Polarized MIMO Channels in 3D: Models, Measurements and Mutual Information

Mansoor Shafi, Min Zhang, Aris L. Moustakas, Peter J. Smith, Andreas F. Molisch, Fredrik Tufvesson and Steven H. Simon

Abstract—The use of cross-polarized antennas for MIMO systems is now receiving considerable attention. However, very few studies have appeared that characterize cross-polarized MIMO channels. Recently, the third generation partnership standards bodies (3GPP/3GPP2) have defined a cross-polarized channel model for MIMO systems but this model neglects the elevation spectrum. In this paper we provide a deeper understanding of the channel model for cross-polarized systems for different environments and propose a composite channel impulse model for the cross-polarized channel that takes into account both azimuth and elevation spectrum. We use the resulting channel impulse response to derive closed from expressions for the spatial correlation. We also present models to describe the dependence of cross-polarization discrimination (XPD) on distance, azimuth and elevation and delay spread. In addition, we study the impact of antenna slant angle on the mutual information (MI) of the system. In particular we present an analytical model for large system mean mutual information values and consider the impact of elevation spectrum on MI. Finally, the impact of multipath delays on XPD and MI is also explored.

Index Terms—MIMO, cross-polarized channels, capacity.

I. INTRODUCTION

The use of cross-polarized antennas for multiple-input-multiple-output (MIMO) systems is now receiving considerable attention. It is well known that under certain conditions MIMO system capacity can be linearly proportional to the minimum of the number of transmit and receive antennas. Cross-polarized systems are of interest since they are able to double the antenna numbers for half the spacing needs of co-polarized antennas. Slant polarized antennas are already used for cellular mobile systems for up link receive diversity and are therefore potential candidates for use on systems that use MIMO receivers. Very few studies are available on propagation models for cross-polarized channels. The papers most often cited are those of [1], [2]. However both these papers consider only transmission from a vertically polarized antenna and also do not consider the impact of elevation spectrum.

For mobile terminals located in indoor buildings and in vehicles, the elevation spectrum must also be considered. There is a real scarcity of published models for the elevation spectrum in these environments that are suitable for MIMO system studies. In a study done in urban Tokyo [3], a Gaussian elevation spectrum is proposed and measurements are used to derive an expression for the mean effective gain of a mobile antenna. In a recent study [4] the elevation spectrum and cross-polarization discrimination (XPD) were measured in different radio propagation environments in Finland. Both the studies in [3] and [4] were also done using a vertically polarized transmitter at the base station. In another recent study a cross-polarized channel model for MIMO systems was developed [4], but the elevation spectrum impacts are neglected.

In this paper we make the following contributions:

• We present a comprehensive model for the cross-polarized channel and extend the 3rd Generation Partnership Project (3GPP) model to include a three-dimensional (3D) component. We call this the composite channel.
• We discuss a model for XPD and discuss its dependence on distance, angles and delay spread, partly derived from a new measurement campaign.
• Using the composite channel, we develop closed form expressions for spatial correlations and verify their accuracy by simulations. The overall spatial correlation is a weighted sum of the constituent 2D and 3D components. We also show that when the angles of arrival and departure are dependent and follow a von-Mises distribution [5], the spatial correlation can be written in a sum-Kronecker form.
• Using the composite channel we determine system mutual information (MI) by simulation and study the impact of XPD and antenna slant angle. In order to show the benefits of using cross-polarized antennas we compare the MI values obtained by cross-polarized and co-polarized antennas for similar total array sizes. We also determine analytic expressions for large system MI values and verify their accuracy by comparing them to the simulations.
• Finally we look at the impact of the delay-dependence of the XPD on the system capacity in frequency selective fading.

Results show that system MI is not sensitive to the relative proportions of the 2D and 3D components. The closed form spatial correlations derived are shown to give good agreement to the more complex models in [4]. Large system approximations to the MI values under these correlation scenarios are also shown to be very accurate. The XPD measurements are...
used to develop models of XPD vs distance and delay and simulations of the relationship between MI and the temporal dependence of XPD suggest the effects are minor.

The paper is organized as follows. Section II develops the new generalized composite channel model and Section III reviews the literature on XPD before presenting new XPD measurements. Section IV derives the correlation structure for the new model. Results and conclusions are given in Sections V and VI.

II. GENERALIZED MODEL FOR CROSS-POLARIZED CHANNELS

In this section we will briefly describe the SCM channel model introduced in the standards [6] and subsequently generalize it to take into account the effects of three-dimensional propagation and polarization mixing, in particular, for indoors and in-vehicle situations.

A. 2D simplified SCM model

The SCM model is a detailed system level model for a variety of environments. It does not consider the elevation spectrum; therefore it is defined for the 2D case. The SCM considers N clusters of scatterers and each simulation, or drop, varies the cluster statistics and array orientations. A cluster corresponds to a separate path and within the path, the subpaths are the unresolvable rays. A simplified sketch of the model is given in Fig. 1.

