First Congress of Greek Mathematicians
Special Session in Differential Equations and Dynamical Systems
June 25-28, 2018

Organizers

Nikolaos Alikakos – Gerassimos Barbatis – Stathis Filippas – Maria Hadjinicolaou
Spyridon Kamvissis – Nikolaos Karachalios – Vasileios Papanikolaou – Achilles Tertikas

Time Schedule of Talks

	June 25	**June 26**	**June 27**	**June 28**
09:00 – 09:30		Savas-Halilai	Rapti	Latos
09:30 – 10:00		Smyrnelis	Bitsouni	Pournashash
10:00 – 10:30		*break*	*break*	*break*
10:30 – 11:00		Karaliolios	Fotiadis	El-Borai
16:00 – 16:40	Hadjinicolaou	Katzourakis	Daniilidis	Giannoulis
16:40 – 17:20	Langford	Mantoulidis	Bourni	Athanassoulis
17:20 – 17:30	*break*	*break*	*break*	*break*
17:30 – 18:00	Roidos	Abugirda	Protopapas	
18:00 – 18:30	Kalimeris			

Speakers

Hussien Abugirda (University of Reading)
Phase separation of n-dimensional ∞-harmonic mappings

Among other interesting results, in a recent paper [On the Structure of ∞-Harmonic Maps, Communications in Partial Differential Equations, 39(11): 2091-2124, 2014], Katzourakis analysed the phenomenon of separation of the solutions $u : \mathbb{R}^2 \supseteq \Omega \longrightarrow \mathbb{R}^N$, to the ∞-Laplace system

$$\Delta_\infty u := \big(\Delta u \otimes \Delta u + |Du|^2 [\![Du]\!]^\perp \otimes I\big) : D^2 u = 0,$$

to phases with qualitatively different behavior in the case of $n = 2 \leq N$. The solutions of the ∞-Laplace system are called the ∞-Harmonic mappings. In this talk I will discuss an extension of Katzourakis result to higher dimensions by studying the phase separation of n-dimensional ∞-Harmonic mappings in the case $N \geq n \geq 2$.

Agis Athanassoulis (University of Dundee)
Landau damping for the Alber equation and observability of unidirectional wave spectra

Ocean waves can be approximately described in terms of the Nonlinear Schrödinger equation (NLS), and considerations based on this approximation are widely used. On the other hand, ocean waves are noisy and in many applications statistical models have to be used.

The Alber equation was originally derived as a way to start from a stationary background spectrum and perform a linear stability analysis on it, assuming NLS dynamics. While this approach combines two of the most successful models for ocean waves to yield an intuitive result, it has remained relatively little-known. The heretofore lack of a rigorous well-posedness framework has been an important reason.

We present the first well-posedness result for the Alber equation and, building on that, we prove linear Landau damping. More specifically, we derive a nonlinear eigenvalue problem which is shown to determine if a given background spectrum is linearly stable or unstable. This explains why certain wave spectra (i.e. the stable ones) are commonly observed in the ocean. The analysis is broadly inspired by L. Landau's and O. Penrose's classical treatment of the Vlasov-Poisson equation.

Joint work with G. Athanassoulis (NTUA), M. Ptashnyk (Heriot-Watt) and T. Sapsis (MIT).

Vasiliki Bitsouni (University of Edinburgh)

Nonlinear differential equations in infectious disease epidemiology and cancer growth

Mathematical biology is used to study the mechanisms which underpin the dynamics of biological systems. A wide range of areas of applied mathematics is used to describe cell biology, ecology, epidemiology, evolutionary biology and genetics, and many other topics of life sciences and medicine. We give a brief introduction of mathematical models formulated in terms of dfferential equations which can be used to study the evolutionary dynamics of various aspects of biological science and we present some recent results on epidemiological consequences of different vaccination strategies against infectious diseases in domestic livestock. Next we focus on modelling of cancer invasion. We present a system of nonlocal partial differential equations coupled with ordinary differential equations, describing cancer cell movement and aggregation as a result of integrin-controlled cell-cell/cell-matrix adhesion and transforming growth factor beta (TGF-β) signalling, for two cancer cell populations with different levels of mutation. We discuss results regarding the existence and boundedness of solution, as well as different possible cell aggregation patterns.

