The Characteristic States of the Magnetotelluric Impedance Tensor for General Earth Conductivity Distributions I: Construction

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SUMMARY

This presentation shows that the (equivalent) *Singular Value Decomposition* and *Canonical Decomposition* of the MT impedance tensor are, in fact, proper rotations in 3-D space based on the topology and operators of the SU(2) rotation group: they comprise a symmetric generalized eigenstate – eigenvalue formulation of the Magnetotelluric (MT) induction problem suitable for the analysis of general Earth conductivity structures. It is also demonstrated that they can be reformulated into an anti-symmetric (characteristic) generalized eigenstate – eigenvalue decomposition consistent with the anti-symmetric interaction between electric and magnetic fields referred to the same coordinate frame. In both cases, the decomposition involves a left operator whose columns comprise the eigenvectors of the electric field and a right operator whose columns comprise the eigenvectors of the total magnetic field; it yields two characteristic states (generalized eigenstates) that comprise simple proportional relationships between linearly polarized generalized eigenvalues of the magnetic and electric field along the locally fastest (resistive) and slowest (conductive) propagation path into the Earth. The proportionality is respectively expressed by the maximum and minimum characteristic values of the impedance tensor (eigen-impedances). The electric and magnetic eigen-fields are 3-D and non-orthogonal in the real 3-space; their tilt is respectively a measure of the local landscape of the total electric and magnetic field. When the eigen-fields are projected on the axes of the horizontal observational coordinate frame their components are superimposed and the resulting mixing of phases is manifest in the form of elliptical polarization. The eigen-impedances provide a succinct analytical representation of the impedance tensor and may be valuable for interpretation. Their utility is explored in a sequel (part II) presentation.

Keywords: Impedance Tensor, Rotation, Decomposition, Characteristic State, Generalized Eigensate

INTRODUCTION

The *equivalent* Singular Value Decomposition (La Torraca et al., 1986) and Canonical Decomposition (Yee and Paulson, 1987) of the impedance tensor offer a unique and powerful analytical and interpretational tool but did not have many applications and are generally not acknowledged for what they actually are. They have been presented as *ad hoc* complete formulations of *polarization states* and not as topologically complete descriptions of the *intrinsic geometry* of the MT field. Arguably, due to their "complicated" form and without due explanation, they may have easily been understood as nothing more than *ad hoc*, not particularly useful descriptions of the tensor.

One difficulty in elucidating the nature of the SVD/CD may be that their theoretical basis is borrowed from quantum mechanics and is 'exotic'. However, one may note that the relationship $\mathbf{E}(\omega)=\mathbf{Z}(\omega)\cdot\mathbf{H}(\omega)$, defines the impedance to be a form of "boson" mediating the exchange of energy between the electric and magnetic fields; this "boson" looks down and has a spin of 1. The MT field can also be viewed as an ensemble of low frequency – large size photons negotiating their way through the inhomogeneous Earth and thinking of it as an anisotropic *birefringent* material: in this view, the rank 2 impedance tensor should encode geometrical information about the fast (resistive) and slow (conductive) directions through this material. With such simple arguments it should be straightforward to see why one may borrow tools from both polarization optics and quantum mechanics to analyse the impedance tensor. The SVD/CD formulation implicitly adopted the former (indirect) approach. Because the analytical tools of polarization optics are based on the mathematics of spin analysis developed for quantum mechanics, the present work attempts to close the circle by taking the second (direct) approach.

SU(2) GROUP AND ROTATION OPERATORS

A basic tool to be used in the ensuing analysis is the SU(2) rotation group and its irreducible representations. A good introduction to the group exists in Arfken and Weber (2005) and advanced presentations in Murnaghan (1938), Wigner (1959) and others. Only absolutely essential information is given here.

