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# John Wallis and the French: his quarrels with Fermat, Pascal, Dulaurens, and Descartes

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**Abstract**

John Wallis, Savilian professor of geometry at Oxford from 1649 to 1703, engaged in a number of disputes with French mathematicians: with Fermat (in 1657–1658), with Pascal (in 1658–1659), with Dulaurens (in 1667–1668), and against Descartes (in the early 1670s). This paper examines not only the mathematical content of the arguments but also Wallis's various strategies of response. Wallis's opinion of French mathematicians became increasingly bitter, but at the same time he was able to use the confrontations to promote his own reputation.


**Résumé**

John Wallis, professeur de géométrie à Oxford de 1649 à 1703, a participé à plusieurs controverses avec des mathématiciens français: avec Fermat (en 1657–1658), avec Pascal (en 1658–1659), avec Dulaurens (en 1667–1668), et avec Descartes (au début des années 1670). Cet article examine non seulement le contexte mathématique des arguments, mais aussi les différentes stratégies utilisées par Wallis dans ses réponses. L'opinion de Wallis sur les mathématiciens français était devenue de plus en plus acerbe, mais dans le même temps il était capable d'utiliser ces confrontations pour asseoir sa propre réputation.


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In the course of his long career as Savilian Professor of Geometry at Oxford (from 1649 to his death in 1703), John Wallis was involved in a great many quarrels and controversies.[1] No other national group, however, came to rile him so much as 'the French'. It was not always so, for he started out without any discernible antipathy toward French mathematicians. In his *Arithmetica infinitorum* of 1656, for instance, he spoke of both Viète and Descartes as 'great men' [Wallis, 1656, 83; Stedall, 2004, 82].[2] Thirty years later, however, his attitude to his French counterparts had become one of unreasoning bitterness. In this paper we will follow the course of this changing relationship by reviewing Wallis's mathematical quarrels with Fermat (in 1657–1658), Pascal (in 1658–1659), Dulaurens (in 1667–1668), and Descartes himself, albeit posthumously (in the early 1670s). These four episodes have not to my knowledge been examined together previously, and indeed the quarrel with Dulaurens is hardly known at all.[3]

The content of the arguments is only part of the story. We will also analyse the nature of the exchanges, and in particular Wallis's various strategies of response, provocation, and attack. These too developed over the years, as Wallis became not only more belligerent but also more skilled in creating distortions of the truth that would crush his adversaries' positions and bolster his own.

## 1. The exchanges with Fermat (1657–1658)

Wallis had very little contact with French mathematicians until the publication of his *Arithmetica infinitorum* in 1656 brought his name to a broader international audience. In 1655, however, he had written to Pierre Gassendi, hoping to gather ammunition for use against his more immediate adversary, Thomas Hobbes [CJW, I, letter 65]. He must have been somewhat shocked when in October 1656, after the publication of the *Arithmetica infinitorum*, Roberval circulated an open letter accusing not only Hobbes but Wallis himself of plagiarism. Wallis wrote two lengthy letters of defence, denying that he had ever seen Roberval's work [CJW, I, letters 83, 84]. Indeed, it is hard to see how he could have done so; even Hobbes thought it unlikely on the grounds that Wallis's work was crude in comparison with Roberval's. Nevertheless, this incident was perhaps a warning to Wallis of the dangers he faced now that his work was reaching wider audiences and more critical readers [for fuller accounts of the Roberval affair see Probst, 1997; Jesseph, 1999, 117–125; Malcolm, 2002, 162–165; Beeley and Scriba, 2008].

[1] This article is based on a paper originally given at a conference entitled 'Wallis as Controversialist and Correspondent' held in Oxford in 2010; for a companion article from that conference see Guicciardini [2012]. Most of the controversies are described in what remains the only full-length biographical study of Wallis to date, Scott [1938], but Scott's style and analysis are now somewhat dated. He writes (p. 65), for instance, that Wallis 'found himself entangled in a maze of controversy, from which he did not completely extricate himself until the closing years of his life', as though controversy, like a spell of ill health, was something for which Wallis himself was not responsible. More recent and more rigorous accounts of some of the individual controversies will be cited in the course of this article.

[2] *Vieta, Oughtredus, Harriotus, Ghetaldus, Cavallerius, Torricellius, Chartesius, aliique magni viri.*

[3] The quarrels with Fermat and Pascal are discussed briefly, and the accusations against Descartes at greater length, in Beeley and Scriba [2005]. The argument with Dulaurens is outlined briefly in Aramov [2002].

In the summer of 1656 Kenelm Digby, who was based in Paris, sent a copy of the *Arithmetica infinitorum* to Fermat in Toulouse. Two years earlier, Fermat had tried without success to interest Blaise Pascal in his discoveries of certain remarkable properties of the natural numbers [Mahoney, 1994, 332–347]. On receiving the *Arithmetica infinitorum* he must have thought that in Wallis he might find a worthy correspondent with whom he could share his findings. Rather than initiating a straightforward exchange, Fermat chose instead to put out an open challenge to the mathematicians of northern France, Belgium, and England. Nevertheless, the version that arrived in England in March 1657 was clearly intended for Wallis and was annotated as such by the messenger, Thomas White, with the words: 'A challenge from M. Fermat, for D. Wallis' [CJW, I, letters 88, 96].