Theodora Bourni (University of Tennessee)

On the existence of translating solutions of mean curvature flow in slab regions

We prove, in all dimensions $n \geq 2$, that there exists a convex translator lying in a slab of width $\pi \sec\theta$ in $\mathbb{R}^{n+1}$ (and in no smaller slab) if and only if $\theta \in [0, \frac{\pi}{2}]$. We also obtain convexity and regularity results for translators which admit appropriate symmetries and study the asymptotics and reflection symmetry of translators lying in slab regions. This is joint work with Mat Langford and Giuseppe Tinaglia.

Aris Daniilidis (University of Chile)

Gradient flows and determination of convexity

We disclose a connection between the gradient flow of a C^2-function f and evanescent orbits of the second order gradient system defined by the square-norm of ∇f, under an adequate convexity assumption. As a consequence, we obtain the following surprising result for two C^2, convex and bounded from below functions f, g : if $\|\nabla f\| = \|\nabla g\|$, then $f = g + k$, for some $k \in \mathbb{R}$. Joint work with T. Boulmezaoud and P. Cieutat.

Mahmoud El-Borai (Alexandria University)

Ultimate behavior of some stochastic non-linear fractional parabolic systems

Problems of the form

$$\frac{\partial^\alpha u_i(x,t)}{\partial t^\alpha} = cL_i u_i(x,t) + cf_i(H(t),u) + c^2 F_i(S(t),x,u)\ , \quad t > 0\ ,\ 0 < \alpha \leq 1$$

$$u_i(x,0) = \phi_i(x), \quad i = 1, \ldots, n$$

are considered, where $L_1,\ L_2, \ldots, L_n$ are elliptic partial differential operators of higher orders and with coefficients depending on $x = \{x_1, \ldots, x_m\}$, $-\infty < x_j < \infty$, $j = 1, 2, \ldots, m$.

It is supposed that $H(t)$ is an $n \times N$ matrix with elements that are statistically stationary processes with the property of ergodicity of the mean. It is also supposed that $S(t)$ is a matrix of order $n \times N$ with stochastic processes elements. The existence and asymptotic behavior of solutions as c tends to zero are studied. Joint work with Khairia El-said El-Nadi.

Anestis Fotiadis (Aristotle University of Thessaloniki)

Estimates of the derivatives of the heat kernel

We obtain pointwise upper bounds on the derivatives of the heat kernel for a class of Riemannian manifolds, and we provide some applications. The aforementioned class includes symmetric spaces of noncompact type and Damek-Ricci spaces. Joint work with E. Papageorgiou.

Ioannis Giannoulis (University of Ioannina)

Interaction of modulated water waves of finite depth

We present results concerning the formal derivation and rigorous justification of the modulation equations describing the macroscopic dynamics in the hyperbolic scaling of small-amplitude gravity and capillary-gravity water waves of finite depth, focusing on their possible interaction.

Maria Hadjinicolaou (Hellenic Open University)

Generalized semi-separable eigenfunctions for the Stokes flow operator in spheroidal geometry

The steady axisymmetric creeping flow of an incompressible viscous fluid around a spheroidal object, is modelled by employing the notion of a stream function, which satisfies the well-known Stokes equation, $\mathrm{E}^4\psi = 0$, where E^2 is the second order Stokes irrotational operator and $\mathrm{E}^4 = \mathrm{E}^2 \circ \mathrm{E}^2$. In spheroidal coordinates, the solution space of the equation $\mathrm{E}^2\psi = 0$, is consisted of eigenfunctions in separable form, given in terms of products of Gegenbauer functions of the first and the second kind. The complete representation of the kernel of E^2 is obtained in closed form, as a series expansion of these separable eigenfunctions. Since the kernel space of the operator E^4 does not admit separable eigenfunctions, the general solution of the equation $\mathrm{E}^4\psi = 0$ is obtained as follows. First we derive a complete set of eigenfunctions of the generalized 0-eigenspace of the operator E^2 in closed form. These generalized eigenfunctions are given in terms of products of Gegenbauer functions of mixed order. The general solution of the equation $\mathrm{E}^4\psi = 0$, is then obtained as the sum of two functions, one that represents the kernel space of the operator E^2 and the other one that represents the generalized 0-eigensapce of the operator E^2. A rearrangement of the terms of this complete expansion of the stream function ψ, in a form appropriate for solving boundary value problem, is also shown, which leads to the notion of semi-separability. The semi-separable form of the stream function ψ has been used in various applications in physical sciences and engineering.