SU(2) is a continuous, compact Lie group comprising 2×2 unitary(unimodular) matrices **U**, such that |**U**|=1 (rotations only) and $\mathbf{U}^{\dagger}=\mathbf{U}^{-1}$ $\forall$ **U** $\in$ SU(2) with (†)denoting Hermitian transposition. In the space $\mathbb{R}^3$ we live in, rotations are specified by representations of the Special Orthogonal Lie group SO(3) of 3×3 real valued unimodular matrices. From any Cartesian tensor in $\mathbb{R}^3$, one can define a mapping onto the set of 2x2 complex matrices, in the Hilbert space of complex-valued squared functions $\mathbb{C}^2$, in the sense

$$\mathbf{P}(x,y,z) = \boldsymbol{\sigma}_1 x + \boldsymbol{\sigma}_2 y + \boldsymbol{\sigma}_3 z = \begin{vmatrix} \mathbf{z} & \mathbf{x}+i\mathbf{y} \\ \mathbf{x}-i\mathbf{y} & -\mathbf{z} \end{vmatrix},$$

with $|\mathbf{P}|=\mathbf{x}^2+\mathbf{y}^2+\mathbf{z}^2=1$. σ_j are the Pauli spin matrices with properties $(\sigma_j)^2=\mathbf{I}$ and $-i[\sigma_i,\ \sigma_j]=2\varepsilon_{ijk}\sigma_k$, so that together with **I** they form an orthonormal vector basis of the 3-D space over the real field. SU(2) enters as a symmetry group in $\mathbb{C}^2$. For any unimodular matrix **U**$\in$SU(2), an arbitrary unitary transformation $\mathbf{P} \rightarrow \mathbf{Q} = \mathbf{U}\cdot\mathbf{P}\cdot\mathbf{U}^{\dagger}$ is also traceless Hermitian, so that $\mathbf{Q}(x',y',z') = \boldsymbol{\sigma}_1 x' + \boldsymbol{\sigma}_2 y' + \boldsymbol{\sigma}_3 z'$. Since |**Q**| = |**P**|, the real linear transformation $\{x, y, z\} \rightarrow \{x', y', z'\}$ induced by $\mathbf{P} \rightarrow \mathbf{Q} = \mathbf{P}(x', y', z')$ is such that $\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = \mathbf{x}'^2 + \mathbf{y}'^2 + \mathbf{z}'^2$. In other words, $[\mathbf{x}'\ \mathbf{y}'\ \mathbf{z}']^T = \mathbf{O}\cdot[\mathbf{x}\ \mathbf{y}\ \mathbf{z}]^T$, where **O** is a *real* orthogonal 3x3 matrix comprising a representation of group SO(3). It is easy to show that any subsequent unitary transformation $\mathbf{Q} \rightarrow \mathbf{R} = \mathbf{V}\cdot\mathbf{Q}\cdot\mathbf{V}^{\dagger}$ induces the orthogonal transformation **O**(**VU**) = **O**(**V**) **O**(**U**). This demonstrates in a naïve albeit instructive manner, that the collection of 3x3 real orthogonal matrices **O**(**U**) obtained by letting **U** wander over the 2x2 unitary group SU(2) constitutes a representation of the 3x3 rotation group SO(3) by 2x2 unitary matrices. The representation is locally isomorphic and globally homeomorphic, meaning that rotations will be unique to within a symmetry of 2π: SU(2) comprises the *universal covering space* of SO(3).

The $\mathbb{R}^3$ electric and magnetic coordinate frames commonly used in MT data acquisition are right-handed with the *y*-axis positive to the right of the *x*-axis and the *z*-axis pointing down. Given the correspondence between rotations in $\mathbb{C}^2$ and $\mathbb{R}^3$, the reasonable choice for would be to rotate the impedance tensor about the real axes of the basis (**x**+*i***y**, **z**). It is, then, simple to show that a clockwise rotation about the *z*-axis followed by a clockwise rotation about the *x*-axis is performed by the operator

$$\mathbf{U}_{zx}(\varphi,\theta) = \mathbf{U}_z(\varphi)\mathbf{U}_x(\theta) \qquad (1)$$

$$= \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}.$$

Further justification for the choice of operator will be given below

ROTATION AND DECOMPOSITION

The impedance tensor may be defined in one of two *right-handed* coordinate systems: In the coordinate system used in the SVD/CD, the horizontal axes of the magnetic reference frame (x_h, y_h) are rotated by 90° clockwise with respect to the horizontal axes of the electric reference frame (x_e, y_e); the relationship between the magnetic and electric fields in this system is denoted by is written $\mathbf{E} = \mathcal{Z}\cdot\underline{\mathbf{H}}$ is apparently *symmetric*. In the coordinate system commonly used in MT practice, the magnetic and electric frames are identical and the familiar relationship **E = Z·H** is *anti-symmetric*. The tensors $\mathcal{Z}$ and **Z** are related as