The largely two-dimensional character of electromagnetic propagation in outdoors environments, with about $1-2^\circ$ elevation spread at the base-station [7], has an important implication in the modeling of the propagation and mixing of the polarization of electromagnetic waves. Specifically, one can decompose the polarization into vertical and in-plane directions. Also, while the in-plane (horizontal) components tend to mix with each other strongly, the mixing between horizontal and vertical components generally is relatively small, with an XPD ratio typically larger than 6dB. Therefore, in general one needs to model four channels between base and mobile antennas, namely those connecting the horizontal/vertical polarization at the base-station to the horizontal/vertical polarization at the mobile station. This results in the decomposition of the antenna patterns at both ends into vertical and horizontal. Thus, for example, an ideal dipole antenna with polarization vector $p$ tilted at angle $\alpha$ from the $z$-axis (see Fig. 2), the vertical and horizontal components of the antenna pattern are proportional to

$$\chi(k) = \begin{bmatrix} \chi^v(k) \\ \chi^h(k) \end{bmatrix} e^{ikr} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \cos \phi \end{bmatrix} e^{ikr}$$

(1)

where for compactness we have included the overall phase factor for the incoming wave in the response of the antenna. And $r$ is the position of the antenna with respect to the reference antenna of the antenna array,

$$k = 2\pi/\lambda[\cos \phi, \sin \phi, 0]$$

(2)

is the (three-dimensional) wave-vector of the direction of the incoming (or outgoing) wave, with the carrier wavelength $\lambda$ and $\phi$ is the azimuth angle of incoming/outgoing waves.

For simplicity, we focus on a single path with Rayleigh fading. The fading channel coefficient $h_{su}^{2D}(t)$ of the path between base-station (BS) antenna $s$ and mobile-station (MS) antenna $u$ as given in [6] is then given by

$$h_{su}^{2D}(t) = \sqrt{\frac{1}{M}} \sum_{i=1}^{M} \left( \chi_{s,BS}^i(k_i,BS)H_i^{2D}X_{u,MS}(k_i,MS)e^{-i(k_i,MS \cdot v)t} \right)$$

(3)

with the superscript $2D$ indicating wave propagation in 2 dimensions. In the above, $\chi$ are the antenna response vectors for the BS and MS antennas as defined in (1). Note that the square norm of $\chi$, $G(k) = |\chi(k)|^2$, is the antenna gain in the direction $k$. Also, $k_{i,MS}$ and $k_{i,BS}$ in (3) are the random plane wavevectors of the incoming and outgoing waves for each wave component $i = 1, \ldots, M$, both assumed to be in the horizontal plane. $v$ is the velocity vector of the mobile and $t$ is time. In addition, $H_i^{2D}$ is a $2 \times 2$ matrix with the random coefficients of the $i = 1, \ldots, M$ wave components given by

$$H_i^{2D} = \begin{bmatrix} z_i^{vh} & \sqrt{z_i^{vv}} \\ \sqrt{z_i^{hv}} & z_i^{hh} \end{bmatrix}$$

(4)

with $z_i$ being the random coefficients of the $i$th wave component of the sum for each of the four polarization channels HH, HV, VH, and HH/HV respectively. In other words, $r_1$ is the power of the VH component relative to the VV component. Note that the fading of the channel component $h_{su}^{2D}(t)$ is manifested by the movement of the mobile with respect to the ground. The incoming plane-waves $k_{i,MS}$ are assumed to be coming from fixed sources toward the mobile.

B. 3D SCM model

The assumption of two-dimensional propagating waves breaks down when in some propagation environments the angular spectrum is significant. This effect is particularly important when analyzing cross-correlations between antennas with very different three-dimensional response patterns. In these cases, focusing only on the radiation arriving in the horizontal plane may produce erroneous results. Therefore, we need to modify the above 2D model to incorporate such effects. In doing so, we should aim to minimize the increase in
therefore the corresponding XPD for these processes are components of the radiation to have been fully mixed, and from which these waves emanate towards the mobile are of the mobile from openings (e.g. windows). Since the surfaces antenna response shown in (1).

\[ \alpha = 2\pi/\lambda \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \] as shown in Fig. 2. Note that in the limit of horizontal wave propagation, i.e. when \( \theta = \pi/2 \), we recover the form of antenna response shown in (1).

The relative strength of the three-dimensional radiation at the mobile to that of the already existing 2-dimensional radiation depends on several factors, including the distance of the mobile from openings (e.g. windows). Since the surfaces from which these waves emanate towards the mobile are of no particular geometry, we expect the horizontal and vertical components of the radiation to have been fully mixed, and therefore the corresponding XPD for these processes are assumed to be equal to unity. As a result, we may write the fading channel coefficient \( h_{su}^{3D}(t) \) for this component of the propagation between BS antenna \( s \) and MS antenna \( u \) as

\[ h_{su}^{3D}(t) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( \chi_{s,BS}^{-1}(\mathbf{k}_i,BS)H_{3D}^{3D} \chi_{u,MS}(\mathbf{k}_i,MS)e^{-i\mathbf{k}_i \cdot \mathbf{r}_{iu}^v} \right)} \]

with the superscript \( \text{3D} \) indicating wave propagation in 3 dimensions. Moreover, \( H_{i}^{3D} \) of the \( i \)th wave component is given by

\[ H_{i}^{3D} = \begin{bmatrix} z_{i}^{v\theta} & z_{i}^{v\phi} \\ z_{i}^{f\theta} & z_{i}^{f\phi} \end{bmatrix} \]

The antenna responses \( \chi \) for the BS are the same for both 2D and 3D models. At the MS the antenna responses are different due to the 3D character of the radiation.