Fermat's first challenge was twofold: (i) to find a cube which added to its divisors makes a square, and (ii) to find a square which added to its divisors makes a cube. As a sample solution to the first problem Fermat offered the cube $7^3 = 343$ (whose divisors are 1, 7, and 49), since $1 + 7 + 49 + 343 = 400$, a square. The question was, were there other such numbers? The problem is typical of many others in what modern mathematicians call 'number theory', which may be simple enough to pose but fiendishly difficult to solve.

The problem was first delivered to William Brouncker in London, who immediately sent it on to Wallis. Brouncker observed that it was likely to be more difficult than it appeared at first sight. Wallis, on the other hand, failed to see that it might have any value and simply dismissed it [CJW, I, letter 97]:

> The question is just about of the same sort as the problems ordinarily posed concerning the numbers called 'perfect', 'deficient' or 'abundant'. . . . Whatever the details of the matter, it finds me too absorbed by numerous occupations for me to be able to devote my attention to it immediately.

It might have been better if Wallis had indeed left it alone. Instead, he could not resist suggesting that the number 1 was an obvious solution. What did he mean? That $1^3 = 1^2$? Surely he cannot have thought that Fermat would be satisfied with such a trivial, and in fact incorrect, answer?

A few days later, Brouncker received a second challenge from Fermat, this time requesting solutions to the equation $Nx^2 + 1 = y^2$ for any value of $N$. This time Brouncker did not even bother to pass the letter on to Wallis but sent Fermat his own reply to both challenges, in English [CJW, I, letters 95, 98, 99].

By September 1657 Wallis had still not seen the second challenge and remained dismissive [CJW, I, letter 112]:

> I looked upon problems of this nature, (of which it is easy to contrive a great many in a little time,) to have more in them of labour than either of Use or Difficulty.

Nevertheless, he asked Brouncker to send him the second challenge, and Brouncker did so, suggesting that Wallis should send all their solutions so far to Fermat in Latin. Wallis was happy enough to do that, indeed at some length, and that became the spokesman in an exchange to which intellectually he had until then contributed nothing [CJW, I, letters 113, 114, 115].

The tension rose dramatically with the arrival in October of a further package of letters from Fermat. The first of those letters carried Fermat's outright rejection of Brouncker's early responses to the challenges. A second expressed his opinions of the first few pages of the *Arithmetica infinitorum* [CJW, I, letters 102, 109, 116, 117, 118]. Fermat had already questioned Wallis's quadrature of the circle, to which Wallis had replied with his own, or rather Brouncker's, arguments [CJW, I, letters 102, 103, 104, 106]. The new criticisms were more difficult to answer and suddenly Wallis must have seen his *Arithmetica infinitorum*, of which he was so proud, under serious attack. Thus the number challenges and the criticisms of the *Arithmetica infinitorum* became inextricably entangled in Wallis's emotions and in the subsequent correspondence.

Now at last Brouncker began to give Fermat's challenges his full attention, and came up with a solution to the second challenge that is one of the gems of 17th-century mathematics [CJW, I, letters 119, 120, 122 Appendix; Wallis, 1685, 365]. It was Wallis, however, who continued to play the role of spokesman. In a letter to Fermat written on 21 November he set out Brouncker's solutions followed by a lengthy reply to the criticisms of the *Arithmetica infinitorum* [CJW, I, letter 121]. By now Wallis's irritation with the challenges and his umbrage at the criticisms had combined into an unpleasant mix of resentment and dismissiveness. Fermat's critique should have taught Wallis that he was facing an adversary whose mind was more subtle and perceptive than his own, but still he refused to acknowledge that the number challenges were anything but a trivial waste of time.

One of the problems that crept into the correspondence was in fact the most elementary case of 'Fermat's Last Theorem' [CJW, I, letter 109][4]:

> It is proposed to split a cube number into two cubes.

Wallis's response once again was that he had neither time nor inclination for such things and anyway he was sure Brouncker could succeed in it if he put his mind to it [CJW, I, letter 121].[5] Fermat knew better: he had already proved that the problem was impossible [CJW, I, letter 144]. Wallis later claimed that he had suspected as much all along even though he had not examined it; both he and Brouncker evidently regarded the setting of such 'negative' problems as rather absurd [CJW, I, letters 156, 157, 158]. This single problem somehow epitomizes the entire correspondence: Fermat had hoped to engage Wallis in intelligent debate, but failed; he succeeded only in flushing out Wallis's ignorance.

The thickness of Wallis's skin was extraordinary, though. In early February 1658 Frenicle de Bressy read Wallis's letter of 21 November and wrote a scathing reply. Even Wallis was now goaded at last into coming up with some proper solutions to the first challenge [CJW, I, letters 129, 130, 138]. But neither Fermat's criticisms nor Frenicle's seem to have dented his self-belief. From as early as February 1658 he already had it in mind to publish the entire correspondence, failing to see how badly he himself came out of it.

