First Congress of Greek Mathematicians
Special Session in Algebra and Combinatorics
June 27-29, 2018

Organizers

Ioannis Emmanouil – Alexis Kouvidakis – Vasileios Metaftsis

Time Schedule of Talks

	Wednesday, June 27	**Thursday, June 28**	**Friday, June 29**
09:00 – 09:40	Pallikaros	Chlouveraki	Tziolas
09:45 – 10:05	Psaroudakis	Tsatsomeros	Papalexiou
10:10 – 10:30	Sklinos	Papadakis	Syrigos
10:35 – 10:55	Manousaki	Tzanaki	Kallipoliti
17:00 – 17:20	Kapetanakis	Douvropoulos	Stergiopoulou
17:25 – 17:45	Michailidis	Paramantzoglou	Savvidou
17:50 – 18:10	Zafeirakopoulos	Sarakasidis	Kosta

Speakers

Maria Chlouveraki (Université de Versailles-St Quentin)
The symmetrising trace conjecture for Hecke algebras

Exactly twenty years ago, Broué, Malle and Rouquier published their seminal paper in which they associated to every complex reflection group two objects which were classically associated to real reflection groups: a braid group and a Hecke algebra. Their work was further motivated by the theory, developed together with Michel, of "Spetses", which are objects that generalise finite reductive groups in the sense that their associated Weyl groups are complex reflection groups. The four of them advocated that several nice properties of braid groups and Hecke algebras generalise from the real to the complex case, culminating in two main conjectures as far as the Hecke algebras are concerned: the "freeness conjecture" and the "symmetrising trace conjecture". The two conjectures are the cornerstones in the study of several subjects that have flourished in the past twenty years, but have remained open until recently for the exceptional complex reflection groups. In the past five years, the proof of the "freeness conjecture" was completed for all exceptional complex reflection groups. In this talk, we will mainly discuss our proof of the symmetrising trace conjecture for the first five exceptional groups. This is joint work with Christina Boura, Eirini Chavli and Konstantinos Karvounis.

Theodosios Douvropoulos (Université Paris Diderot)
Enumeration of reflection factorizations of a Coxeter element via Malle's permutation $\Psi \in \mathrm{Perm}(\mathrm{Irrep}(W))$

Hurwitz showed, already in 1892, that there are n^{n-2} shortest-length factorizations of the long cycle $(12\cdots n) \in S_n$, in transpositions. By the end of the 80's, Jackson and others had computed the exponential generating series $F(t)$ of such factorizations of arbitrary length. It can be written as

$$F(t) = \frac{e^{t\binom{n}{2}}}{n!}\,(1 - e^{-tn})^{n-1},$$

in this way relating the product structure in the reduced (shortest-length) and arbitrary length cases.

As with plenty more fascinating results for the symmetric group, these have beautiful analogs for other reflection groups W. The number of shortest length reflection factorizations of a Coxeter element c is given by the Deligne-Arnol'd-Bessis formula $\frac{h^n n!}{|W|}$, where h is the Coxeter number of W. The exponential generating series of arbitrary length factorizations is given by the Chapuy-Stump formula:

$$F_W(t) = \frac{e^{t|R|}}{|W|}(1 - e^{-th})^n,$$

where R is the set of reflections. Again, one observes the common product structures in the reduced and arbitrary length cases.

One disadvantage of this last more general formula is that it lacks a uniform proof and hence understanding. We will present here a derivation (and further generalization of it via Malle's permutation Ψ_W of the irreducible representations of W. This approach is uniform up to a reliance on the freeness conjecture (now case-by-case theorem) of Broué, Malle, Rouquier for generic Hecke algebras.

Myrto Kallipoliti (University of Vienna)

Edgewise Cohen-Macaulay connectivity for partially ordered sets

In this talk we discuss a (new) notion of connectivity for partially ordered sets, called edgewise Cohen-Macaulay connectivity, which is motivated by the edge connectivity for graphs. After giving the necessary definitions, we focus on an important class of partially ordered sets with respect to edgewise Cohen-Macaulay connectivity and present some open questions. This is a joint work with Christos Athanasiadis.