Konstantinos Kalimeris (Cambridge University)

The water wave problem through a non-local formulation

We discuss a non-local formulation for the classical equations of rotational water waves, which is based on the so-called unified transform or the Fokas method, which provides a novel approach for the analysis of linear and integrable nonlinear boundary value problems. After reviewing some of the basic aspects of this approach we analyse inviscid, irrotational, two dimensional water waves in a bounded domain, and in particular we study the generation of waves by a moving bottom, as it occurs in tsunamis. We show that this problem is characterised by two equations which involve only first order derivatives. Under the assumption of "small amplitude" waves, these equations yield a new generalisation of the Boussinesq system which is valid without the long wave approximation.

Nikolaos Karaliolios (Imperial College, London)

Normal form theorems in Elliptic and Parabolic dynamics

We will discuss some normal form theorems for perturbations of the main examples of elliptic and parabolic dynamics, generalizing the classical theorem due to Arnol'd and Moser, concerning perturbations of Diophantine translations in tori. Such theorems try to establish that the orbit of a certain type of diffeomorphism under the relevant notion of conjugation is locally a closed submanifold of a certain codimension. Subsequently, one tries to interpret the perturbations in the directions transversal to the manifold as perturbations that modify the dynamics of the studied diffeomorphism.

We will discuss normal form theorems in the following settings:

(1) commuting diffeomorphisms of the torus, close to Simultaneously Diophantine rotations. This provides a new, more general and stronger proof of a theorem by Moser on the simultaneous linearizability of such dynamical systems.
(2) in the case of a single diffeomorphism, this theorem dengenerates to stronger version of the classical normal form theorem for perturbations of Diophantine translations in tori, which provides a unified argument for the rigidity. of such systems independently of the dimension of the torus
(3) periodic diffeomorphisms of arbitrary compact manifolds.
(4) resonant Diophantine translations, which are intermediate between the periodic rotations and Diophantine ones.
(5) the parabolic mapping of the torus $\mathbb{T}^2$.

Nikos Katzourakis (University of Reading)

On the existence and uniqueness of vectorial absolute minimisers in Calculus of Variations in L_∞

Calculus of Variations in the space L_∞ has a relatively short history in Analysis. The scalar-valued theory was pioneered by the Swedish mathematician Gunnar Aronsson in the 1960s and since then has developed enormously. The general vector-valued case delayed a lot to be developed and its systematic development began in the 2010s. One of the most fundamental problems in the area which was open until today (and has been attempted by many researchers) concerned that of the title. In this talk I will discuss the first result in this direction, which is based on joint work with my research associate Giles Shaw.

Mat Langford (University of Tennessee)

Ancient solutions of geometric flows

I will survey recent developments which have led to a number of new existence, uniqueness and rigidity results for ancient solutions to geometric flows of mean curvature type.

Evangelos Latos (Universität Mannheim)

Long-time behaviour of some nonlocal problems

We study the asymptotic behavior of the solutions to some nonlocal reaction-diffusion problems on a bounded domain. We provide conditions for which the problem admits a unique non-negative classical solution, as well as, conditions under which the problem blows up in finite time.

Christos Mantoulidis (Massachusetts Institute of Technology)

Allen–Cahn min-max in 3-manifolds

The Allen–Cahn equation is a semi-linear PDE that produces minimal surfaces via a certain singular limit. We will describe recent work proving index, multiplicity, and curvature estimates in the context of an Allen–Cahn min-max construction in a 3-manifold. Our results imply, for generic Riemannian metrics, the Allen–Cahn analogs of the Marques–Neves "multiplicity one" and "index lower bound" conjectures, as well as give a new proof of Yau's conjecture for the existence of infinitely many minimal surfaces in Riemannian 3-manifolds. This is joint work with Otis Chodosh.

Hossein Pourbashash (University of Garmsar)

Mathematical analysis of a class of HIV infection models of cd4+ t-cells and center manifold analysis

HIV can infect cells via virus-to-cell infection or cell-to-cell viral transmission. These two infection modes may occur in a synergistic way and facilitate viral spread within an infected individual. In this paper, we developed an HIV latent infection model including both modes of transmission and time delays between viral entry and integration or viral production. We analyzed the model by defining the basic re-productive number. Secondly, an analysis of local center manifold shows that when $R0 = 1$, the disease free equilibrium is asymptotically stable and a trans-critical bifurcation occurs. The result of this study can be used in disease control. Joint work with Hossein Mohebbi.

