First Congress of Greek Mathematicians
Special Session in Mathematics Education
June 27-28, 2018

Organizers

Fragkiskos Kalavasis – Chronis Kynigos – Despina Potari
Theodossios Zachariades

Time Schedule of Talks

	Wednesday, June 27		**Wednesday, June 27**
09:00 – 09:05	Welcome	17:00 – 17:05	Welcome
09:05 – 09:20	Argyri	17:05 – 17:20	Moutsios-Rentzos
09:20 – 09:35	Rizos	17:20 – 17:35	Voskoglou
09:35 – 09:50	Remoundou	17:35 – 17:45	Discussion
09:50 – 10:00	Discussion	17:45 – 18:00	Kafousi
10:00 – 10:15	Chasiotis	18:00 – 18:15	Gialamas
10:15 – 10:30	Misailidou	18:15 – 18:30	Discussion
10:30 – 10:45	Maragou		
10:45 – 11:00	Discussion		

	Thursday, June 28
09:00 – 09:05	Welcome
09:05 – 09:20	Biza
09:20 – 09:35	Mali
09:35 – 09:50	Discussion
09:50 – 11:00	Workshop for secondary mathematics teachers

Speakers

Panagiota Argyri (National Kapodistrian University of Athens)
Mathematical models of Game Theory
(joint work with Christos Iraklidis)

Game Theory helps us to understand situations in which decision-makers interact and it includes a set of Models. In general, in a game there is a set of players competing on a predetermined set of rules. Players must make decisions in competitive conditions, taking into account the possible moves of opponents. A game under mathematical framework focus on the problems deals with the maximization (or minimization) of a function (e.g. profit or cost) under constraints. The one that maximizes (or minimizes) the function is called a "controller", and the variables whose values they choose to achieve their goal are called "control variables". This paper presents examples of game theory such as "The prisoner's dilemma", explaining the meaning of Nash Equilibrium in mathematical models and under the framework of Combinatorics it analyzes the strategy of Game Theory in NIM game as a specific mathematical operation in the binary numbering system.

Irene Biza (University of East Anglia, Norwich, UK)
MathTASK: Transforming Aspirations into Strategies in Context

MathTASK is a collaborative research and development programme on secondary mathematics teachers' pedagogical and mathematical discourses and the transformation of

teacher aspirations into pedagogical practice. MathTASK has four strands (1) mathematical thinking (e.g. pedagogical and didactical practices in relation to the teaching of specific mathematical topics); (2) classroom management and mathematics learning (e.g. interference of disruptive behaviour with aspirations for high quality teaching); (3) CAPTeaM*: changing ableist perspective on the teaching of Mathematics (e.g. inclusion of disabled learners in the mathematics classroom); and (4) the role of digital technology and other resources in the teaching and learning of mathematics (e.g. shifts in mathematics teachers' practices when they use technology). Our research sets out from the assumption that teachers' mathematical and pedagogical discourses are better explored and developed in situation-specific contexts. To this aim, we design situation-specific tasks and then use them for research and teacher education purposes. MathTASK and CAPTeaM are projects of the RME Group at UEA in collaboration with researchers in Brazil and Greece. In the session I will present examples of situation-specific tasks and samples of teachers' written and interview responses. I will also outline two theoretical constructs that have emerged from data analysis: a Toulmin and Freeman informed classification of teacher warrants; and, a set of four characteristics (specificity, consistency, reification of mathematics and mathematics education discourses) for diagnosing mathematics teachers' competences.

* CAPTeaM is funded by the British Academy.

Christos Chasiotis (University of Ioannina)

Logical reasoning games and Mathematics Education

It is the purpose of my contribution the presentation of an open research project based on the utilisation of some logical reasoning games for the assessment, the investigation and the improvement of logical reasoning and its relation to mathematics education.

The above games, which have been devised by the author and adapted for research and classroom use, are referred to a set of ordinary playing cards, which are manipulated according to predetermined rules, formulated in everyday life language, free of logical and mathematical terminology. So these games can be utilized with students of all levels, teachers of all subjects, researchers and all citizens.

The variation of different variables of the above situation, the number of different colors, the number of cards, the logical form of propositions, the type of logical tasks and the rules for the determination of the winners, leads to different forms, equivalent or not, of these games.

So the above games have been utilized in the concrete card version, in which the correction can be done by the students themselves, by direct comparison of their responses with the concrete cards, or in written-test version, in which the cards are only described and the correction of student's responses, must be done by the teacher.