$$\mathcal{Z} = \mathbf{Z}\cdot\mathbf{R}(\pi/2) \qquad (2)$$

A rotation by a single operator of the form $\mathbf{U}_{zx}(\theta, \varphi)$ *cannot* reduce the *regular* complex $\mathcal{Z}$ to a diagonal form because $[\mathcal{Z},\mathcal{Z}^{\dagger}]\neq 0$: the tensor depends on eight degrees of freedom (topological dimensions), where each of $\Re\{\mathcal{Z}_{ij}\}$ and $\Im\{\mathcal{Z}_{ij}\}$ is assigned with one degree of freedom, but the rotation would depend on only six and would be *incomplete*. Exactly two operators $\mathbf{U}(\theta_1, \varphi_1)$ and $\mathbf{V}(\theta_2, \varphi_2)$ are required, thereby providing an eight parameter set that completely describes the tensor: four in the two complex principal impedances and four rotation angles.

The products $\mathcal{C}_1 = \mathcal{Z}\cdot\mathcal{Z}^{\dagger}$ and $\mathcal{C}_2 = \mathcal{Z}^{\dagger}\cdot\mathcal{Z}$ are normal (Hermitian) matrices with and constitute mappings of $\mathcal{Z}$ onto $\mathbb{C}^2$. Each of $\mathcal{C}_j$ depends on only four degrees of freedom and can be diagonalized with a single unitary rotation operator of the form (1). Their *eigenvalue-eigenvector* decompositions are:

$\mathcal{C}_1 = \mathbf{U}(\theta_1, \varphi_1)\cdot\mathcal{D}\cdot\mathbf{U}^{\dagger}(\theta_1, \varphi_1)$

and

$\mathcal{C}_2 = \mathbf{V}(\theta_2, \varphi_2)\cdot\mathcal{D}\cdot\mathbf{V}^{\dagger}(\theta_2, \varphi_2)$

where $\mathcal{D} = \tilde{\mathcal{Z}}\cdot\tilde{\mathcal{Z}}^{\dagger} = \tilde{\mathcal{Z}}^{\dagger}\cdot\tilde{\mathcal{Z}}$ and

$$\tilde{\mathcal{Z}} = \begin{bmatrix} \zeta_1 & 0 \\ 0 & \zeta_2 \end{bmatrix}, \quad |\zeta_1| > |\zeta_2|, \quad \zeta_j\zeta_j^* = r_j^2, \; j = 1,2$$

is the *characteristic impedance* (*eigen-impedance*) tensor. Then it is easily shown that

$$\tilde{\mathcal{Z}} = \mathbf{U}^{\dagger}(\theta_1,\varphi_1)\cdot\mathcal{Z}\cdot\mathbf{V}(\theta_2,\varphi_2) \qquad (3)$$

which is precisely the SVD/CD of the impedance tensor.

Now, right multiply Equation 3 by **R**(π/2) to rotate the eigen-impedance tensor 90° counter-clockwise and on substituting Equation 2 obtain

$$\tilde{\mathbf{Z}} = \tilde{\mathcal{Z}} \cdot \mathbf{R}(-\pi/2) = \\ = \mathbf{U}^{\dagger}(\theta_1,\varphi_1)\cdot \mathbf{Z}\cdot \mathbf{R}(\pi/2)\cdot \mathbf{V}(\theta_2,\varphi_2)\mathbf{R}(-\pi/2) , \quad (4)$$

with $\tilde{\mathbf{Z}} = \tilde{\mathcal{Z}} \cdot \mathbf{R}(-\pi/2)$ being the eigen-impedance tensor in the practical MT reference frame. Moreover, $\mathbf{V}(\theta_2,\varphi_2) = \mathbf{V}_z(\varphi_2)\cdot\mathbf{V}_x(\theta_2)$ and because $\mathbf{R}(\pm\pi/2) \in SO(2) \subseteq SU(2)$ and $\mathbf{V}_z \in SO(2) \subseteq SU(2)$, the operators commute. It is thus straightforward to show that, $\mathbf{R}(\pi/2)\cdot\mathbf{V}(\theta_2,\varphi_2)\cdot\mathbf{R}(-\pi/2) = \mathbf{V}^*(\theta_2,\varphi_2)$. Substituting in Equation 4 one obtains,