Georgios Kapetanakis (Sabanci University)

On the existence of primitive completely normal bases of finite fields

Let $\mathbb{F}_q$ be the finite field of cardinality q and $\mathbb{F}_{q^n}$ its extension of degree n, where q is a power of the prime p and n is a positive integer. A generator of the multiplicative group $\mathbb{F}_{(q^n)^*}$ is called primitive. An $\mathbb{F}_q$-normal basis of $\mathbb{F}_{q^n}$ is an $\mathbb{F}_q$-basis of $\mathbb{F}_{q^n}$ of the form $\{x, x^q, \ldots, x^{q^{n-1}}\}$ and the element x is called normal over $\mathbb{F}_q$. An element of $\mathbb{F}_{q^n}$ that is simultaneously normal over $\mathbb{F}_{q^l}$ for all $l \mid n$ is called completely normal over $\mathbb{F}_q$. It is well-known that primitive and normal elements exist for every q and n. The existence of elements that are simultaneously primitive and normal is also well-known for every q and n. Further, it is also known that for all q and n there exist completely normal elements of $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$. Morgan and Mullen [Util. Math. 49, 21-43, 1996] took the next step and conjectured that for any q and n, there exists a primitive completely normal element of $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$. This conjecture is yet to be established for arbitrary q and n, but instead there are partial results, covering special types of extensions. Recently, Hachenberger [Des. Codes Cryptogr. 80(3), 577-586, 2016] using elementary methods, proved the validity of the Morgan-Mullen conjecture for $q \geq n^3$ and $n \geq 37$. In this work, we use character sum techniques and prove the validity of the Morgan-Mullen conjecture for all q and n, provided that $q > m$, where $n = p^l m$ and $\gcd(m, p) = 1$. In the talk, the previous results will be briefly presented, our proof will be outlined and possible improvements will be discussed.

Dimitra Kosta (University of Glasgow)

Symmetric phylogenetic group-based models using numerical algebraic geometry

Phylogenetic models have polynomial parametrization maps. For symmetric group-based models, Matsen studied the polynomial inequalities that characterize the joint probabilities in the image of these parametrizations. We employ this description for maximum likelihood estimation via numerical algebraic geometry. In particular, we explore an example where the maximum likelihood estimate does not exist, which would be difficult to discover without using algebraic methods. We also study the embedding problem for symmetric group-based models, i.e. we identify which mutation matrices are matrix exponentials of rate matrices that are invariant under a group action.

Panagiota Manousaki (National and Kapodistrian University of Athens)

On the injective completion of the homological functor Tor

This presentation is based on joint work with Ioannis Emmanouil and concerns amongst others the study of the existence of the notion mentioned. We study both the stable homology, defined by Vogel, and the complete homology, defined by Triulzi, and compute the kernel of the natural surjection between the theories in order to find conditions under which these are isomorphic providing the injective completion of the homological functor Tor. The Tate cohomology of a finite group G and the Tate-Farrell cohomology of a group G with finite virtual cohomological dimension are generalized by a complete cohomology theory expressed via complete resolutions in the strong sense. The notion of the projective completion of the cohomological functor $\mathrm{Ext}R(A,-)$ for a left R-module A, dually the injective completion of the functor $\mathrm{Ext}R(-,B)$ for a left R-module B presented by Nucinkis, arises from the above as an aspect of a complete cohomology theory making the existence of such resolutions uncompulsory, as a cohomological functor with a universal property as Mislin's construction asserts. A natural equivalent expression is given by Benson and Carlson. The Vogel groups express the isomorphic theory of the stable cohomology. The homology version is further studied by Christensen; Celikbas, Liang and Piepmeyer. Our study is threefold since it can be concentrated in the comparison of the stable and the complete homology as mentioned above, vanishing criteria that detect modules of finite injective or at dimension over Noetherian rings and the balance of the stable homology over R characterizing coherent rings in terms of the finiteness of the invariant concerning the at dimension of injective modules and Iwanaga-Gorenstein rings.