Some forms also of the above games have been addressed to students, teachers and researchers, in an intermediate version of working sheets, in which the games were accompanied by questions about their didactical analysis.

Sonia Kafoussi (University of the Aegean)

Investigating the role of the Greek primary school students' parents in their mathematical identity formation

(joint work with Petros Chaviaris and Andreas Moutsios-Rentzos)

Many researchers have investigated the role of parents in the students' developing of positive attitudes about mathematics and their better achievement in mathematics. This interest is related to the socio-cultural and socio-political approaches to mathematics education and is in line with the attempt to gain deeper understanding of the identified variation in students' learning mathematics.

In this paper, we synthesise our previous and ongoing research in order to discuss the relationships that emerge amongst and through the interactions of the triad school-student-family in Greece. Parental involvement may be conceptualised (Cao et.al.,2006) as being direct (e.g. engagement with students homework) or indirect (expectations about attainment, attitudes towards mathematics). Moreover, the notion of identity can be used as a tool for investigating this involvement as a culturally shaped activity (Sfard & Prusak,2005).

Specifically, we studied: a) the school-family daily communication, b) cultural aspects of parental involvement and c) the perceived parental influences in students' mathematical identity. The results seem to converge to the conclusion that the educational design should be reformed to include practices that would facilitate the teachers', parents' and students' awareness of the existence and the importance of these interactions.

Angeliki Mali (School of Education, University of Michigan)

Instructor and student uses of technologically enhanced textbooks

(joint work with Vilma Mesa)

With new technological developments, textbooks have been changing from paper to digital open-source formats, which include dynamic features (e.g., computing cells, navigation tools) easing access at relatively low cost. Research on uses of technologically enhanced, free textbooks is nevertheless in its infancy, so the potential of such textbooks to improve mathematics teaching and learning has not been realized yet. Undergraduate Teaching and Learning in Mathematics with Open Software and Textbooks (UTMOST, Beezer et al., 2016) seeks to describe how university instructors and students use two open-source digital textbooks, First Course in Linear Algebra and Abstract Algebra: Theory and Applications. Drawing on computer-generated data of use of those textbooks at the classroom- and individual- user levels, bi-weekly surveys to six instructors and their students in six different states, research literature on textbooks, and the documentational approach (Gueudet & Trouche, 2009), the question investigated is: How do instructors and students use the textbooks for their courses? Findings across the six instructors include a continuum of textbook use from extensive to minimal with textbooks residing in instructor's network of teaching resources. Importantly, instructors used the notation and definitions presented in the textbooks, while theorems and proofs sometimes came from other resources. The analysis of student responses revealed methodological challenges in gathering meaningful data from students, suggesting that students used certain dynamic features and textbook elements (e.g., examples, definitions) over others in order to prepare for or follow lessons, to understand the material by defining concepts, and to study for quizzes, exams, and homework.

Georgia Maragou (1^{st} Junior High School of Skala Oropou Attikis)

The laboratory of Mathematics at 1^{st} Junior High School of Skala Oropou Attikis, Greece

Mathematics and Art have a long historical relationship. Artists have used Mathematics since the 4th century BC when the Greek sculptor Polykleitos wrote his Canon, prescribing proportions based on the golden ratio for the ideal male nude. Besides, in all seasons highlighted prominent art forms used mathematics as the main component of their art. The Greeks were aware of that from the antiquity and without Geometry they could not have gone further within Art. Geometry having always been the way to both practical and artistical needs and due to the fact that it can offer the students the opportunity to acquire the capacity of understanding the space and provide empirical procedures that promote creativity and logic, it was utilized at the 1st Junior High School of Skala Oropou Attikis, Greece. As the students are overwhelmed by daily information, education has to be effective but at the same time attractive as well. Being also aware of the wider effort

of the research education community to mitigate the relationship between Mathematics and Art through alternative teaching methods, teaching the courses, it was also settled a Laboratory of Mathematics by the professor at this school above. She tried explaining to the students how she designed, constructed her several art-objects packed in her personal collection years ago. As her constructions apart from wealthy geometric structures they had also a significant aesthetic interest, she realized that the students observed the hidden Maths beauty and they've got great inspiration and interest in Maths.

