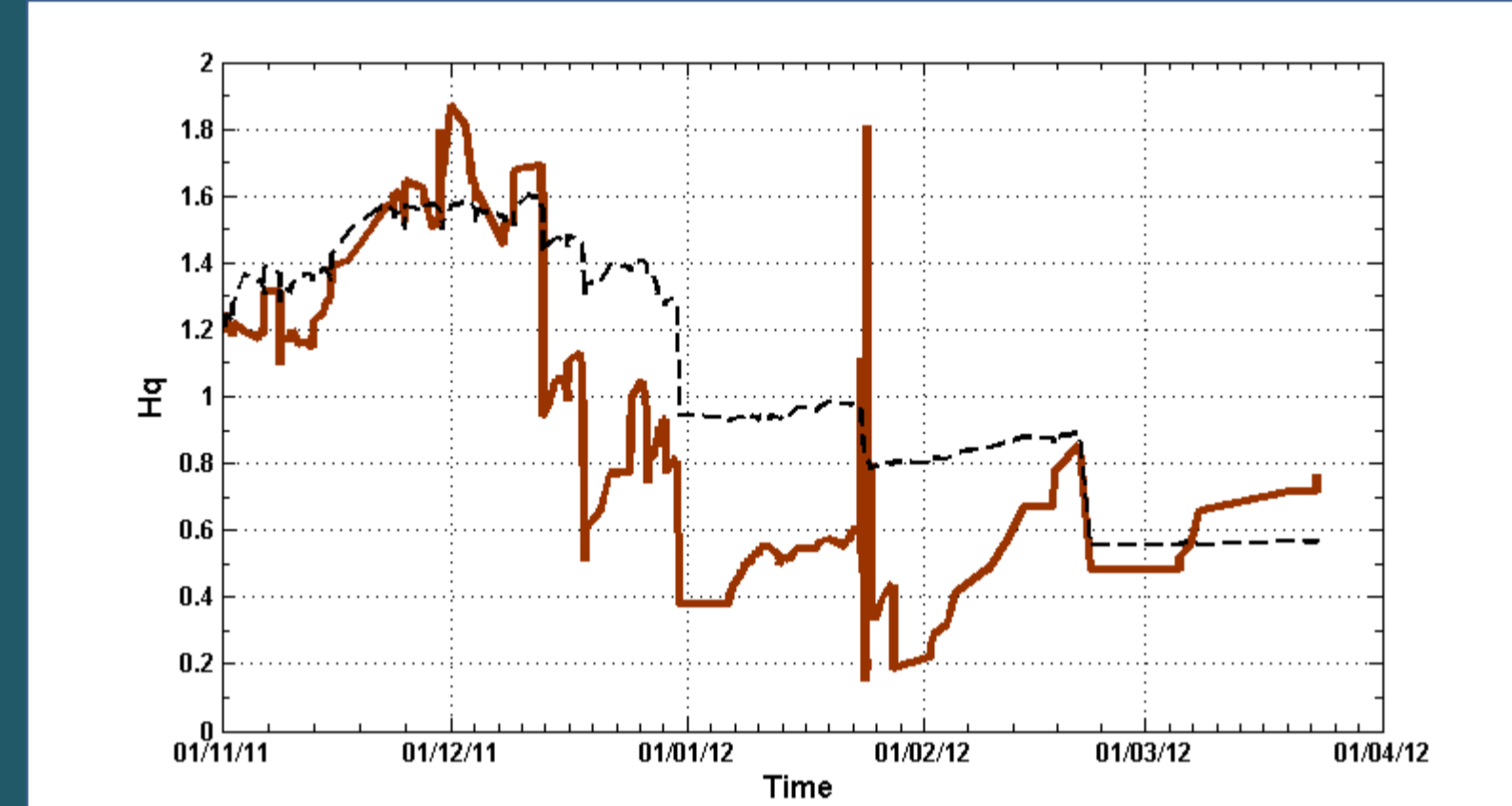
are generally performed to study pattern, structure and correlation of systems and processes and provide us with an information tool that can detect possible alterations in the dynamics of the system.

A well-known information measure is Shannon Entropy (Shannon 1948), which have been used to quantify the degree of disorder in dynamical systems. As we have seen, the inter-event time distribution of Santorini seismicity can well be described using the NESP concept. For this reason, we use a generalized information measure that includes Shannon information measure as a particular case. This generalized information measure, which has been derived in the NESP concept, is called *Tsallis' information measure* due to Tsallis (non-additive) entropy  $S_q$  and reads as (Cappuro et al., 1999):

$$H_q = \frac{1}{q-1} \sum_{i=1}^M (p_i - p_i^q) \quad (4)$$

This relationship describes a system  $\mathcal{L}$  with  $M$  possible microscopic configurations and  $p_i$  is the probability of finding the system in the configuration  $i$ . The parameter  $q$  is a real parameter and  $\sum_i p_i = 1$ .

Figure 6: Tsallis' information measure ( $H_q$ ) with time. The temporal sliding windows have a width of 50 events and a sliding factor of 1. The dashed line represents  $H_q$  for increasing temporal windows.



We use the Tsallis' information measure to study the complexity of the inter-event time distribution through time. For this purpose, we have defined sliding temporal windows depending on the width and the sliding factor. We have set the width of the temporal windows as 50, considering this number of events as the minimum number for having statistical significant results. In order to get the maximum resolution for  $H_q$  with time, we have set the sliding factor as 1. So, for each temporal window of 50 events we have calculated the inter-event times, the value of the entropic index  $q$  that describes better the inter-event time distribution in the NESP context and the Tsallis' information measure  $H_q$  that takes the time value of the last event in each temporal window.

The results are presented at Fig.6 for the dataset  $\geq M_c$ . For comparison, are also plotted the values of  $H_q$  (dashed line) for increasing time windows (initial width=50 and increasing factor=1), containing the cumulative number of events. From the diagram of Fig.6 we observe that higher complexity characterizes the inter-event time distribution during 11/11 and 12/11. Then  $H_q$  decreases and increases again with a sharp peak during the 23/1/12-24/1/12 sudden seismicity rate rise. From that time, the seismicity rate decreases and lower complexity characterizes the inter-event time distribution until the end of our dataset.

## 5. Conclusions

In this study we have investigated the magnitude and temporal distribution of seismicity during the recent unrest at the Santorini volcanic complex. The frequency-magnitude distribution departs from the power-law distribution at higher magnitudes, exhibiting a bimodal character. The low b-value of 0.746 along with higher b-values found for higher magnitudes imply the complexity of the seismicity process, controlled by both tectonic and volcanic processes. We have used the Non-Extensive Statistical Physics concept to describe the inter-event time distribution between the successive earthquakes. The observed distribution is well described (fitted) by the NESP model. Moreover, it exhibits different clustering and correlations with time. By applying the Tsallis' information measure to the inter-event time distributions for different temporal windows, we have seen that higher complexity characterizes the observed distributions when the seismicity rate is rising, most likely being linked with magma or fluids movement. All the results discussed in this study imply the inherent complexity of the Santorini volcanic seismicity, the applicability of NESP concepts to volcanic earthquake activity and the usefulness of NESP in investigating complex dynamical systems exhibiting multifractality and long-range coupling effects.

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