First Congress of Greek Mathematicians
Special Session in Geometry
June 26-27, 2018

Organizers

Andreas Arvanitoyeorgos – Georgios Tsapogas – Theodoros Vlachos

Time Schedule of Talks

	Tuesday, June 26	**Wednesday, June 27**
09:00 – 09:25	Charalambous	
09:25 – 09:50	Galanis	
09:50 – 10:15	Onti	
10:15 – 10:40	Bourni	
10:40 – 11:05	Polymerakis	
17:00 – 17:25	Calvaruso	Batakidis
17:25 – 17:50	Samiou	Xiong
17:50 – 18:15	Spilioti	Kasioumis

Speakers

Panagiotis Batakidis (University of Cyprus)
Poisson geometry, Seiberg-Witten invariants and the classification of 4 *manifolds*

Witten introduced topological quantum field theory to understand the physical meaning of Donaldson's invariants, which were unveiled earlier towards the classification of smooth 4-manifolds. This lead to Seiberg-Witten equations whose solutions define smooth 4-manifold invariants, the SW-invariants. If the 4-manifold carries a symplectic form, things become better and Taubes proved that the SW-invariants represent a count of pseudoholomorphic curves (Gromov's invariant), associating the classification of smooth structures to symplectic structures. To broaden the scope of the symplectic category, Taubes presented near-symplectic structures by relaxing the non-degeneracy condition. Lefschetz pencils offer another tool to study symplectic manifolds. This is a map that one can put on a 4-manifold and which is equivalent to the condition of admitting a symplectic form. Blowing up certain points creates then what is called Lefschetz fibrations. These mappings have a finite number of isolated points as singularities and allowing a second type of singularity, we get broken Lefschetz fibrations (bLfs). Auroux, Donaldson and Katzarkov showed a correspondence between near-symplectic structures and bLfs on a 4-manifold. The talk brings in Poisson geometry for near-symplectic structures and bLfs. We construct a singular Poisson structure from a near-symplectic form, (a model for bLfs was known) and compute the Poisson cohomology for both. This is the complex of spaces of solutions to certain systems of linear partial differential equations on the exterior algebra of the tangent bundle. It turns out that the two cohomologies are different, exhibiting the distinctive position of the two structures in open classification problems.

Theodora Bourni (University of Tennessee, Knoxville)

Ancient Pancakes

We construct a compact, convex ancient solution of mean curvature flow in $\mathbb{R}^{n+1}$ with $O(1) \times O(n)$ symmetry that lies in a slab of width π. We provide detailed asymptotics for this solution and show that, up to rigid motions, it is the only compact, convex, $O(n)$-invariant ancient solution that lies in a slab of width π and in no smaller slab. This work is joint with Mat Langford and Giuseppe Tinaglia.

Giovanni Calvaruso (University of Salento, Lecce)

Pseudo-Riemannian g.o. and two-step g.o. spaces

Let (M, g) denote a homogeneous pseudo-Riemannian manifold and G a connected Lie group of isometries acting transitively on it, with H the isotropy group at a point $o \in M$. A geodesic $\gamma : I \to G/H$ through o is called homogeneous if it is the orbit of a one-parameter subgroup, that is, it can be expressed in the form $\gamma(t) = \exp(tX) \cdot o$ for some vector X in the Lie algebra of G. If every geodesic in M is homogeneous, then $(G/H, g)$ is called a g.o. space. Two-step homogeneous geodesics generalize homogeneous geodesics, considering in $(G/H, g)$ geodesics of the form $\gamma(t) = \exp(tX)\exp(tY) \cdot o$. A two-step g.o. space is a homogeneous pseudo-Riemannian space $(G/H, g)$, whose geodesics are all two-step homogeneous. We shall review some recent results concerning pseudo-Riemannian g.o. and two-step g.o. spaces, with particular regard to the affine method used for the non-reducive cases, the classification of four-dimensional pseudo-Riemannian g.o. spaces and the first examples of pseudo-Riemannian two-step g.o. spaces.

Nelia Charalambous (University of Cyprus)

The spectrum of the Laplacian on forms

The computation of the essential spectrum of the Laplacian requires the construction of a large class of test differential forms. On a general open manifold this is a difficult task, since there exists only a small collection of canonically defined differential forms to work with. In our work with Zhiqin Lu, we compute the essential k-form spectrum over asymptotically flat manifolds by combining two methods: First, we introduce a new version of the generalized Weyl criterion, which greatly reduces the regularity and smoothness of the test differential forms; second, we make use of Cheeger-Fukaya-Gromov theory and Cheeger-Colding theory to obtain a new type of test differential forms at the ends of the manifold. We also use the generalized Weyl criterion to obtain other interesting facts about the k-form essential spectrum over an open manifold.

George Galanis (Hellenic Naval Academy)

Statistical manifolds and Wasserstein spaces for data analysis and environmental applications

A new geometrical framework based on recent advances in Information, and non Euclidean in general, Geometry is proposed for the analysis and study of big data sets and the optimization of numerical modeling systems. Trying to take some steps beyond the classical methodologies and to avoid simplifications that are usually adopted, the data under study are categorized in statistical manifolds and the corresponding geometric tools including minimum length curves are defined accordingly. Moreover, the problem of the optimal combination of numerical modeling systems results is approached by utilizing optimal transportation theory over Wasserstein spaces towards the quantification and minimization of forecasting uncertainty. The proposed new geometrical framework aims at the better understanding of potential physical characteristics of the available data sets, avoiding simplifications induced by classical approaches that treat the data as simple numerical values, failing to take into consideration potential intrinsic entities. The obtained tools are applied to the results of numerical weather and wave prediction models supporting several applications that are highly impacted today by such prediction systems including renewable energy monitoring and assessment, search and rescue operations and others. It is highlighted, in this way, the added value of the use of solid mathematical tools for the support of activities and problems that surpass the borders of the pure scientific research having, however, critical impact to socio-economic actions.