Christina Misailidou (National and Kapodistrian University of Athens, Department of Education)

Promoting meaningful mathematics learning in primary school

Despite its importance, mathematics is very often perceived as a boring, difficult and many times meaningless subject. Designing mathematical experiences which are meaningful to children, as opposed to the ones offered traditionally in schools, can be a challenging task. This paper presents selected results from a teaching experiment that involved the design and implementation of a "Maths Day" (MD) in a Greek primary school. The duration of the MD was the same as a normal school day. Throughout that time, all the pupils from all the classes, instead of being taught their normal subjects, they were involved in mathematical activities, quite different than the ones they were used to. The activities were designed to be entertaining and challenging and, on the same time, to provoke "inquiry based" mathematics learning. The paper focuses on the activities designed for the 5th and 6th Year classes of the school (10-12 years old). The theme of their day was "Travel the World" and their day was divided in three phases: the "Introductory Activity", the "Circular Activities" and the "Being a Designer" activity.

Data were collected from observations and interviews and were then analysed by adopting as a unit of analysis the "thematic episode". The analysis of the results indicates that the "MD" activities provoked "inquiry-based learning" through the pupils' active participation. Moreover, the pupils perceived the activities as challenging and very entertaining.

Andreas Moutsios-Rentzos (University of the Aegean)

Interdisciplinary Mathematics Education and systemic school organisation

(joint work with Georgios Kritikos and Fragkiskos Kalavasis)

In this paper, we adopt an interdisciplinary approach to mathematics education to argue that the learning of mathematics and the sciences needs to be realised through making explicit their mutual linkings, thus allowing for the construction of a collaborative teaching space in which math and science educators may collaborate to understand the implicit ways (epistemological, historical and didactical) that each discipline enriches the other and at the same time differs from the other. Within this approach the mathematics knowledge is constructed in contrast and in relation to the physics knowledge and vice versa. Furthermore, we posit that such an approach requires the transformation of the organisational structure of the school towards an open learning organisation within which the educational protagonists (including students, teachers, principals, parents) act and interact through a multiplicity of roles, functions, intentionalities and meanings. Drawing upon this perspective, we present results from an ongoing research project developed in the Learning Technology and Educational Engineering laboratory of the University of the Aegean in Rhodes (LTEE; http://ltee.aegean.gr), with a particular focus on the meanings of each school course associated to mathematical signs appearing in both courses. Through a spiral of reflective activities the educational protagonists will actively experience the divergences and the convergences between the associated meanings for each course. It is posited that this process will reveal both unidisciplinary and interdisciplinary obstacles

(which would be otherwise remain hidden), thus allowing for a novel, quality of learning as linking links to emerge.

Dimitra Remoundou (University of the Aegean)

Rate of change: Necessity, epistemological obstacles and teaching proposals

(joint work with Evgenios Avgerinos)

What is the speed of a runner who runs 500 meters in 1,6 minutes? How fast does the volume of an inflated balloon change? What is the rate of change of water volume in relation to the level of water in a cube tank? What is the meaning of a positive, decreasing rate of change? These are a few of questions related to rate of change that confuse students. The solution is introduced in the last grade of high school: the rate of change is the derivative. An algorithmical solution is introduced, with the use of formulas and value substitution and that's it.

Rate of change is used continuously in everyday life and sciences and could be a familiar concept. It is firstly introduced in physics, by speed and acceleration. The opportunity to relate mathematics and physics and clarify formulas and concepts already used seems to cause more misconceptions. Some of the difficulties in conception of rate of change arise from epistemological obstacles related to the concept of limit, infinity and instantaneous. The language used in problems with rate of change is quite complicated with long phrases and ambiguous words. Moreover, the main point and core of the concept is usually suppressed in order to learn an algorithm for problem solving. In the current study, the concept of rate of change is considered from three points of view: the necessity to emphasize its meaning, the epistemological obstacles related to its understanding and the methods they can be overcome.

Ioannis Rizos (University of Patras)

Teaching scenarios and their role in the interdisciplinary approach. Case study: The Minkowskian Metric

One of the demands of modern teaching is the interdisciplinary approach of cognitive subjects and in particular that of the natural sciences, with simultaneous engagement of teachers and students. At the same time, teaching scenarios seem to be gaining ground in the methodology of teaching Mathematics in school. In this paper we analyze the concepts of the teaching scenario and interdisciplinarity, and we present the basic results of the conducting of a teaching experiment using teaching scenario, focused on the teaching of Minkowskian Metric in two dimensions.