Thus, we can write the composite channel coefficient between antennas \( s \) and \( u \) as

\[ h_{su}(t) = \sqrt{\frac{1}{1+g} h_{su}^{2D}(t) + \frac{g}{1+g} h_{su}^{3D}(t)} \]

where \( g \) is the ratio of powers of the 3D to 2D components of the channel. There are very few measurements of \( g \). For the case of indoor mobiles, which are far from open spaces, e.g. windows, one can assume that \( g = \infty \), i.e. one can keep only the 3D components of the channel. For indoor channels close to a window, a reasonable value for \( g \) is \( g = -4dB \) [8]. A similar value should be valid for in-vehicle mobiles.

One should point out a distinction in the fading behavior of the indoor and in-vehicle cases. In the former, the fading fluctuations are created by the relative motion of the mobile with respect to all surrounding scatterers, near and far, which are assumed to be stationary. In contrast, the nearby scatterers of a mobile in a vehicle, which give rise to the three-dimensional behavior discussed above, do not move with respect to the mobile. Instead, the fading of these waves arises from the fact that they are scattered from stationary sources outside the vehicle. The result of this discrepancy is that the temporal correlations of these waves deviate from the conventional two-dimensional Jakes form, \( J_0(2\pi v t/\lambda) \) to its three-dimensional counterpart, \( \text{sin}c(2\pi v t\lambda) \). For \( g \) values which are not too large, (which is the case for in-vehicle mobiles) the effect of this discrepancy is small and therefore not important. In contrast, for indoor environments, where \( g \) is large, the channel correctly approaches the sinc-function form for three-dimensional wave propagation [9].

### III. Models and Measurements for XPD

As shown in (3) and (4), the polarization mixing for the entire path is governed by the inverse XPD values \( r_1 \) and \( r_2 \). Most models make the assumption that \( r_1 = r_2 \), or more explicitly, that \( VV = HH \) and \( VH = HV \). While this might be an oversimplification, only a few measurements are available that do not make this assumption. In this section, we discuss measurements and statistical models for the XPD in the literature, as well as the results from a new measurement campaign.
There is a general consensus in the literature that the XPD, when expressed in dB, has a non-zero-mean Gaussian distribution (though many papers only give a mean and variance, without analyzing the exact shape, or giving a goodness-of-fit test). Hence, we may write

\[ \text{XPD} \sim \mathcal{N}(\mu, \sigma) \] (10)

Depending on the environment and the existence of a line-of-sight (LOS) connection, the mean XPDs measured in the literature vary from 0 to 18dB, with standard deviations typically of the order of 3-8 dB.

From the physical propagation processes, we can also conclude that the XPD can depend on other channel parameters, namely the total attenuation (or equivalently, the distance between transmitter and receiver), the angles of arrival and departure (both azimuth and elevation), as well as the delay of the multipath components. We first review the literature for measurements of those dependencies, and then present measurement results in an indoor environment. From the results, we can obtain XPD values that are relevant for different environments, distances, and delays. These values can then be used, for example, in the MI computations of Section V.

1) Dependence of XPD on distance: For indoor environments, extensive measurements are given in [10]. These authors found that for a corridor environment with LOS, the decay with distance can best be modeled as an exponential decay, although the traditional \( n \log(d) \) law also gives satisfactory performance. Decay exponents are different for all polarizations, namely 1.07, 1.20, 1.49 and 1.47 for VV, HV, VH and HH respectively. The XPD is large (around 15 dB), and increases with distance. Other indoor measurements are reported by [11], [12]. For outdoor environments, distance dependence of XPD measurements are reported by [13], [14], [15], and [16].

2) Dependence of the XPD on azimuth and elevation: In [17], measurements were performed with vertically and horizontally polarized antennas of different antenna patterns, and a definite dependence of XPD on antenna pattern was found. However, it is difficult to extract azimuth and/or elevation-dependent XPDs from these results.

For macrocells, results in [18] suggest that the azimuth spread is independent of the polarization when the polarization directions \( \pm 45^\circ \) are considered. However, for vertical transmission from the MS, the XPD has been shown to have a weak \( (\rho = -0.15) \) negative correlation with the azimuth spread [19]; in other words, a larger azimuth spread at the base leads to a lower XPD. Again, this result is intuitive, as a larger azimuth spread indicates stronger scattering.

Extensive investigations of the elevation at the mobile station, and its dependence on the polarization can be found in [16]. For simplicity, in the case of indoor and in-vehicle environments we assume that the elevation spectrum is such that every point on the sphere in Fig.2 is equally likely and the elevation spectrum for both polarizations is the same.

3) Dependence of XPD on delay spread: Results in [20] did not find a dependence between delay spread and polarization in outdoor environments, a result that was also confirmed by [19]. A similar result was obtained for indoor environments by [11]. However, some of the measurements in [19] showed a difference in the shape of the power delay profile for the two polarizations.