Eleftherios Protopapas (National Technical University of Athens)

R-semiseparable solutions for Stokes blood flow

Stokes flow is the axisymmetric flow of a Newtonian incompressible fluid, where viscous forces dominate over the inertial ones. The governing equation is $\mathrm{E}^4\psi = 0$, where E^2 is the Stokes operator, $\mathrm{E}^4 = \mathrm{E}^2 \circ \mathrm{E}^2$ is the Stokes bistream operator and ψ is the stream function. It has been shown that in the spheroidal system of coordinates, equation $\mathrm{E}^4\psi = 0$ semiseparates variables. In the case of the inverted spheroidal system of coordinates we prove that the 0-eigenfunctions of the operator are expressed in R-separable closed form, with R denoting the Euclidean distance. Furthermore the generalized eigenfunctions of the operator E^2 are obtained in a non-closed R-semiseparable form, where R stands, this time, for the Euclidean distance on the third power. Unfortunately, the obtained form for the stream function is not suitable for solving boundary value problems. Instead, by taking advantage of the particular form of the R-semiseparation, we transform the problem we formulated in the inverted spheroidal coordinates, to an equivalent one, expressed in the spheroidal coordinates. We then employ the known expression for the stream function, which is in semiseparable form. An interesting application of the above results, is to employ this obtained expansion of ψ, for solving the problem of the blood plasma flow past a Red Blood Cell (RBC). The geometrical representation of the RBC, as an inverted prolate spheroid, is indicated by its shape, which is a biconcave disk. Significant hydrodynamic quantities are calculated using the stream function such us the drag coefficient and the settling terminal velocity of an RBC.

Zoi Rapti (University of Illinois at Urbana-Champaign)

Stability problems arising in biologically motivated PDEs

This talk is focused on the spectral properties of certain classes of nonlinear PDEs with applications to epidemic and animal dispersal models. We will present results that show the existence of only real spectrum in the corresponding non self-adjoint eigenvalue problem. Our proof relies on the theory of operator pencils and Herglotz functions. Concrete applications will be demonstrated in models of rabies infection in fox populations, plant-herbivore interactions and morphogen diffusion.

Nikolaos Roidos (Leibniz Universität Hannover)

Conic manifolds under the Yamabe flow

We consider the unnormalized Yamabe flow on manifolds with conical singularities. Under certain geometric assumption on the initial cross-section we show well posedness of the short time solution in the L^q-setting. Moreover, we give a picture of the deformation of the conical tips under the flow by providing an asymptotic expansion of the evolving metric close to the boundary in terms of the initial local geometry. Due to the blow up of the scalar curvature close to the singularities we use maximal L^q-regularity theory for conically degenerate operators.

Andreas Savas-Halilaj (Universität Hannover)

Lagrangian Mean Curvature Flow of Whitney Spheres

We investigate the evolution under the mean curvature flow of an equivariant Lagrangian sphere. Moreover, under a convexity condition on the Ricci curvature, we are able to describe the singularity. This is a joint work with K. Smoczyk (Hannover).

Panayotis Smyrnelis (University of Chile)

Symmetry breaking and restoration in the Ginzburg-Landau model of nematic liquid crystals

We study qualitative properties of global minimizers of the Ginzburg-Landau energy which describes light-matter interaction in the theory of nematic liquid crystals. This model depends on two parameters: $\epsilon > 0$ which is small and represents the coherence scale of the system and $a \geq 0$ which represents the intensity of the applied laser light. In particular we are interested in the phenomenon of symmetry breaking as a and ϵ vary. We show that when $a = 0$ the global minimizer is radially symmetric and unique and that its symmetry is instantly broken as $a > 0$ and then restored for sufficiently large values of a. Symmetry breaking is associated with the presence of a new type of topological defect which we named the shadow vortex. We also discovered that the profile of the global minimizers is given on the boundary of the illuminated region by the universal equation of Painlevé. The symmetry breaking scenario is a rigorous confirmation of experimental and numerical results obtained earlier. Joint work with Michal Kowalczyk and Marcel Clerc.