Angeliki Stamati (American Community Schools)

Mathematics Morfosis Educational Philosophy: Transforming High School Teaching and Learning Mathematics

(joint work with Stefanos Gialamas)

Through our presentation we will show ways to expose students to a new pedagogical methodology in which they are encouraged and guided to recognize how mathematical concepts and their applications relate to everyday life so that connections between the abstract and the concrete are established and solidified. This purposeful connection encourages students to identify the practicality of mathematics even in the most unlikely places. Thus, through this teaching methodology, the concepts become crystallized in the minds of the students. Our presentation titled, "Mathematics Morfosis Educational Philosophy: Transforming High School Teaching and Learning Mathematics" participants will be engaged in exploring and reflecting on how students learning can be meaningful and exciting. Therefore, holistic teaching and learning of mathematics transcends merely

regurgitating mathematics concepts and allows the student to dive deeper into the "why." For instance, why we do we need Venn Diagrams or why have we come up with factorials? The math curriculum does not only need to emphasize arithmetic, computational accuracy, or processes memorization, but also provide a foundational grounding into the "why's" or "how's" of mathematics and their real world connections.

Meaningful teaching of mathematics, after all, is about revealing to the students that mathematics is not an isolated and meaningless discipline, but being able to look at a "problem" in reverse. For example, helping a student understand what makes one drawing of a face more beautiful than others. clearly lies in the comprehension and application of mathematical proportions and symmetry. Using this methodology, students are exposed to different perspectives of mathematical thinking. At times, students who excel in other courses perform subpar in mathematics, as they do not actively seek engagement with mathematical ideas for a lack of relevance to their personal world. Such students can be great-learners and creative-thinkers, but weak students.

This presentation renders practical daily teaching strategies; as simple as the kinds of questions that spark students' curiosity in mathematics, in order to make mathematics authentic and enable students to develop an appreciation the infinite possibilities of mathematics. When students ask meaningful questions, for example, they can invoke their own strategies. The skillful teacher purposefully bridges the gap between being a curious student and a student who excels in math.

The Morfosis Educational Philosophy transforms traditional teaching to a process that produces harmonious mathematical learning. Strategies on how to integrate mathematics into life beyond the classroom by means of this model, are provided. The i^2 Flex delivery methodology encompasses three components in its instructional methodology: independent learning, learning guided by a faculty mentor, and face-to-face. The underlying goal of the implementation of this systematic pedagogy is effective learner-centered methodology via the development of higher order cognitive skills as is specified in Bloom's revised Taxonomy. Students simultaneously engage both mind and heart, and real-world relevance unfolds.

Michael Voskoglou (TEI of Western Greece)

Teaching the Graphical Representation of the Derivative with the APOS/ACE Instructional Treatment for Mathematics

The APOS/ACE instructional treatment of Mathematics, developed by Ed Dubinsky and his collaborators in the USA during the 1990s, is applied in the present work for teaching/learning the concept of the derivative with emphasis on its graphical understanding. For this, a Genetic Decomposition is developed based on the outcomes of previous researches and on our personal teaching insights on the subject. Further, an ACE cycle is designed with the help of the proper software and implemented on a group of freshmen students of the School of Technological Applications of the TEI of Western Greece in the city of Patras (experimental group). The outcomes of this implementation are evaluated by comparing the performance of the experimental group to the performance of another equivalent student group (control group), to which the same subject was taught in the traditional, lecture-based way. At the end our conclusions are stated together with some hints for further research on the subject.

Workshop for secondary mathematics teachers

Irene Biza and Elena Nardi (University of East Anglia, UK)

MathTASK: Transforming Aspirations of Mathematics Teachers Into Strategies In Context

The MathTASK research programme engages secondary mathematics teachers (pre- and in- service) with situation-specific tasks that aim to trigger reflection on pedagogical aspirations as well as actual practice. Our tasks draw on real-classroom situations and address key learning and teaching issues such as fostering mathematical thinking, managing the mathematics classroom, inclusion of students with disability and integrating digital resources in mathematics teaching. In this 70-minute workshop you will be introduced to the MathTASK principles and task design (10 minutes); engage in a group activity with two situation-specific tasks (40m); become acquainted with summaries of our research findings including theoretical constructs which have emerged from these findings (10 minutes); and, share your thoughts in a plenary discussion about the tasks, their utility and purpose as well as other teacher priorities that they may focus in the future (10 minutes).