Precursory Acceleration of Seismicity: From the Theoretical Elegance to the Practical Difficulties

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It has been credibly argued that the earthquake generation process is a critical phenomenon culminating with a large event that corresponds to some critical point. In this view, a great earthquake represents the end of a cycle on its associated fault network and the beginning of a new one.

The dynamic organization of the fault network evolves as the cycle progresses and a great earthquake becomes more probable, thereby rendering possible the prediction of the cycle’s end by monitoring the approach of the fault network toward a critical state.

This process may be described by a power-law time-to-failure scaling of the cumulative Benioff strain:

\[
\frac{d \sum \varepsilon(t)}{dt} = k(t_c - t)^{-\alpha} \Rightarrow \sum \varepsilon(t) = K + A(t_c - t)^m
\]

where \( t_c \) is the time when the critical state is attained, \( A < 0, m < 1 \) and \( K = \sum \varepsilon @ t=t_c \).

- Observational evidence has corroborated the power-law scaling in many cases and has empirically determined that \( m \approx 0.3 \).
- Ben-Zion and Lyakhovsky (2002) theoretically predict give \( m=1/3 \).
- Rundle et al. (2000) show that the power-law activation associated with the excitation in proximity of a spinodal instability is essentially identical to the power-law acceleration of Benioff strain with \( m=0.25 \).
PHYSICAL MODELS

- More recently, the CP earthquake concept has gained support from the development of regional seismicity models with realistic fault geometry that show accelerating seismicity before large events.
- Essentially, these models involve stress transfer to the fault network during the cycle such that the region of accelerating seismicity will scale with the size of the culminating event, as for instance in Bowman and King (2001).
- It is thus possible to understand the observed characteristics of distributed accelerating seismicity in terms of a simple process of increasing tectonic stress in a region already subjected to stress inhomogeneities at all scale lengths.
- Then, the region of accelerating seismic release is associated with the region defined by the stress field required to rupture a fault with a specified orientation and rake; it is thus possible to incorporate tectonic information into the analysis.
A NEW THEORETICAL APPROACH

• Consider a crustal volume \( V(t) \), enclosed by a surface \( S_V(t) \), into which energy is transferred and stored as elastic deformation or dissipated by seismic and aseismic release.

• Because seismic release occurs only at faults, seismic dissipation involves only an effective sub-volume \( V_{\text{eff}} \subset V \).

• Let \( E_S(t) \) be the surface density of the energy flowing into \( V(t) \):

\[
E_S(t) = \frac{dE(t)}{dS_V(t)}
\]

• Let \( E_V(t) \) be the volume density of energy out flowing due to seismic release.

• Let \( R(t) \) be the volume density of the energy not related to seismic release.

• Conservation of energy demands

\[
E_S(t) \cdot S_V(t) = E_V(t) \cdot V_{\text{eff}}(t) + R(t) \cdot V(t)
\]

• Let \( L(t) \) be the characteristic size of \( V(t) \):

\[
S_V(t) \propto L^2(t) \quad \text{and} \quad V(t) \propto L^3(t).
\]

• Assume a fractal / hierarchical fault system with \( 2 < D < 3 \), so that \( V_{\text{eff}}(t) \propto L^D(t) \).

• Finally, assume that the seismic release rates scale with \( E_V(t) \):

\[
\frac{d\varepsilon(t)}{dt} = \gamma_0 [E_V(t)]^\alpha \quad \rightarrow \quad \frac{d\varepsilon(t)}{dt} = \gamma \left[ E_S(t) \frac{1}{L^{D-2}(t)} - R(t) \cdot L^{3-D}(t) \right]^\alpha
\]
A NEW THEORETICAL APPROACH ... continued

Because $E_S(t=t_c) \neq 0$,

$$E_S(t) = E_S(t_c) + E'_S(t_c) \cdot T_c \cdot \left( \frac{t_c - t}{T_c} \right) + E''_S(t_c) \frac{T_c^2}{2} \left( \frac{t_c - t}{T_c} \right)^2 + E'''_S(t_c) \frac{T_c^3}{6} \left( \frac{t_c - t}{T_c} \right)^3 + O_4 \left( \frac{t_c - t}{T_c} \right)^4$$

where $T_c$ is a characteristic time for setting out the preparation of the global event.