Dimitrios Michailidis (University of Kent)

Bases of simple modules for the Temperley-Lieb algebra of type B

We give a combinatorial construction for the cellular basis of the simple modules for the Temperley-Lieb algebra of type B, which is sometimes referred as blob algebra. Apart from the definition of the blob algebra with generators and relations, we can view it as a quotient of the Khovanov-Lauda-Rouquier (KLR) algebra, which allows us to use combinatorics. Our method follows A. Mathas techniques for the construction of simple modules of the Hecke algebra.

Christos Pallikaros (University of Cyprus)

On degenerations of algebras over an arbitrary field

Since the second half of the twentieth century a lot of attention has been given to the study of different types of limit processes (called contractions) between various physical or geometrical theories. The claim is that if two physical theories are related by a limit process, then the associated invariance algebras should also be related by some limit process. This led to a wide investigation of contractions of Lie algebras from the physical point of view. Working over $\mathbb{C}$ or $\mathbb{R}$, the statement "Lie algebra h_1 is a contraction of Lie algebra h_2" can be rephrased as "h_1 lies in the closure, in the metric topology, of the orbit of h_2 under the "change of basis" action of the group of invertible linear transformations". The corresponding orbit closures with respect to the Zariski topology are called degenerations. The notion of degeneration is well-defined not only over the fields $\mathbb{C}$ or $\mathbb{R}$ but also over an arbitrary ground field.

In this joint work with N. M. Ivanova we explored the possibility of investigating degenerations over an arbitrary field using elementary algebraic techniques. For this we needed to extend or modify techniques already used over the fields $\mathbb{C}, \mathbb{R}$. As an application, we have classified all n-dimensional algebras (over an arbitrary field) which have the Abelian Lie algebra as their only proper degeneration.

Stavros Papadakis (University of Ioannina)

A result on Stanley-Reisner Rings of Simplicial Spheres

A simplicial sphere is a simplicial complex whose underlying topological space is homeomorphic to a sphere. The talk will be about the question of whether for a simplicial sphere the Weak Lefschetz Property of its Stanley-Reisner ring is equivalent to the same property of a stellar subdivision.

Nikolaos Papalexiou (University of the Aegean)

W-algebras and the orbit method

The theory of W-algebras gave a new impulse in the representation theory of semisimple Lie algebras. We provide a new realization of W-algebras via the Cattaneo-Felder-Torossian reduction algebra. We show how we can compute the generators of this reduction algebra. We also give the connection with the orbit method and the classification of the completely prime, primitive spectrum of the enveloping algebra of a semi-simple Lie algebra.

Panagiotis Paramantzoglou (National and Kapodistrian University of Athens)

Group actions on curves

We give an explanation of the MKR dictionary in Arithmetic topology using Ihara's theory of profinite braid groups. Motivated by the analogy we perform explicit computations for representations of both braid groups and the absolute Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ for cyclic covers of the projective line and generalized Fermat curves.

Chrysostomos Psaroudakis (University of Stuttgart)

Reduction techniques for the finitistic dimension

One of the longstanding open problems in Representation Theory of Finite Dimensional Algebras is the so called "Finitistic Dimension Conjecture". The latter homological conjecture is known to be related with other important problems concerning the homological behaviour and the structure theory of finite dimensional algebras. In this talk we present a reduction technique for the finitistic dimension using recollements of abelian categories. We will show that we can remove some vertices from a finite dimensional algebra (viewed as a quotient of a path algebra) such that the problem of computing the finitistic dimension can be reduced to a possible simpler ("homologically compact") algebra. This is joint work with Edward L. Green and Øyvind Solberg.