In [21] it was found that the co-polarized and cross-polarized components had different decay time constants. Analyzing cluster decay constants in a microcellular scenario, they showed that the VV component decayed with 8.9 dB/\( \mu s \), while the co-polarized component decayed with 11.8 dB/\( \mu s \). This indicates that the XPD increases with increasing delay - a somewhat surprising result. However, the 3GPP model finds for macrocellular environments that the mean (in dB) is equal to 0.34 \((\text{mean relative path power in dB}) + 7.2 \) so that the XPD decreases with increasing delay; in microcellular environments, the XPD is a constant 8dB.

A. XPD Measurements

Here we briefly report on the measurements that were performed with the RUSK LUND MIMO channel sounder from MEDAV at a center frequency of 2.6 GHz and a bandwidth of 200MHz. Due to limitations of space we are unable to report on the measurement set up, details can be found in [22]. We have measured two indoor office scenarios at various distances: LOS in a corridor and NLOS between the corridor and the offices, where the walls consist mainly of gypsum wallboards. We evaluated the measurement results to obtain the entries of the polarization matrix, both as a function of the distance, and as a function of the delay. Figures 3 and 4 show the power of the different components \( P_{vv}, P_{vh}, P_{hv}, \) and \( P_{hh} \) with the best fitting linear curves as a function of the distance. The curves are obtained by the minimum squared error integrated over the logarithm of the distance. Figure 3 shows the results for the LOS scenario. Similarly, Fig. 4 shows the results for the corridor-to-office (NLOS) scenarios.

<table>
<thead>
<tr>
<th></th>
<th>HH</th>
<th>VV</th>
<th>VH</th>
<th>HV</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>-1.0</td>
<td>-1.3</td>
<td>-1.7</td>
<td>-1.6</td>
</tr>
<tr>
<td>NLOS</td>
<td>-2.4</td>
<td>-2.2</td>
<td>-2.6</td>
<td>-2.6</td>
</tr>
</tbody>
</table>
As expected, decay components are much higher for the NLOS case than for the LOS case. We also see that there is a difference in the decay exponent $n$ between the co-polarized and the cross-polarized components. As a consequence, the XPD increases with increasing distance. We find a small difference between the decay exponents of VV and HH components, but the values are quite sensitive to large scale fading in the measurements.

Comparisons with the results of [10] show that most of the general trends are similar. However, while [10] found that the VV exponent in the LOS case is smaller than the VV exponent, we find the reverse. Also, our measurement results indicate larger differences in the decay exponents of the different polarization components than [10]. We attribute those differences to the somewhat different environment (building materials) in the scenarios. As opposed to [10], the power decay of the cross- and co-polar components can best be described by a power law. The XPD as a function of distance can be modeled as

$$XPD(d)_{dB} = XPD_{d_0=1m} + n_1 \cdot 10 \log_{10}\left(\frac{d}{d_0}\right)$$

where $2 \leq d \leq 50$ is the distance in meters and the constants are given by the measurements as

<table>
<thead>
<tr>
<th></th>
<th>$XPD_{d_0=1m}$ (dB)</th>
<th>$n_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>7.9</td>
<td>0.55</td>
</tr>
<tr>
<td>NLOS</td>
<td>2.6</td>
<td>0.29</td>
</tr>
</tbody>
</table>

We also analyzed the power of the components as a function of their delay. Figure 5 shows the results in the LOS corridor. We find that both for the co-polarized and the cross-polarized components, the power delay profile decays approximately exponentially. The decay exponents are different for the co- and cross-polarized components, with the cross-polarized components decaying faster; thus, the XPD increases with the delay of the multipath components. This is a noteworthy contrast to the assumptions of the 3GPP model. However, it is also important to note that for very small delays (LOS component), the cross-polarized components are small (compared to the pure exponential law), so that the XPD at this small delay is high. Figure 6 shows the results in the office NLOS environment. The conclusions are very similar.

Based on the measurements we propose the following model for the XPD as a function of the delay:

$$XPD(\tau)_{dB} = (XPD_{HI} - XPD_{LO}) \delta(\tau) + XPD_{LO} + n_2\tau,$$

where $0 \leq \tau \leq 0.2\mu s$ is the excess delay and $\delta(\tau)$ is the delta function. The constants are given by the measurements as

<table>
<thead>
<tr>
<th></th>
<th>$XPD_{HI}$ (dB)</th>
<th>$XPD_{LO}$ (dB)</th>
<th>$n_2$ ($\mu s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>15.5</td>
<td>1.9</td>
<td>60</td>
</tr>
<tr>
<td>NLOS</td>
<td>5.9</td>
<td>2.1</td>
<td>16</td>
</tr>
</tbody>
</table>

IV. SPATIAL CORRELATION PROPERTIES

It is well known that MIMO mutual information degrades in the presence of spatial correlation. In order to investigate...
this behavior we study the spatial correlation of the composite channel in this Section. The correlation can be expressed as a weighted sum of the constituent 2D and 3D components. Under some special conditions, analytic expressions for the constituent 2D and 3D components can be derived.