$$\tilde{\mathbf{Z}} = \begin{bmatrix} 0 & \zeta_1 \\ -\zeta_2 & 0 \end{bmatrix} = \mathbf{U}^{\dagger}(\theta_1,\varphi_1)\cdot \mathbf{Z}\cdot \mathbf{V}^*(\theta_2,\varphi_2) \quad (5)$$

which is the anti-symmetric decomposition of **Z** in the practical MT reference frame; it comprises an adaption of the generalized (complex) SVD to physical systems with anti-symmetric intrinsic geometry and will henceforth be referred to as the *Anti-symmetric SVD* or ASVD.

THE CHARACTERISTIC STATES

Henceforth we concentrate on the ASVD. For reasons to be immediately apparent use the notation $\mathcal{E}(\theta_E,\varphi_E) \equiv \mathbf{U}(\theta_1,\varphi_1)$ and $\mathcal{H}(\theta_H,\varphi_H) \equiv \mathbf{V}^*(\theta_2,\varphi_2)$. Re-arrange terms to write Equation 5 as

$$\mathbf{Z} = \mathcal{E}(\theta_E,\varphi_E)\cdot \tilde{\mathbf{Z}}\cdot \mathcal{H}^{\dagger}(\theta_H,\varphi_H) \quad (6)$$

and substitute in $\mathbf{E} = \mathbf{Z}\cdot\mathbf{H}$ to obtain

$$\mathcal{E}^{\dagger}(\theta_E,\varphi_E)\cdot \mathbf{E} = \tilde{\mathbf{Z}}\cdot \mathcal{H}^{\dagger}(\theta_H,\varphi_H)\cdot \mathbf{H} \quad (7)$$

The column vectors of the rotation operators $\mathcal{E}$ and $\mathcal{H}$ describe rotations of opposite handedness and constitute, in themselves, orthogonal rotation operators for 2-component orthogonal vectors. Denote

$$\mathcal{E}(\theta_E,\varphi_E) = \left[\mathbf{e}_1(\theta_E,\varphi_E) \quad \mathbf{e}_2(\theta_E,\varphi_E+\tfrac{\pi}{2})\right],$$

$$\mathcal{H}(\theta_H,\varphi_H) = \left[\mathbf{h}_1(\theta_H,\varphi_H) \quad \mathbf{h}_2(\theta_H,\varphi_H+\tfrac{\pi}{2})\right],$$

such that $\mathbf{e}_i^{\dagger}\bullet\mathbf{e}_j = \delta_{ij}$ and $\mathbf{h}_i^{\dagger}\bullet\mathbf{h}_j = \delta_{ij}$, whereupon Equation 7 yields:

$$\begin{bmatrix} \mathbf{e}_1^{\dagger}\mathbf{E} \\ \mathbf{e}_2^{\dagger}\mathbf{E} \end{bmatrix} = \begin{bmatrix} 0 & \zeta_1 \\ -\zeta_2 & 0 \end{bmatrix}\cdot \begin{bmatrix} \mathbf{h}_1^{\dagger}\mathbf{H} \\ \mathbf{h}_2^{\dagger}\mathbf{H} \end{bmatrix}, \quad (8)$$

or,

$$\begin{bmatrix} E_1(\theta_E,\varphi_E) \\ E_2(\theta_E,\varphi_E+\tfrac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 & \zeta_1 \\ -\zeta_2 & 0 \end{bmatrix}\cdot \begin{bmatrix} H_1(\theta_H,\varphi_H) \\ H_2(\theta_H,\varphi_H+\tfrac{\pi}{2}) \end{bmatrix},$$

which can be succinctly written $\tilde{\mathbf{E}} = \tilde{\mathbf{Z}}\cdot\tilde{\mathbf{H}}$. It is apparent that

$$\tilde{\mathbf{Z}} = \begin{bmatrix} 0 & E_1/H_2 \\ -E_2/H_1 & 0 \end{bmatrix}. \quad (9)$$