One cannot but wonder at and admire Wallis's ability to promote himself, with a skill that a modern politician might envy. Not only did he publish the entire correspondence, as the *Commercium epistolicum* in 1658, but also somehow contrived to make himself the lead figure in the story, triumphant over the cavils of the French. In this he was helped in no small way by Kenelm Digby's effusiveness [CJW, I, letter 151]:

> And I doubt not that your last Letters will make [Fermat and Frenicle] and all the world give as large and as full a deference to you. . . . I see enough of the redundant light in them to reverence, not a rising, but a noon day Sun in its very vertical point and highest Zenith.

[4] *Proponatur itaque, datum numerum cubum in duos cubos rationales dividere.*

[5] *Quas quidem si adhuc aggredi velit Honoratissimus Vicecomes Brouncker (qui & modo velit aggredi, non dubito quin feliciter sit assecuturus, saltem quatenus rei natura patitutr,) vel etiam quivis alius, ego id minime aversor; mihi saltem neque vacat, neque animo est.*

Digby, however, understood almost nothing of the substance of the matter. Fermat knew better [CJW, I, letter 156][6]:

> So that from now on we may proceed frankly on both sides, the French will acknowledge that the English satisfied the proposed problems. But let the English acknowledge in turn that the problems were worthy of being proposed to them.

Fermat's hope was never realized. Like Pascal before them, Wallis and Brouncker wanted nothing more to do with Fermat and his number problems.

I first wrote about this episode some years ago with the particular aim of re-examining Brouncker's role in it [Stedall, 2000]. What strikes me most as I return to it now is Wallis's profound sense of self-righteousness. It is true that in 1657 Wallis was at his 'highest Zenith', having recently published his first collected works, and having had what Tom Whiteside once described to me as his single real piece of mathematical luck: the discovery of his infinite fraction for $\pi$. But compared with Fermat, who was some 20 years more experienced and far more able, Wallis was a mere novice. Someone temperamentally or mathematically more sensitive might have realised as much and behaved with greater humility, but humility was never Wallis's style.

At the same time, it has to be said that Fermat stirred in elements of nationalism that could only inflame the situation. Fermat invited Wallis personally to participate but at the same time set his challenge as a contest between England, Belgium, and northern France against his native Narbonne. Since Fermat was working alone and indeed was searching for collaborators it was absurd to strike this regional pose, but it set the tone for what was to come. Nor did it help that he invoked the image of 'the English' and 'the French' as opposing sides on a field of battle [CJW, I, letter 109][7]:

> It is not that I pretend by this to renew the jousts and ancient lance blows that the English have at other times carried out against the French.

This was hardly the way to achieve the mathematical collaboration he so badly craved. If anyone set the scene for Wallis's later dislike of his French counterparts, it was surely Fermat.

## 2. The quarrel with Pascal (1658–1659)

Barely was the ink dry on the correspondence with Fermat when another French mathematician entered Wallis's orbit. In June 1658 Blaise Pascal anonymously put out a challenge to the mathematicians of Europe to solve a set of problems on the cycloid, the solutions to be sent to Carcavi in Paris by 1 October.[8] As with Fermat's number challenges, this was not so much an invitation to mathematicians to engage in new research as a roundabout way for Pascal to demonstrate what he himself had already discovered. His first letter was followed by another a month later containing clarifications and extensions to the first [CJW, I, letters 166, 170]. Pascal must have recognized that there was by now little time for

[6] *Verum ut deinceps ingenue utrimque agamus, fatentur Galli propositis quaestionibus satisfecisse Anglos: Sed fateantur vicissim Angli quaestiones ipsas dignas fuisse quae ipsis proponerentur.*

[7] *Ce n'est pas que je pretende par là renouveller les joustes & les anciens coups de lances, que les Anglois ont autrefois fait contre les Francois.*

[8] Throughout this section I am indebted to Kokiti Hara's translation and analysis of the relevant documents in Hara [1969].

anyone to complete a full set of solutions before the end of September, and so decreed that Carcavi would accept either partial solutions to the full set of problems, or a full solution to one particular problem, a centre of gravity of a solid of revolution.

Wallis received the June challenge on 31 July [10 August] and sent his solutions in 55 numbered paragraphs to Carcavi on 19 [29] August [CJW, I, letter 172].[9] On 3 [13] September he wrote to Carcavi again. The letter is missing but it contained what Wallis later described as explanations and modifications of some minor points [CJW, I, letter 174; Wallis, 1659, sigs a2$^{v}$–a3$^{r}$]. We will look at these in a moment. On 30 September [10 October] Wallis wrote to Paris yet again, and again the letter is missing, but according to Wallis he noted in it that there were further errors in his calculations. He did not specify what they were, because, he said later, he had by now realized that Pascal was the author of the challenge and did not want to offer him any advantage. Besides, his communication was already too late for the 1 October deadline. He was therefore prepared to wait for Carcavi to invite a full solution from him [CJW, I, letter 176].