A. Spatial correlation of the 2D component

The SCM model [6] proposes distributions for the random angles and phases which define the channel coefficient. These distributions do not result in a closed form expression for the spatial correlation. In [23], a new co-polarized MIMO channel model was introduced which extends the work of Abdi and Kaveh [5] by allowing varying degrees of correlation between AoAs and AoDs. The parameters which allow this variation in correlation are $\kappa_{BS}$ and $\kappa_{MS}$ which are inverse angle spread parameters similar to those in [5]. This model spans the full range of possibilities from perfect correlation between AoA and AoD (as in one-ring models) and zero correlation (as in Kronecker and SCM models [6]). In addition it leads to closed form expressions for spatial correlation. For these reasons, this model is used for the 2D component throughout this paper.

Below, we describe the spatial correlation which results from extending the model in [23] to the cross-polarized case. We show that the resulting correlation has a sum-Kronecker structure. The following notation is used to label the antennas which we assume are pairs of co-located co-polarized antennas. The particular antenna in the pair $s$ or $u$ is labeled by the superscript $p, q$ in $BS^p, MS^q$ with $p$ and $q$ equal to 1 or 2 representing the left or right antenna of the antenna pair.

Repeating the derivation in [23], we can obtain the cross-correlation coefficients for the co-polarized MIMO channel using either exact results or approximations. For reasons of space, the derivation is omitted and only one approximation is given here. The resulting correlation is the sum of four terms due to the polarization paths (VV, VH, HV, HH) and is given by

$$
\rho_{ss', uu'}^{pq, p'q'} = E[(h_{ss}^{pq})(h_{ uu'}^{p'q'})^\dagger] = \begin{bmatrix} \cos(\alpha_p) \cos(\alpha_{p'}) r_{ss'}^{v^p v'} \\ \sin(\alpha_p) \sin(\alpha_{p'}) r_{ss'}^{h^p h'} \end{bmatrix}^T 
\times \begin{bmatrix} \cos(\alpha_q) \cos(\alpha_{q'}) r_{uu'}^{v^q v'} \\ \sin(\alpha_q) \sin(\alpha_{q'}) r_{uu'}^{h^q h'} \end{bmatrix}
$$

(11)

The coefficients, $r^{v^p v', h^p h'}$ are defined by

$$
\begin{bmatrix} r^{v^p v'} \\ r^{h^p h'} \end{bmatrix} = E \begin{bmatrix} 1 \\ p_1 \\ p_2 \end{bmatrix}
$$

(12)

In order to evaluate the integrals defining $r^{v^h_{ss'}/uu'}$, we use the results below [3.937,p488,[24]]

$$
F_s(p, q, a, b, m) = \int_0^{2\pi} \exp(p \cos x + q \sin x) \sin(a \cos x + b \sin x - mx) dx
$$

(13)

$$
F_c(p, q, a, b, m) = \int_0^{2\pi} \exp(p \cos x + q \sin x) \cos(a \cos x + b \sin x - mx) dx
$$

noting that [24] gives closed form expressions for both $F_s(.)$ and $F_c(.)$. With this notation, the remaining terms in (11) are given by

$$
\begin{align*}
 r_{ss'}^{v} &= \frac{I_0(\sqrt{p_{ss'}^2 + q_{ss'}^2})}{I_0(\kappa_{BS})} \\
 r_{ss'}^{h} &= \frac{1}{4\pi I_0(\kappa_{BS})} \times \{ F_c(p_{ss'}, q_{ss'}, 0, 0, 0) + \cos(2\phi_{BS})F_c(p_{ss'}, q_{ss'}, 0, 0, 2) - \sin(2\phi_{BS})F_s(p_{ss'}, q_{ss'}, 0, 0, 2) \} \\
 r_{uu'}^{v} &= \frac{I_0(\sqrt{p_{uu'}^2 + q_{uu'}^2})}{I_0(\kappa_{MS})} \\
 r_{uu'}^{h} &= \frac{1}{4\pi I_0(\kappa_{MS})} \times \{ F_c(p_{uu'}, q_{uu'}, 0, 0, 0) + \cos(2\phi_{MS})F_c(p_{uu'}, q_{uu'}, 0, 0, 2) - \sin(2\phi_{MS})F_s(p_{uu'}, q_{uu'}, 0, 0, 2) \}
\end{align*}
$$

(14)

where

$$
\begin{align*}
 p_{ss'} &= \kappa_{BS} + j2\pi d_{ss'} \sin(\phi_{BS})/\lambda \\
 q_{ss'} &= -j2\pi d_{ss'} \cos(\phi_{BS})/\lambda \\
 p_{uu'} &= \kappa_{MS} \cos(\phi_{AoA}) + j2\pi d_{uu'} \sin(\phi_{MS})/\lambda \\
 q_{uu'} &= \kappa_{MS} \sin(\phi_{AoA}) - j2\pi d_{uu'} \cos(\phi_{MS})/\lambda \\
 d_{ss'} &= \text{distance between BS antenna pair s and s'} in \lambda \\
 d_{uu'} &= \text{distance between MS antenna pair u and u'} in \lambda \\
 \phi_{AoA} &= \text{mean angle of arrival}
\end{align*}
$$

(15)