Joint work with A. Yannacopoulos and G. Papayiannis.

Theodoros Kasioumis (University of Ioannina)

Isometric immersions and relative nullity

We discuss minimal isometric immersions with relative nullity and we provide a classification of complete minimal isometric immersions with large index of relative nullity using tools from analysis. Under the mild assumption that the Omori-Yau maximum principle holds, we deduce that the immersion must be a cylinder over a minimal surface.

Christos-Raent Onti (IMPA)

Holonomic submanifolds with applications

A remarkable class of submanifolds in space forms are those that enjoy the property of being holonomic. An isometric immersion $f : M^n \to \mathbb{Q}_c^N$ of a Riemannian manifold into a space form of constant sectional curvature c is said to be *holonomic* if M^n carries a global system of orthogonal coordinates such that at any point the coordinate vector fields diagonalize its second fundamental form $\alpha\colon TM \times TM \to N_fM$ with values in the normal bundle.

There are several conditions that imply that a submanifold of a space form has to be locally holonomic. By locally we mean along connected components of an open dense subset of the manifold. For instance, this is the case of any isometric immersion $f : M_c^n \to \mathbb{Q}_c^N$ with flat normal bundle of a manifold with the same constant sectional curvature as the ambient space form provided that index of relative nullity vanishes at any point. Recall that the *index of relative nullity* $\nu(x)$ of $f : M^n \to \mathbb{Q}_c^N$ at $x \in M^n$ is the dimension of the *relative nullity subspace* $\Delta(x) \subset T_xM$ given by

$$\Delta(x) = \{X \in T_xM : \alpha(X, Y) = 0 \ \text{ for all } \ Y \in T_xM\}.$$

Isometric immersions $f\colon M_c^n \to \mathbb{Q}_{\tilde{c}}^{n+p}$ with sectional curvatures $c < \tilde{c}$ and in the least possible codimension $p = n - 1$ have flat normal bundle and thus are always locally holonomic. This was already known to Cartan who made an exhaustive study of the subject and determined the degree of generality of such submanifolds. Moreover, being holonomic is also necessarily the case for $c > \tilde{c}$ but now under the extra condition that the submanifold is free of weak-umbilic points.

In our talk, we will see that the results discussed above still hold for isometric immersions of the larger class of Einstein manifolds. In fact, this turns out to be the case even in the presence of a constant positive index of relative nullity, thus in the case of submanifolds of manifolds with the same constant sectional curvature the restriction mentioned above can be dropped.

This is a joint work with M. Dajczer and Th. Vlachos.

Panagiotis Polymerakis (Humboldt University of Berlin)

On the spectrum of the Laplacian under Riemannian coverings

The spectrum of the Laplace-Beltrami operator on a Riemannian manifold is an isometric invariant, whose behavior under natural maps remains unclear. In this talk, we discuss some recent results on the behavior of the spectrum under Riemannian coverings. In particular, we focus on the relation between the amenability of a covering and the behavior of the bottom of the spectrum under the covering.

Evangelia Samiou (University of Cyprus)

The X-ray transform on manifolds with conjugate points

The X-ray transform on a Riemannian manifold maps a compactly supported function f to a function Xf on the set of escaping geodesics. It is defined by $Xf(\gamma) = \int_{-\infty}^{\infty} f(\gamma(t))\, dt$. Support theorems (and thus injectivity) of the X-ray transform have been established by Helgason for symmetric spaces of non compact type. This has been extended to Damek-Ricci spaces by Rouviere. For simple manifolds there is a support theorem by Krishnan. These spaces do not have conjugate points.

We prove a support theorem for the X-ray transform on manifolds each of whose points lies in a totally geodesic euclidean or hyperbolic plane. This applies to most nilpotent Lie groups and to all 2-step nilpotent Lie groups, except the Heisenberg group. On the Heisenberg group we obtain a support theorem by more direct methods. These are the first known examples of support theorems on manifolds with conjugate points.

Polyxeni Spilioti (University of Tübingen)

Dynamical zeta functions, Lefschetz formulae and applications

In this talk, we will present powerful tools from the field of spectral geometry, such as trace formulae and Lefschetz formulae, and the machinery that they provide to study the analytic properties of the dynamical zeta functions and their relation to spectral invariants. In addition, we will present other applications of the Lefschetz formula, such as the prime geodesic theorem for locally symmetric spaces of higher rank.

Min Xiong (Institute of Computer Application, CAEP, China)

Some Properties for the Abreu Equation

Abreu equation appears in the study of differential geometry of toric varieties and it is a real four order equation. Fortunately, this equation has a differential geometric background and we can use some geometric theoretical knowledge and affine techniques to study it. For this equation, we introduce a Bernstein type theorem on a special complete Riemannian manifold and a regularity theorem under the assumption that Ricci curvature is bounded.