That invitation never came. Instead, in November 1658, Pascal published his 'Récit de l'examen et du jugement des écrits énvoyés', his judgment on Wallis's submission and others, in which he said that no one had won the prize because no correct solution had been received [Pascal, 1658b]. According to Pascal, the error that Wallis had committed in his first attempt was the calculation of a certain ratio as 23 to 2. In early September Wallis had corrected this to a new ratio of 37 to 4, which was still wrong. Mistakes are useful to historians, however. Kokiti Hara, by reconstructing the calculations that led to these two ratios, has shown that far from being trivial numerical lapses, as Wallis would have us believe, these ratios betray a crucial error in his approach, an incorrect assumption that certain infinitesimal quantities are equal when they are not [Hara, 1969, 47–49]. Hara comments wryly on Wallis's remark that the ratio was the sole point he had to correct before later publishing his solutions: this 'sole point' was in fact the heart of the problem and Wallis spent a further 29 paragraphs trying to improve on it. Wallis's grasp of the mathematics of curves generated by moving points was indeed not strong; he had already made similarly serious errors in the *Arithmetica infinitorum* in his attempt to discover the rectification of the Archimedean spiral [see Wallis, 1656, Propositions 5 to 13; Probst, 1997; Jesseph, 1999, 117–125; Malcolm, 2002, 162–165; Stedall, 2004, 16–22; Beeley and Scriba, 2008]. Hara, like Pascal before him, describes Wallis's errors as 'paralogisms', superficially logical but fundamentally wrong.

Thus Pascal arrived at the only possible conclusion: that Wallis's entry was worthless and should be eliminated. Pascal does not seem, however, to have shared the nationalistic feelings expressed by Fermat a few years earlier, because he spoke well of the rectification of the cycloid sent by another Englishman, Christopher Wren [see Pascal, 1658a]. At the same time, he dismissed an incorrect entry by the French Jesuit Antoine de Lalouvère, who in this way became the one French mathematician for whom Wallis afterwards had some sympathy [CJW, letter 249].

Pascal published his own solutions in *Lettres de A Dettonville* in February 1659 [see Pascal, 1659]. Before Wallis read the *Lettres*, he too had published his own solutions, much extended but still not completely correct, in his *Tractatus duo*. Wallis's preface to that volume says much about his view of the affair, but one has to ask oneself whether even he

[9] During the 17th-century, the Gregorian calendar used in France was 10 days ahead of the Julian calendar still in use in England; hence the need for a double dating system for letters between England and France at this time.

really believed everything he wrote. Was he serious in claiming, for example, that his published version was expressed in ‘almost the same words’ (*eisdem fere verbis*) as he had sent to Pascal, even though he had now changed two key paragraphs twice? Did he really believe that his corrections had been no more than modifications of ‘minor points’ (*minutoria*)? If so, why did Wallis, who in the exchanges with Fermat had been so keen to expose every detail of the correspondence, now conceal the contents of two key letters? Hara concludes, and I find it impossible not to agree with him, that Wallis may not actually have lied but certainly did everything he could to conceal the truth [Hara, 1969, 52]: ‘Thus, to reduce his weakness in the eyes of his readers, Wallis did whatever he could without forcing the facts.’[10]

Constructing his own story without ‘forcing the facts’ was something Wallis became very good at; so good that he not only presented himself favourably to his contemporaries but also managed to project a similar image to posterity. Just as in the exchanges with Fermat Wallis had somehow emerged as the star player, so in the controversy with Pascal he has been seen as the mistreated victim of French scheming. René Taton, for example, thought that Wallis’s heated reaction to Pascal’s judgement was partially justified [Taton, 1970; see also Beeley and Scriba, 2008, 288]. Even Hara, after thoroughly demolishing Wallis’s mathematical arguments, hesitates to suggest that Wallis’s mathematical talent was actually inferior to Pascal’s. What did Wallis himself think? Is it possible that by the end of the 1650s he saw himself as a second-rate mathematician, considerably less able than Fermat, Roberval, or Pascal in France, or than Brouncker and Wren in England? It seems unlikely. Just as in the exchanges with Fermat, Wallis never conceded publicly that Pascal had been right.

## 3. The quarrel with Dulaurens [1667–1668]

In the exchanges with Fermat and Pascal we see two of Wallis’s persistent traits of character: first, a deep-rooted inability to see or admit that he might be wrong; second, a propensity to manipulate the truth to put across his own version of reality. In his quarrel with François Dulaurens in the late 1660s, we see another aspect of his personality: irrational anger out of all proportion to the matter at hand.