For normal antenna configurations with symmetric antenna pairs and BS/MS slant offset angle $\alpha_{BS}/\alpha_{MS}$, from (11) the channel correlation matrix $R^{2D}$ can be expressed as

$$
R^{2D} = E \left( [vec(H^{2D}) vec(H^{2D})^\dagger] \right) = \sum_{x,y=v,h} \bar{r}^{xy} \times
$$

$$
(\mathbf{R}_{BS}^{x} \otimes \mathbf{\Omega}_{BS}^{x} \otimes \mathbf{R}_{MS}^{y} \otimes \mathbf{\Omega}_{MS}^{y})
$$

(16)

where $R_{BS/MS}^{x/y}$ is the matrix form of (14) and

$$
\mathbf{\Omega}_{BS/MS} = \cos^2(\alpha_{BS/MS}) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

(17)

B. Spatial correlation of the 3D component

Here we describe the correlation properties of $h^{3D}$. For simplicity, we focus on ideal dipole antennas, as they carry many of the properties of most antennas used and we assume that the number of scatterers is sufficiently large to replace the sums in (7) with integrals. The spatial correlations of two pairs of antennas at transmitter and receiver ends take the following Kronecker product form for $r_1 = r_2 = 1$. This is
functions given by vectors of the MS antennas \( m \) the \( n, n' \) can be found in Table I. In these results and (22) matched simulations of the SCM model extremely well. Analytical results from (16), (20) and (22) matched simulations of the SCM model extremely well. In these results \( \phi_{BS} = \phi_{MS} = 0^\circ \) and other parameters can be found in Table I.

C. Overall correlation

The overall correlation matrix for the MIMO system is given by a weighted sum as follows:

\[
R = \frac{1}{1 + g} R^{2D} + \frac{g}{1 + g} R^{3D} \tag{22}
\]

The form of the correlations at the mobile have two interesting implications. First, since \( f_1(0) = 0 \), for co-located antennas, we can have 3 mutually independent antennas with polarizations orthogonal to each other [25]. Second, one can form an uncorrelated uniform linear array with minimum distance \( \lambda/2 \) only if the antennas are tilted in a special way. Otherwise, the term proportional to \( f_1 \) in (20) will not vanish. The simplest way to make eliminate this term is by tilting the antennas by 54.73° with respect to the line connecting them [4].

V. RESULTS AND DISCUSSIONS

The ergodic mutual information (MI) is given by the classic equation

\[
\mathcal{I} = E \left[ \log_2 \det \left( R_{IM} + \frac{\text{SNR}}{n_{BS}} HH^\dagger \right) \right] \tag{23}
\]

where the expectation is over the realizations of the channel matrix \( H \) and we assume the BS is the transmitter. And the mean MI (MMI) is average value over different orientations of BS and MS. We assume unit antenna gain at both the BS and MS. Both the SCM model (9) and the sum-Kronecker correlation structure (22) with Rayleigh fading are used to calculate MI. Important parameter values and distributional assumptions used in the numerical results can be found in Table I. Where parameter values are not given, we have used the suburban macro environment parameters in [6]. The values of \( \kappa_{BS}, \kappa_{MS} \) were chosen to approximate the SCM model.

A. Impact of slant angle on correlation for different XPD values

In order to study the relationship between the optimal slant offset angle (with maximal MI) and other channel parameters we consider a (2,2) cross-polarized MIMO system with zero 3D component and the Kronecker structure (16). Note that at high SNR, the log-determinant in (23) is almost linear so that maximizing MI over slant offset angle \( \alpha \) is similar to
maximizing $\tilde{\rho} = E[\det(I_2 + \frac{SNR}{2}HH^t)]$. Also at low SNR, the first order approximation $I \approx \tilde{\rho}$ is accurate since the variance of the determinant in (23) is small. Hence, for these special cases, we can focus on $\tilde{\rho}$ rather than $I$. Here, some routine analysis shows that $\tilde{\rho}$ can be expressed as

$$\tilde{\rho} = 1 + SNR \left\{ 2\cos^4 \alpha + \left(1 + \frac{I_2(\kappa_{BS})}{I_0(\kappa_{BS})}\right) \tilde{\rho}^{vh} \sin^2 \alpha \cos^2 \alpha + \left(1 + \frac{I_2(\kappa_{MS})}{I_0(\kappa_{MS})}\right) \tilde{\rho}^{hv} \sin^2 \alpha \cos^2 \alpha + \frac{1}{2} \left(1 + \frac{I_2(\kappa_{BS})}{I_0(\kappa_{BS})}\right) \left(1 + \frac{I_2(\kappa_{MS})}{I_0(\kappa_{MS})}\right) \sin^4 \alpha \right\}$$

(24)

Equation (24) reveals several important conclusions:

- At low SNR, the second term is the major factor. Using $0 < \tilde{\rho}^{vh} = \tilde{\rho}^{hv} < 1$ and Bessel function approximations for large arguments ($\kappa_{BS}$) and small arguments ($\kappa_{MS}$), close inspection of (24) shows that the optimal slant offset angle is always $0^\circ$ implying perfect correlation. This matches the well-known fact that at small SNR, highly correlated channels perform better since the signal to noise ratio is increased. Fig. 8 verifies this remark numerically for $\kappa_{BS} = 500$ and $\kappa_{MS} = 0.5$.
- When the SNR is large, the third term will dominate and the optimal slant offset angle is always $45^\circ$ irrespective of any other parameters such as $\kappa_{BS}, \kappa_{MS}, XPD$. Again, this makes sense since the slant angle of $45^\circ$ reduces correlation and at large SNR, uncorrelated channels are better since we can transmit multiple streams.