As $t \to t_c$, $V(t)$ shrinks toward the EQ focus. Assuming analyticity of $L(t)$ at $t \to t_c$,

$$L(t) = L'(t_c) \cdot T_c \cdot \left( \frac{t_c - t}{T_c} \right) + L''(t_c) \frac{T_c^2}{2} \left( \frac{t_c - t}{T_c} \right)^2 + L'''(t_c) \frac{T_c^3}{6} \left( \frac{t_c - t}{T_c} \right)^3 + O_4 \left( \frac{t_c - t}{T_c} \right)^4$$

Note that due to stress localization as $t \to t_c$, $L(t_c) \to 0$ and $L(t = t_c - T_c) = L'(t_c)$.

Keeping only the first order approximations above:

$$\frac{d \varepsilon(t)}{dt} = \gamma \left[ \frac{E_S(t_c)}{L'(t_c)^{D-2}} \left( \frac{t_c - t}{T_c} \right)^{2-D} + \frac{E'_S(t_c)}{L'(t_c)^{D-2}} \left( \frac{t_c - t}{T_c} \right)^{3-D} + R(t) \cdot L'(t_c)^{3-D} \left( \frac{t_c - t}{T_c} \right)^{3-D} \right]^\alpha$$

Observe that for $2 \leq D \leq 3$

$$\lim_{t \to t_c} \left( \frac{t_c - t}{T_c} \right)^{3-D} = 0$$
On integrating the first term only,

\[ \varepsilon(t) = \varepsilon(t = t_c) - \gamma \frac{T_c E_0^\alpha}{L'(t_c)^{\alpha(D-2)}} \frac{1}{\alpha(2 - D) + 1} \left( \frac{t_c - t}{T_c} \right)^{1+\alpha(2-D)} \]

Letting:

\[ K = \varepsilon(t = t_c), \]

\[ A(t) = -\gamma \frac{T_c^{\alpha(D-2)-1} E_S(t_c)^\alpha}{[L'(t_c)]^{\alpha(D-2)}} \cdot \frac{1}{\alpha(2 - D) + 1} \]

and

\[ m = \alpha \cdot (2-D) + 1, \]

- If \( \alpha > 0 \) and \( 2 < D \leq 2 + (1/\alpha) \), then \( m < 1 \) : the celebrated time-to-failure equation
- If \( \alpha < 0 \) the strain rate decreases with time (deceleration)
- For \( m \approx 0.25 \) (critical state) and \( D \approx 2.3 - 2.4 \) (runaway fracturing), \( \alpha \approx 1.8 - 2.5 \).
- The power-law scaling process depends on:
  - The scaling between deformation rate and seismic energy release (i.e. material properties)
  - The geometry (fractal distribution and dimension) of the fault system
• From a *theoretical* point of view, matters appear to be quite illuminated:

☞ It should be possible to predict the earthquake cycle’s end by monitoring the approach of the fault network toward a critical state.

☞ However, would this always be feasible or can reality be more complicated than our expectation?

☞ We will attempt an answer,

1. First by presenting an example (out of a few),
2. Then by appealing to the theory.
The Southwest segment of the Hellenic Arc is the most active plate margin of the Mediterranean area, with correspondingly high seismicity and relatively frequent occurrences of large earthquakes.


It has been shown that $M_w \approx M_{L(NOA)} + 0.5$, for $3.6 \leq M_L \leq 6.5$ (e.g. Papazachos et al, 2002)

The magnitude of completeness is $M_L = 3.6$
The Benioff strain, defined as \( \varepsilon(t) = \sum_{i=1}^{N(t)} \sqrt{E_i(t)} \) with \( E_i(t) \) being the energy of the \( i \)th event, \( N(t) \) the total number of events at time \( t \) and \( \log_{10}E_i(t) = 4.7 + 1.5 \times M_W \) (e.g. Papazachos and Papazachos, 2000).