After some years of calm, at least as far as relations with the French were concerned, Wallis was provoked once more when in 1667 he read Vincent Leotaud’s *Cyclomathia* (1662), with its detailed refutation of Wallis’s *De angulo contactus* (1656). Wallis wrote a long letter of response to Leotaud in February 1668 [CJW, II, letter 183], but Leotaud never replied, and the matter must have rankled with Wallis, and perhaps influenced his behaviour towards Dulaurens later that year. In August, for example, he mentioned both Leotaud and Dulaurens by name in a complaint to Huygens that ‘your French’ (*Gallis vestris*) flew at him at every possible opportunity; at the same time he expressed puzzlement as to why they did so [CJW, II, letter 242; Loget, 2002, n 62].[11] The tone of his grievance suggests that Wallis no longer saw arguments from French mathematicians as isolated incidents, but as organized opposition from a nation that had turned against him.

[10] *Ainsi donc, pour diminuer sa faiblesse aux yeux de ses lecteurs, Wallis a fait ce qu’il pouvait sans forcer les faits.*

[11] *Me autem quod spectat; miror ego quid Gallis vestris in mentem venerit, quod in me, omni data occasione, (vel non data,) involant.*

The beginnings of the quarrel with Dulaurens were completely trivial and can be traced back to the end of the correspondence with Fermat in 1658. One of the many problems that had made its way into that exchange of letters was one concerning an ellipse, posed to English mathematicians by one Jean de Monfert. A printed version of the problem circulated in London in early 1658, and it was solved by, among others, Christopher Wren and Jonas Moore. Wallis sent his own solution of it to Brouncker in May 1658 in a letter that ended up in the *Commercium epistolicum* [CJW, I, letters 159, 160].[12] In the late 1660s Dulaurens was living in Paris and was acquainted with Frenicle, from whom he learned about the *Commercium epistolicum* and the ellipse problem, though perhaps without seeing either for himself.

Dulaurens brought out his *Specimina mathematica* in 1667. It is for the most part an unremarkable book, an elementary introduction to geometry and basic algebra. At the end, however, Dulaurens added his own solution of de Montfert's problem, which, he said, had been set by Wallis to all the mathematicians of Europe. What most might have regarded as a trivial misattribution, easily corrected, sent Wallis into an inexplicable rage.

Wallis's response was addressed in a lengthy letter to Henry Oldenburg, editor of the *Philosophical Transactions*, and the first part of it was published in April 1668 under the following title [see CJW, II, letters 195, 196; Wallis, 1668a]:

> Concerning some mistakes to be found in a book lately published under the title of Specimina Mathematica Francisci Du Laurens, especially touching a certain problem affirm'd to have been proposed by Dr Wallis, to the Mathematicians of all Europe, to solve it.

Wallis began by giving his opinion of the *Specimina*: that it promised more than it offered; that much of the first part was taken from Wallis's own work or Oughtred's, though without mentioning either, and that thereafter it borrowed from Viète, van Schooten, and others; that he had failed to find in it the true principles of geometry that the book promised; and that the book contained several things that were unsound or inaccurate.

The remainder of the two-page piece in the *Philosophical Transactions* was devoted to a complaint about the 'manifest injury' that Dulaurens had done Wallis in ascribing the ellipse problem to him. At considerable length Wallis proclaimed that he had never proposed a problem to all the mathematicians of Europe; that if he had, he would have posed a more difficult one than this; that he had never set this problem to anyone; on the contrary, a similar problem had been given to him by Richard Rawlinson and he had given a solution. In his letter to Oldenburg, Wallis had also produced a long list of errors in the *Specimina*, but Oldenburg declined for the time being to print those.

Dulaurens, who had been eagerly awaiting the opinions of English mathematicians, wrote with some sadness to Oldenburg in May, to say that his information that Wallis was the author of the problem had come from Frenicle; even if Wallis was not, Dulaurens went on, 'the mistake seems a pardonable one to me, and hardly deserved the trouble he took to get so angry with me' [CHO, IV, letter 859]. Further, he would like to know what Wallis meant by speaking of things in his book that were 'hardly sound' (*parum sana*)? By the beginning of June, Dulaurens had written a much longer response, which Henri Justel described as 'very salty' (*pleine de sel*), and had it printed under the title 'Responsio' [CHO, IV, letter 870; Dulaurens, 1668]. Wallis received a copy on 2 July and wrote back the same day to Oldenburg [see CJW, II, letters 203, 204; Wallis, 1668b].

[12] Solutions by Wren and Moore are to be found in Bodleian Library MS Aubrey 10.

Wallis's letter is telling as to the reasons for his anger: 'thinking to triumph over me, by showing he could solve it' (*quo de me triumphum ageret, monstrando, quam ille potis sit solvere*). That would have been bad enough, but even more hurtful to Wallis, it seems, was the accusation of boastfulness: 'it cannot be denied that he has gone astray, when he depicted me as some bragging Thraso' (*dum me tanquam Thrasonem aliquem inducit ... Non diffitetur, errasse se*). Dulaurens, who had done no more than acknowledge Wallis as the supposed source of the ellipse problem, saw to the heart of the matter: that his 'very grave sin' (*mea gravissima culpa*) of wrongly attributing the problem to Wallis had in turn provoked all Wallis's other complaints [Dulaurens, 1668].