Christos Sarakasidis (University of Kent)

Low dimensional modular invariant theory of the finite Heisenberg group $\mathrm{UT3}(F_p)$

Invariant Theory has been a very active area of research over the last century with its origins coming directly from David Hilbert. The work that has been done so far gives a deep understanding for the case where the underlying field has characteristic zero, or more generally when the characteristic is positive and doesn't divide the cardinality of the group G. During this talk, I will illustrate the general idea behind the so-called modular case (where the characteristic $p > 0$ of the underlying field F divides the cardinality of the group G) when $G = \mathrm{UT3}(F_p)$, the finite Heisenberg group, for dimensions $n = 3, 4$. Moreover, by using the theory of SAGBI bases we will give examples showing why the non-Cohen-Macaulayness of the modular invariant ring $F[V]^G$ implies hard computational complexity for the generating set of $F[V]^G$.

Christina Savvidou (University of Central Lancashire, Cyprus)

On super-strong Wilf equivalence classes of permutations

Super-strong Wilf equivalence is a type of Wilf equivalence on words that was originally introduced as strong Wilf equivalence by Kitaev et al. [Electron. J. Combin. 16(2)] in 2009. We provide a necessary and sufficient condition for two permutations in n letters to be super-strongly Wilf equivalent, using distances between letters within a permutation. In this way super-strong Wilf equivalence classes in the symmetric group S_n on n letters are in bijection with pyramidal sequences of consecutive differences.

Giving a characterization of each such equivalence class via two-colored binary trees we are able to prove, in the special case of super-strong Wilf equivalence, a conjecture stated in the aforementioned article by Kitaev et al., that the cardinality of each Wilf equivalence class is a power of 2.

Furthermore, we enumerate all super-strong Wilf equivalence classes of S_n by giving a recursive formula in terms of a two-dimensional analogue of the sequence of the number of non-interval permutations in S_n. By a refined version of this formula we also enumerate the super-strong Wilf classes of given order 2^j in S_n. Finally, as a by-product, we give a recursively defined set of representatives of super-strong Wilf equivalence classes.

Rizos Sklinos (Université Claude Bernard Lyon 1)

Limit groups

Limit groups have been introduced by Z. Sela in his celebrated solution of Tarski's problem (2001). These groups coincide with the long studied class of finitely generated fully residually free groups introduced by Baumslag in 1967. Limit groups have a natural interpretation in many mathematical disciplines: topology, geometric group theory, algebraic geometry and logic to name a few. This fact makes them an interesting class to study and also provides us with an abundance of tools to do so.

In this talk, I will present all different points of view on limit groups and survey some basic results about them. Moreover, I will use model theory and in particular Frass limits in order to construct new groups with interesting universal and homogeneous properties with respect to the class of limit groups. This is joint work with O. Kharlampovich and A. Miasnikov.

Dionysia Stergiopoulou (National and Kapodistrian University of Athens)

On extensions of hook Weyl modules

In this talk we will discuss our results on the computation of integral extension groups between hook Weyl modules for the general linear group. Using computations with weight spaces, long exact sequences in cohomology and a result of Kulkarni, we study Ext_1, Ext_2

and the highest possible non vanishing Ext. From this, the dimensions of certain modular Ext groups are obtained. Joint work with M. Maliakas.

Dionysios Syrigos (University of the Aegean)