The power-law model is fitted with a non-linear non-linear Nelder-Mead optimisation procedure, operating on the \( L_2 \) norm.

The suitability of the power-law behaviour is tested using the CURVATURE

\[ C = \frac{\text{Powerlaw fit RMS}}{\text{Linearfit RMS}} \]

which should be significantly less than 1 if power-law affords a better approximation.

The analysis entails the following procedure:

- Power-law model if fitted to earthquake data within concentric circular areas.
- Radius at which \( C \) is minimum and corresponding model parameters are deemed optimal
- The procedure is repeated on a regular grid and maps of the curvature, the critical exponent, the critical time and the predicted magnitude are compiled
The curvature shows areas of strong(er) / weak(er) power-law behaviour.

Power-law behaviour will nonetheless be observed both when seismicity is accelerating ($m<1$) or decelerating ($m>1$).

The answer is given by the distribution of the critical exponent, which shows a well structured butterfly pattern with nearly sharp boundaries between exponents greater or smaller than unity.
Power-law models are computed with earthquakes within the areas of stress increase / decrease.

Examples for earthquakes within the +1kPa contour (acceleration) and the -1kPa contour (deceleration) are shown.

The critical exponent in the accelerating branch is $\approx \frac{1}{4}$, consistent with the view of power-law acceleration as a Self-Organising Spinodal (SOS – a CP system undergoing a repetitive series of first order phase transitions - Rundle et al., 2000).
One year later, the overall pattern of acceleration / deceleration remains unchanged, but the seismic release rates appear to have slowed down and an updated model indicates a critical time deferred to 2004.6.

The updated critical exponent is still consistent with the SOS concept (0.27).

There’s no significant change as to the size of the predicted event.
One year later, the overall pattern of acceleration / deceleration is still the same but an updated model indicates a critical time deferred to 2006.3. The slowed seismic release rates appear to have recently picked up again.

The critical exponent is has increased ($m=0.31$) but is still consistent with the CP concept.

The predicted size of the “event” appears to have increased.
The observations were consistent with almost all of the predictions of the CP / stress transfer model and every time, it appeared as if a \textit{bona-fide} prediction had been made.

\textit{Re-evaluation} of seismicity changes shows that the activated area may actually be relaxing, or the crustal material has stiffened and does not release as much seismic energy; the time of failure has been deferred to approximately 2006.3.

Empirically speaking:

\textbullet Time-to-failure modelling of accelerated seismicity is a relatively \textit{new field} of study with \textit{few} cases-histories whence to draw experience, \textit{most of which, comprise retrospective analyses of past earthquakes}.

\textbullet Still, very little is known as to the development of real-time situations and their probability of success or failure.
Fault systems are not isolated - *deformation depends on the time-varying rate of energy transfer from without*.  

We have shown that the time-to-failure power law *depends* on *time-varying factors: Fault system geometry and Material Properties*.  

The variation of these parameters may drive the system *back and forth*, between a *subcritical self-organising mode* with *reduced* seismic release rates and a *full-scale self-organising mode* with *accelerated* release rates.  

The variation is uncontrollable and unpredictable, as also are the factors pertaining to the nucleation process, which have their own time dependence.  

Therefore, even if a full-scale self-organising process operates in some area, it is not at all necessary that a large earthquake will occur as soon as the system goes critical. *The critical point model merely predicts that past the critical time an earthquake is possible, not certain.*  

In spite of the theoretical elegance it enjoys, *the real-time predictive capacity* of the CP / stress transfer earthquake concept *is still to be “critically” tested*. Such testing would be important in assessing the feasibility of using the model to evaluate short and intermediate term seismic hazard.

*WE THINK YES!*