Next we simulate a larger (8,8) cross-polarized MIMO system with $\phi_{AoA} = 0$, $\phi_{BS} = \phi_{MS} = 0$, $\kappa_{BS} = 100$ or 500, $\kappa_{MS} = 0.5$, $d_{ss'} = \lambda$, $d_{uu'} = 0.5\lambda$. The XPD’s are independently generated by $r_1 = r_2 = 10^{\frac{-8.5 + 5.5\eta(0,1)}{10}}$ where $\eta(0, 1)$ is a standard normal variable [6]. These equations enable us to vary the XPD and study the impact on MI. Varying offset angles in both the BS and the MS, we obtain the results in Fig. 9. This shows that the optimal offset angle will vary from $35^\circ$ to $45^\circ$ at large SNR and it will be closer to $45^\circ$ when the power is increased.

B. Impact of array width on co-polarized and cross-polarized correlation

The slant offset angle used here is $45^\circ$ in the BS and the MS and the MI is spatially averaged over 10,000 drops with uniformly distributed orientations of BS and MS. The results of MI simulation for both models (9), (22) are shown in Fig 10. Both models perform similarly and the benefits and drawbacks of cross-polarized MIMO are clear. At very small spacings the cross-polarized system outperforms the co-polarized system. This occurs for BS antenna spacings less than about 2.5 wavelengths for (4-4) systems. The (4-4) cross-polarized system is worse than the (3-3) co-polarized system when the BS spacing is larger than 6 wavelengths. This is reasonable since when the spacing is increased, the correlation between BS antennas always decreases in a co-polarized MIMO system. However, half of the antennas are still strongly correlated and the benefits of cross-polarization are maintained.
correlated in the cross-polarized MIMO system. For a given spacing, the MIMO system benefits from increased numbers of antenna elements but is impaired by higher correlation.

C. Large system MI analysis

It would be useful to have an analytic expression for the mutual information for the channels discussed thus far. For the case of Gaussian channels with Kronecker product correlated channels, the statistics of the mutual information are well understood, since exact closed form expressions for the moment generating function exist for both semi-correlated [26], [27] and fully correlated [28], [29], [30] channels. However, these methods cannot be generalized to non-Kronecker product correlations. Therefore, to make analytic progress, one needs to employ asymptotic methods. In [31] such methods were used to calculate the first few moments of the distribution of the mutual information for non-Kronecker product correlations in the limit of large antenna numbers. Fortunately, these methods are accurate also for a few (2-3) antennas [32].

In this section we apply these methods to calculate the ergodic mutual information of the composite cross-polarized channels developed here. For the case of outdoor propagation one can simply disregard $g$ by setting it to zero. The correlations of the composite channel matrix may be written as

$$E[\mathbf{H}_{in} \mathbf{H}_{jn}^T] = \frac{P}{(1 + g)n_t} \begin{bmatrix} R_{BS,\alpha\beta}^{2D,v} & R_{BS,h\beta}^{2D,v} \\ R_{BS,\alpha\beta}^{2D,h} & R_{BS,h\beta}^{2D,h} \end{bmatrix}^T \mathbf{M} \begin{bmatrix} R_{MS,ij}^{2D,v} \\ R_{MS,ij}^{2D,h} \end{bmatrix} + \frac{gP}{(1 + g)n_t} R_{BS,\alpha\beta}^{3D} R_{MS,ij}^{3D}$$

(25)

where $P$ is the signal to noise ratio of each path, $g$ is the 3-dimensional mixing parameter, $R_{BS,ij}^{2D,v}$ are the correlations between antennas $i, j$ at the base for the 3-dimensional propagation (with a corresponding expression for the mobile), $R_{BS,ij}^{3D}$ are the correlations of the vertical polarization (and similarly for the horizontal polarization and the mobile) and $\mathbf{M}$ is the mixing matrix

$$\mathbf{M} = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$$

(26)

where we assume $r = E[r_1] = E[r_2]$.

Using the replica approach and saddle-point analysis, the ergodic mutual information can be written as:

$$I = \log \det \left( \mathbf{I}_{n_t} + \tilde{\mathbf{T}} \right) + \log \det \left( \mathbf{I}_{n_r} + \tilde{\mathbf{R}} \right)$$

(27)

where the matrices $\tilde{\mathbf{T}}, \tilde{\mathbf{R}}$ are given by

$$\tilde{\mathbf{T}} = \frac{P}{1 + g} \begin{bmatrix} (y_1 + ry_2) R_{BS}^{2D,v} \\ (ry_1 + y_2) R_{BS}^{2D,h} \end{bmatrix} + \frac{gP}{1 + g} y_1 R_{BS}^{3D}$$