After receipt of the 'Responsio', Oldenburg felt he had little choice but to publish Wallis's 'Animadversions' in full, in letters that eventually dominated three issues of the *Philosophical Transactions* in August, September, and November 1668 [CJW, II, letters 204, 196, 217; Wallis, 1668b,c,d]. Oldenburg was clearly embarrassed [see Wallis, 1668b, 747]:

> ... the Publisher wished very much, that he might not be necessitated to say any more of this subject, ...

Wallis had no such scruples; Dulaurens was neither an astute mathematician nor a careful writer, and Wallis hammered him mercilessly [CJW, II, letters 196, 450; Wallis, 1668b, 748]:[13]

> With like negligence he puts in transversa ejus Diametro instead of in Axe transverso; ...
>
> And so, instead of datis Ellypseos diametris Maximis, he should have sayd, Ellipseos Diametris Extremis (not Maximis) ...

And so it went on for several paragraphs as Wallis pointed out similar infelicities from many other parts of the *Specimina.*

A more serious accusation was that a 'great part' of Dulaurens' book 'seemed to be taken' out of the writings of Oughtred and Wallis. Dulaurens responded that he had never read Wallis; and that one page of his work was indeed copied from Oughtred but word for word so that the borrowing was obvious. Wallis dissected every detail of this statement, arguing that 'great part' meant not the number of words but the importance of the content; in that sense a 'great part' of the content had indeed been expounded by Oughtred and others. Wallis insisted that he had not meant to make a charge of plagiarism, only to point out that the same matters had previously been treated by others. This was a fine distinction. What Wallis for all his logical niceties could not see or bring himself to acknowledge was that Dulaurens was only writing a textbook, and so, as for any such writer, including Oughtred himself, much of what he wrote was bound to be frequently repeated material. Just as in his exchanges with Fermat, Wallis's obsession with detail blinded him to a larger and more intelligent picture.

What blinded him even more, though, was his outrage at what he perceived as a personal insult. In accusing Dulaurens of expressing his 'impotent rage in writing' (*impotentem scribendis animum*) he could have been just as much describing himself.

[13] The quotations in English given here are from the original letter [CJW, II, letter 195]; in the *Philosophical Transactions* the letter was translated into Latin: *Similiter; Ubi substituitur in transversa ejus diametro, pro, in Axe transverso ...; Adeoque pro, Datis Ellypseos Diamteris maximis, dixisset potius, Ellipseos Diametris Extremis (non maximis,) ....*

## 4. The attacks on Descartes (early 1670s)

The last of the French mathematicians against whom Wallis took up arms was René Descartes, who by then had already been dead for over 20 years. Once again Wallis did so in the context of defending an English mathematician who, in his view, had been slighted by the French: in the 1650s it had been Brouncker; in the 1660s it had been Oughtred; now in the early 1670s, it was Thomas Harriot.

One of the purposes of Wallis's *Treatise of algebra*, composed at this time though not published until 1685, was to trumpet the contributions of English mathematicians from the medieval period onwards. To this end, the achievements of Oughtred, Harriot, Newton, and Wallis himself take up a very large part of his book. Wallis's informant on Harriot was John Pell, an able but difficult person. Like Wallis, he was disinclined to be generous about the efforts of others unless they happened to be his friends, and like Wallis, he had become increasingly shortsighted about changes and advances in European mathematics, preferring to argue about minor details rather than to acknowledge major achievements.

Very near the beginning of his twenty-five chapters on Harriot's algebra, Wallis set the tone for what was to come [Wallis, 1685, 126]:

> [Harriot] hath made very many advantageous improvements in this Art; and hath laid the foundations on which *Des Cartes* (though without naming him,) hath built the greatest part (if not the whole,) of his *Algebra* or *Geometry*. Without which, that whole Superstructure of *Des Cartes* (I doubt) had never been.

A statement as strongly worded as this inevitably sets in the reader's mind a subtext for all that follows. Take, for example, this paragraph, which follows a page later [Wallis, 1685, 127]:

> In both these Expedients (putting small Letters for Capitals, and *aa* for *Aq*, &c,) he is followed by *Des Cartes*, whose *Geometry* or *Algebra*, was first published in *French* in the year 1637.

Read in isolation this sentence is harmless, but in the context Wallis has established for it, the words 'followed by' acquire a new level of meaning. So does Wallis's careful inclusion of a date, 1637, six years *after* the publication of Harriot's *Praxis* in 1631. So does his seemingly innocent conflation of *Geometry* with *Algebra*. In short, Wallis's text contains exactly the kind of insinuations that he had used against Dulaurens in accusing him of plagiarizing Oughtred. It also betrays Wallis's limited view of first-past-the-post precedence, in which the complexities of parallel discovery or of mathematicians building on the work of others counted for nothing. Yet some 15 years earlier, following his quarrel with Pascal, Wallis had argued that several mathematicians might well arrive at the same results, and that happening to be first was simply a matter of good luck, indeed that a later discoverer required no less skill and acuity than the first [Wallis, 1659, 79].[14]