Royden's Theorem for Free products

Let G be a group which admits a Kurosh decomposition i.e. it splits as $G = F_n * G_1 * \cdots * G_k$, where every G_i is freely indecomposable and not isomorphic to the group of integers and by F_n is denoted the free group on n generators (every finitely generated group admits a Kurosh decomposition). Guirardel and Levitt generalised the Culler–Vogtmann Outer space of a free group (on which the outer automorphism group $\mathrm{Out}(F_n)$ acts by isometries) by introducing an Outer space for G as above, on which $\mathrm{Out}(G)$ acts by isometries. Francaviglia and Martino introduced the Lipschitz metric for the Culler–Vogtmann space and later for the general Outer space. The same authors proved the analogues of Royden's Theorem for the Lipschitz metric of Culler–Vogtmann space, by proving that the group of isometries of CV_n with respect to the Lipschitz metric, is exactly $\mathrm{Out}(F_n)$. This result can be seen as an indication that Culler–Vogtmann space equipped with the Lipschitz metric, is an accurate model for $\mathrm{Out}(F_n)$, as it has the correct isometries and no others. In a joint paper with Francaviglia and Martino, we show that the previous result can be generalised for the general free product case, as we prove that the group of isometries of the Outer space corresponding to G, with respect to the Lipschitz metric, is $\mathrm{Out}(G)$. In this talk, we will describe the construction of the general Outer space and the corresponding Lipschitz metric in order to present the result about the isometries.

Michail Tsatsomeros (Washington State University)

Semipositive Matrices

Semipositive matrices map by definition an (entrywise) positive vector to another positive vector. Results and challenges will be presented regarding factorizations of semipositive matrices and characterizations of their semipositivity cones. The relation to other matrix classes that are pertinent to the linear complementarity problem will be examined.

Eleni Tzanaki (University of Crete)

Upper bounds on the number of faces of the Minkowski sum of polytopes

Given two convex polytopes P, Q, their Minkowski sum, which is again a polytope, is defined as $P + Q = \{p + q : p \in P, q \in Q\}$. In this talk, I will present tight expressions for the maximum number of k-dimensional faces, $0 \leq k \leq d-1$, of the Minkowski sum $P_1 + \cdots + P_r$ of r convex d-dimensional polytopes $P_1, \ldots, P_r$ in $\mathbb{R}^d$ where $d \geq 2$ and $r < d$, as a (recursively defined) function of the number of vertices of the polytopes. These upper bounds are proved making use of basic notions such as $f-$ and $h-$vector calculus, stellar-subdivisions and shellings, and generalize the steps used by McMullen to prove the Upper Bound Theorem for polytopes.

The key idea behind the approach is to express the Minkowski sum $P_1 + \cdots + P_r$ as a section of the Cayley polytope C of the summands; bounding the $k-$faces of $P_1 + \cdots + P_r$ reduces to bounding the subset of the $(k + r - 1)-$faces of C that contain vertices from each of the r polytopes.

I will first present the steps of the proof for the Minkowski sum of two polytopes. I will then explain how these steps should be adjusted when one considers the Minkowski sum of three polytopes. Finally, I will briefly describe the general method for more than three summands. This is joint work with M. Karavelas and C. Konaxis.

Nikolaos Tziolas (University of Cyprus)

Vector fields and the classification of surfaces of general type in characteristic $p > 0$

We describe the geometry of surfaces of general type, which are defined over an algebraically closed field of characteristic $p > 0$ and have a non-trivial global vector field. We also describe the consequences that the existence of these surfaces has the classification problem of surfaces of general type.

Zafeirakis Zafeirakopoulos (Gebze Technical University)

Polyhedral Omega: a new algorithm for the solution of Linear Diophantine Systems

Polyhedral Omega is a new algorithm for solving linear Diophantine systems (LDS), i.e., for computing a multivariate rational function representation of the set of all non-negative integer solutions to a system of linear equations and inequalities. Polyhedral Omega combines methods from partition analysis with methods from polyhedral geometry. In particular, we combine MacMahons iterative approach based on the Omega operator and explicit formulas for its evaluation with geometric tools such as Brion decomposition and Barvinok's short rational function representations. In this way, we connect two branches of research that have so far remained separate, unified by the concept of symbolic cones which we introduce. The resulting LDS solver (Polyhedral Omega) is significantly faster than previous solvers based on partition analysis and it is competitive with state-of-the-art LDS solvers based on geometric methods. Most importantly, this synthesis of ideas makes Polyhedral Omega by far the simplest algorithm for solving linear Diophantine systems available to date. This is joint work with Felix Breuer.