(28)

$$\tilde{\mathbf{R}} = x_1 R_{MS}^{2D,v} + x_2 R_{MS}^{2D,h} + x_3 R_{MS}^{3D}$$

with the scalars $x_a, y_a$ for $a = 1, 2, 3$ given by

$$x_1 = \frac{P}{(1 + g)n_t} \text{tr} \left[ \left( R_{BS}^{2D,v} + r R_{BS}^{2D,h} \right) \left( \mathbf{I}_{n_t} + \tilde{\mathbf{T}} \right)^{-1} \right]$$

$$x_2 = \frac{P}{(1 + g)n_t} \text{tr} \left[ \left( R_{BS}^{2D,h} + r R_{BS}^{2D,v} \right) \left( \mathbf{I}_{n_t} + \tilde{\mathbf{T}} \right)^{-1} \right]$$

(29)

$$x_3 = \frac{gP}{(1 + g)n_t} \text{tr} \left[ R_{BS}^{3D} \left( \mathbf{I}_{n_t} + \tilde{\mathbf{T}} \right)^{-1} \right]$$

$$y_1 = \frac{1}{n_t} \text{tr} \left[ R_{MS}^{2D,v} \left( \mathbf{I}_{n_r} + \tilde{\mathbf{R}} \right)^{-1} \right]$$

$$y_2 = \frac{1}{n_t} \text{tr} \left[ R_{MS}^{2D,h} \left( \mathbf{I}_{n_r} + \tilde{\mathbf{R}} \right)^{-1} \right]$$

(30)

$$y_3 = \frac{1}{n_t} \text{tr} \left[ R_{MS}^{3D} \left( \mathbf{I}_{n_r} + \tilde{\mathbf{R}} \right)^{-1} \right]$$

Results in Fig. 11 show how closely the large sample results follow the simulation for an (8-8) system. Furthermore, the lack of sensitivity of MI to $g$ is also shown.

D. Temporal dependence of XPD

In this Section, we analyze the impact that the temporal dependence of the XPD has on the capacity of a polarization-based MIMO-CDMA system. In order to characterize the capacity of a MIMO-CDMA system, we investigate the capacity
The capacity of the matched-filter bound, i.e., the capacity that would be achievable if no intersymbol interference occurs. For a system with a high spreading factor $N_{sp}$ and sufficiently low data rate, the capacity of each single user is given by $C_{mf}/N_{sp}$. We show that for such a system, operating in the 3GPP channel model, the impact of the delay dependence of the XPD on the system capacity is negligible. Further simulations (not shown here) show that this result also holds for the 3D model, and for systems that use OFDM instead of CDMA.

$C_{mf}$ can be computed in a straightforward way: each Rake finger collecting energy from one antenna element can be considered as a "virtual" antenna, so that a $n_{MS} \times n_{BS}$ polarization-based MIMO system with $N_{R}$ Rake fingers corresponds to a virtual $n_{MS}N_{R} \times n_{BS}$ system. The entries in the channel matrix are obtained directly from the 3GPP model or from our new model, where each path leads to a $n_{MS} \times n_{BS}$ submatrix; the submatrices are stacked horizontally into a total matrix $H$. For a WSSUS channel, the entries of one submatrix are statistically independent of the other submatrices. Note that the power carried by each of the submatrices (i.e., its Frobenius norm) is proportional to the power delay profile at delay $\tau_n$. The capacity of this "equivalent" or "virtual" system can be computed by the classical capacity equation (23).

Both due to the different powers collected by the different Rake taps, and because of the cross-polarization discrimination, the entries in the matrix $H$ are non-identically distributed (i.e., all of the entries have a complex Gaussian distribution, but with different variances). Fig. 12 shows the capacity of a system with $n_{BS} = n_{MS} = 2$ where TX and RX transmit and receive, respectively, on both the horizontal and vertical polarizations. The system operates in a 3GPP urban channel. Subfigure (a) shows the capacity when using only a single Rake finger (always matched to the first path), while subfigure (b) shows the result when 6 Rake fingers are used (i.e., all energy from the paths can be collected). Note that in the 3GPP urban model, the XPD decreases with the power of the multipath components (and thus, with their delay). The results for this case are compared to the cases where the XPD is independent of the delay; the value in that case is chosen in such a way that the narrowband XPD is the same in both cases. We see that the difference between the two cases is rather minor. However, it is noteworthy that for the case of a single Rake finger, the constant-XPD is slightly better than the delay-dependent XPD, while for the full Rake receiver (6 fingers) the results are virtually identical. We also note that a difference occurs only for the outage probability at low outage levels, while the mean capacity is indistinguishable.

VI. CONCLUSIONS

In this paper we have brought together two novel approaches to give a MIMO cross-polarized channel model in 3D. The 2D component of the model is able to bridge the gap between Kronecker and one-ring models and has been extended here to the cross-polarized case. The 3D component is new and aims to model indoor or in-vehicle situations. The composite model has a closed form, sum-Kronecker, correlation structure which we use to gain insight into optimal slant angles. In addition, the correlation structure is used to derive large system approximations to MI, which are remarkably accurate. We also review the literature on XPD studies and present the results of a new measurement campaign from which we are able to propose XPD models for distance and delay relationships. Lastly, the temporal dependence of the XPD and its effect on capacity is investigated.
ACKNOWLEDGEMENTS

The authors would like to thank Johan Kåredal, Lund University, for his assistance with respect to the measurements.

REFERENCES