I end with another example that demonstrates Wallis's propensity to attack not so much in what he said as in what he allowed to be understood. Consider the equation $x^4 - 4x^3 - 19x^2 + 106x - 120 = 0$, which has roots 2, 3, 4, and $-5$. Three of the roots are positive (or as Descartes called them, 'true'), one is negative (or 'false'). Descartes in

[14] *Nam* Invenisse, *quidem Acuminis est; at,* primum invenisse, *Fortunae: neque enim minore vel subtilitate vel acumine posterio idem non raro invenit, quod alius (se nescio) invenerat primus.*

1637 offered the following rule, which has since become known as the Rule of Signs [Descartes, 1637, 373][15]:

> The Rule of Signs
> one may have as many true roots as the number of times the signs + and – are found to change; and as many false roots as the number of times the two signs + or the two signs – are found to follow one another.

Unfortunately Descartes neglected an important caveat. In the example above the rule gives *exactly* the number of positive and negative roots because all the roots happen to be real. If any of the roots of an equation is imaginary, however, the rule gives only upper bounds. All this was well understood by the mid-17th century.

Until Wallis wrote his *Treatise of algebra*, there was never any question of attributing the original rule to anyone but Descartes, but Wallis introduced confusion. First he claimed that for any equation, Harriot had been able to estimate the number of its *real* roots [Wallis, 1685, 158]:

> 'Harriot's Rule'
> And in this manner, in any Common equation proposed, ... it will appear what number of real roots it hath.

This was already an exaggeration, because Harriot had investigated only equations of degree 3 or 4. Nevertheless, Wallis referred to it as 'Harriot's Rule', though in truth it was not actually a rule at all: rather, as Wallis himself described it, a 'manner' or method of investigation.

Now, immediately following this claim, Wallis turned to the rule of signs, but without any mention of Descartes [Wallis, 1685, 158]:

> Now, (upon a survey of the several forms,) it will be found, that ... as many times as in the order of Signs + –, you pass from + to –, and contrariwise; so many are the Affirmative Roots: But as many times as + follows +, or – follows –; so many are the Negative Roots.

Wallis went on to offer the caution that this rule holds as stated only when all the roots are real. And, he claimed, to discover how many roots are real or imaginary, what one needs is 'Harriot's Rule' [Wallis, 1685, 158]:

> But how many of these be Real, and how many but Imaginary will depend upon ... *Harriot*'s Rule;

Only now did Wallis mention Descartes, but in such a way as to suggest that Descartes merely *agreed* with the rule of signs, rather than that he discovered it. At the same time Wallis could not resist also castigating him for not warning that all the roots must be real [Wallis, 1685, 158]:

> As to the former of these [the Rule of Signs], we have *Des Cartes* concurrence, (but without the caution interposed, which is a defect:) Of the latter ['Harriot's rule'] for the number of real roots, (if I do not mis-remember) he is wholly silent.

One can argue that nowhere in this exposition was Wallis actually telling a lie. But for him to present the Rule of Signs inside an account of Harriot's work and then claim that Des-

[15] *il y en peut auoir autant de vrayes, que les signes + & – s'y trouuent de fois estre changés; & autant de fausses qu'il s'y trouue de fois deux signes + ou deux signes – qui s'entresuiuent.*

cartes simply ‘concurred’ with it was at best misleading, and at worst deliberately duplicitous.

Two decades earlier Wallis had successfully portrayed himself and Brouncker as the true victors in the Fermat saga despite the scheming and dissembling of the French. Now he pursued the same strategy with Harriot, and again he succeeded: the misapprehension that Harriot was the author of the Rule of Signs persisted. In 1686, Leibiniz, anonymously reviewing Wallis’s *Treatise of algebra* for the *Acta eruditorum*, wrote [Leibniz, 1686, 285][16]

> [Harriot] was the first to observe, by induction, as it seems, that there are as many negative roots as there are changes of sign immediately following each other; and as many positive roots as agreements of the same (at least in an equation having its roots purely real or possible, a warning that Descartes in the rest of his writings incorrectly omits).

Leibniz had been somewhat careless here. Not only did he state the Rule of Signs the wrong way round, but at the same time attributed to Harriot something that is not to be found in any of his writings, manuscript or published. It is not difficult to see, however, how such misunderstanding arose from Wallis’s text, especially if the reader was not entirely fluent in English. This attribution of the rule of signs to Harriot became the accepted story and has persisted to the present day. Only 10 years ago I was asked by an eminent historian of mathematics where exactly in Harriot’s writings it was to be found.

Wallis’s obfuscation was all the more unfortunate because there were in fact several pertinent questions to be asked and answered about Descartes’ precise relationship to Harriot; there still are. Wallis, however, rather than contributing intelligently to the discussion, merely set a tinderbox alight and left it to blaze, arousing antagonisms that did little good to the English cause he was so anxious to promote.