Equation 8 says that **H** rotated to (θ_H, φ_H+π/2), is mapped onto **E** rotated to (θ_E, φ_E) along the least conductive (fast) path through the Earth. This corresponds to the *maximum state* of **Z**. Likewise, **H** rotated to the direction (θ_H, φ_H) is mapped onto **E** rotated to the direction (θ_E, φ_E+π/2) along the most conductive (slow) path. This is the *minimum state* of **Z**. The angles (θ_E, φ_E) define the orientation of the *characteristic* coordinate frame or *eigen-frame* $\{x_E, y_E, z_E\}$ of the *electric eigen-field* $\tilde{\mathbf{E}}$, so that x_E is rotated by φ_E clockwise with respect to the *x*-axis of the experimental coordinate frame and the plane $\{x_E, y_E\}$ is tilted by θ_E clockwise with respect to the horizontal plane $\{x, y\}$. Likewise, the angles (θ_H, φ_H) define the orientation of the characteristic eigen-frame $\{x_H, y_H, z_H\}$ of the *magnetic eigen-field* $\tilde{\mathbf{H}}$, so that x_H is rotated by φ_H clockwise with respect to the *x*-axis of the experimental coordinate frame and the plane $\{x_H, y_H\}$ is tilted by θ_H clockwise with respect to the horizontal plane $\{x, y\}$. Each eigen-frame contains orthogonal, *linearly polarized* components. However, $\varphi_E \neq \varphi_H$ in general and the electric and magnetic eigen-frames are not mutually orthogonal. The electric and magnetic eigen-frames are also not horizontal: the tilt angles θ_E and θ_H are a measure of the local landscape of the electric and magnetic field respectively.

It is now imperative to show how the eigen-fields relate to the source (external) and induced (internal) magnetic and electric fields and to justify the prefix “eigen” used above. Following Berdichevsky and Zhdanov (1984) and Egbert (1990), the tangential total magnetic and electric output fields at a given location on the surface of the Earth may be expressed as

$\mathbf{H} = \mathbf{H}_i + \mathbf{H}_S = [\mathbf{k_H} + \mathbf{I}]\cdot \mathbf{H}_S$

$\mathbf{E} = \mathbf{k_E}\cdot \mathbf{H}_S$

where $\mathbf{H}_i$ is the internal (induced) magnetic field, $\mathbf{H}_S$ is the source (external) magnetic field and $\mathbf{k_E}$, $\mathbf{k_H}$ are *excitation operators* that represent the electric properties of the Earth. Thus, the impedance tensor is obtained as $\mathbf{Z} = \mathbf{k_E}\cdot[\mathbf{k_H} + \mathbf{I}]^{-1}$. Substituting Equation 9 in Equation 6 and after elementary rearrangements:

$$\mathbf{E} = \left(\mathcal{E}\begin{bmatrix} 0 & E_1 \\ -E_2 & 0 \end{bmatrix}\mathcal{H}^{\dagger}\right)\cdot\left(\mathcal{H}\begin{bmatrix} H_1^{-1} & 0 \\ 0 & H_2^{-1} \end{bmatrix}\cdot\mathcal{H}^{\dagger}\mathbf{H}\right).$$

Therefore, letting

$$\mathbf{k_E} = \mathcal{E}\begin{bmatrix} 0 & E_1 \\ -E_2 & 0 \end{bmatrix}\mathcal{H}^{\dagger},$$

shows that the electric eigen-fields are the generalized eigenvalues of $\mathbf{k_E}$ and, simultaneously, the eigen-values of the electric field. Also letting

$$[\mathbf{k_H} + \mathbf{I}]^{-1} = \mathcal{H}\begin{bmatrix} H_1^{-1} & 0 \\ 0 & H_2^{-1} \end{bmatrix}\cdot\mathcal{H}^{\dagger}$$

shows that the magnetic eigen-fields are the eigenvalues of the total magnetic field.