## 5. Conclusion

A comprehensive analysis of 17th-century mathematical controversies remains a *desideratum*. This article has presented just one piece of the picture by examining the disagreements between Wallis and his French counterparts. None was a priority dispute in the usual sense, except that Wallis fought belatedly for Harriot against Descartes, both of whom were long since dead. Rather, these were clashes that arose from misunderstandings and hurt pride, and in particular from aspects of Wallis’s own temperament and circumstances.

When Wallis was appointed to the Savilian Chair in 1649 he had only the most elementary of mathematical backgrounds. Nevertheless, conscientious and hard-working, he held his own for five or six years. In the very small world of English mathematics he could shine, and where he could not shine he could hide behind Brouncker. The success of his *Arithmetica infinitorum* in 1656, however, brought him into meaningful contact for the first time not only with French mathematicians but with mathematicians who were very much more experienced and skilled than he was. Suddenly he was exposed to a level of discourse for

[16] *Observavit primus, ex inductione, ut videtur, tot esse radices privativas (in aequatione scilicet meras radices reales seu possibiles habente, quam cautionem Cartesius caeteris descriptis non recte omisit) quot sunt mutationes signorum immediate sibi succedentium; tot positivas, quot eorundem consensus.* The review was not signed, but circumstantial evidence points to Leibniz as the author, see Beeley 2005, notes 46, 47.

which he was wholly unprepared. Hence his first and thoroughly mistaken reaction, that the questions sent to him by Fermat were below his dignity. Only when Fermat criticized the *Arithmetica infinitorum* did Wallis begin to recognize him as a formidable adversary, though he never acknowledged the intrinsic value of the challenge questions.

Against Pascal, Wallis embarked on a different defensive strategy: instead of dismissing the problems as unimportant he chose to gloss over the difficulties, insisting through distortions of the truth that he had done all that was required. As in the arguments with Fermat, one must question whether Wallis fully understood the level of mathematics he was encountering. The *Tractatus duo* of 1659, like the *Commercium epistolicum* of 1658, displays a verbose self-importance that is strangely at odds with the underlying weakness of Wallis's mathematical arguments.

Verbosity was Wallis's main weapon against Dulaurens too, in an argument that arose in the heat of the moment over almost nothing. His attack on Descartes in the 1670s, on the other hand, was much more carefully considered. It shows Wallis returning to the methods he had first employed against Pascal in the 1650s, manufacturing almost imperceptible distortions of the truth to suit the story he wanted to tell.

No quarrel is one-sided and one must ask what aspects of his opponents' behaviour most angered or provoked Wallis. Fermat was certainly the first to introduce a tone of national rivalry, which may well have resonated with Wallis's own sense of national pride. Of course, Wallis fell out with Englishmen too, notably with Thomas Hobbes and William Holder, but these were individual arguments, whereas he came to see 'the French' ranged against him as a nation.

In keeping with his desire to promote English mathematics, Wallis consistently favoured open publication as a method of claiming and proclaiming results. His early experiences with Fermat and Pascal, especially, made the idea of challenge questions, which generally concealed more than they revealed of what the instigator knew, thoroughly irksome to him. This perhaps partly accounts for his otherwise inexplicable antagonism to Dulaurens.

Finally, it may be observed that animosity rarely restricts itself to a simple argument over a single topic. Fermat's initial challenge escalated into a lengthy and wide-ranging discussion of almost every aspect of Wallis's work; while Wallis took Dulaurens to task not just over the supposed challenge questions but over every detail of his *Specimina*, from spelling to possible plagiarism. The same embroiling of disparate arguments can be seen in other 17th-century (or later) disputes, not least the Newton–Leibniz controversy.

Two characteristics of Wallis emerge from this study of his battles with French mathematicians. The first is his propensity to overwhelm his opponents with a deluge of words. In his polemic as in his mathematics, Wallis's first line of attack was as often as not brute force. More insidious was Wallis's second strategy, of twisting the truth to suit his own ends. One rarely, perhaps never, catches Wallis telling an outright lie, but one quite frequently discovers him telling very much less than the whole truth. This does not sit well with the picture of Wallis as a highly respected mathematical professor, logician, and Protestant divine.

Perhaps the most remarkable outcome of the French disputes, however, was Wallis's presentation of himself as vindicated against the ploys of his rivals. To the last, Wallis portrayed himself, and perhaps really saw himself, as an honourable participant who had fallen foul of the dishonourable French for no obvious reason. His letter to Huygens in August 1668 is telling: 'I do not know how I have offended them, except perhaps by sometimes solving problems they have proposed' [CJW, II, letter 242].[17] This last claim appears more than a little disingenous, when in the exchanges with Fermat Wallis had contributed far less than Brouncker, while faced with Pascal's challenges he had failed altogether. Nevertheless, Wallis contrived to create for himself an image of moral superiority and insulted genius, an image so powerful that for the most part it has continued to protect him from attacks upon his ineptitudes up to the present day.

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[17] *Quid ego certum in illos peccaverim, nescio: nisi forte quod Problemata ab ipsis proposita, solverim aliquoties.*

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