Finally, it is important to note that the *projection* of the eigen-fields on the *horizontal plane* comprise elliptically polarized components. The rotation $\mathcal{E}^{\dagger}(\theta_E, \varphi_E)\mathbf{E}$ can be explicitly written as:

$$E_1 = \left(E_x \cos\varphi_E + E_y \sin\varphi_E\right)\cos\theta_E$$
$$-i\left(E_x \sin\varphi_E - E_y \cos\varphi_E\right)\sin\theta_E$$

and

$$E_2 = -\left(E_x \sin\varphi_E - E_y \cos\varphi_E\right)\cos\theta_E$$
$$+i\left(E_x \cos\varphi_E + E_y \sin\varphi_E\right)\sin\theta_E$$

For a given θ_E, the variation of the azimuthal angle φ_E forces the rotating field vector to trace an ellipse on the *horizontal* frame $\mathbf{x} \pm i\mathbf{y}$, so that the normalized vector will have a major axis equal to $\cos\theta_E$ and a minor axis equal to $\sin\theta_E$. The ratio of the minor to the major axis is the *ellipticity*, given by $b_E = \tan\theta_E$. The same holds for the rotation of the magnetic field vector so that $b_H = \tan\theta_H$. In either case $\theta > 0$ implies a counter-clockwise sense of rotation and $\theta < 0$ a clockwise sense. Thus, ellipticity on the horizontal plane is defined in terms of a rotation in higher dimensional space! This also provides a heuristic means of determining bounds for the variation for θ_E and θ_H: they are $-\pi/4 \le \theta_E \le \pi/4$ and $-\pi/4 \le \theta_H \le \pi/4$, because in a given ellipse, the range of the minor axis is bounded by the maximum value of the major axis.

CONCLUSION

The mathematical physics of spin analysis was used to show that the impedance tensor computed from electric and magnetic fields measured in identical coordinate frames can be rotated to an exact anti-diagonal form and that this is tantamount to a generalized eigenstate – eigenvalue decomposition. The operation results in two characteristic states (generalized eigenstates) of the EM field, which accommodate the interactions between the linearly polarized eigenvalues of the electric and total magnetic fields (eigenfields). The interactions are mediated by the characteristic values (generalized eigenvalues) of the impedance tensor.

The eigenvalues of the electric field are carried by the intrinsic electric coordinate frame (eigen-frame) which is configured in three-dimensions so that the maximum eigenvalue is transverse to the resistive (fast) local direction of the conductivity structure and the minimum eigenvalue is transverse to the conductive (slow) direction through the Earth; the tilt of the electric eigen-frame is a measure of the local landscape of the electric field. The eigenvalues of the magnetic field are respectively carried by the magnetic eigen-frame which is also configured in three dimensions. The electric and magnetic eigen-fields are generally not orthogonal in the real 3-space. When the eigen-fields are projected on the axes of the experimental coordinate frame their components are superimposed and the resulting mixing of phases is manifest in the form of elliptical polarization.

The eigen-impedances provide a succinct frame and unique means by which to investigate the configuration, characterize theoretical and experimental impedance tensors and appraise their suitability for interpretation. Given all the effects that may distort Magnetotelluric measurements, this would be a particularly helpful utility, but would also require adequate understanding of the conditions under which the eigen-impedances are interpretable and which are generally governed by their analytic structure and properties. The analytic properties of the eigen-impedances and their utility will be investigated in a sequel presentation.

REFERENCES

Arfken, G.B. and Weber, H.J. (1985). Mathematical methods for physicists, 6th edition, Elsevier – Academic Press.

Berdichevsky, M. N. and Zhdanov, M. S. (1984). Advanced Theory of Deep Geomagnetic Sounding, Elsevier, Amsterdam, pp 136-155.

Egbert, G.D. (1990). Comments on ‘Concerning dispersion relations for the magnetotelluric impedance tensor’ by E. Yee and K. V. Paulson, Geophys. J. Int., 102: 1-8.

LaTorraca, G., Madden, T. and Korringa, J. (1986). An analysis of the magnetotelluric impedance tensor for three-dimensional structures, Geophysics, 51: 1819–1829.

Murnaghan, F.D. (1938). The theory of group representations, Johns Hopkins Press, Baltimore.

Wigner, E.P. (1959. Group theory and its applications to the Quantum Mechanics of atomic spectra, Academic Press, New York.

Yee, E., and Paulson, K.V. (1987). The canonical decomposition and its relationship to other forms of magnetotelluric impedance tensor analysis, J. Geophys., 61: 173–189.