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Thomas Simpson: Weaving fluxions in 18th-century London

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Résumé

En 1750, Thomas Simpson (1710–1761) a publié *The Doctrine and Application of Fluxions*, que l'auteur considérait comme un nouveau livre, plutôt que comme une deuxième édition de son *A New Treatise of Fluxions* (1737). En se concentrant sur la méthode directe des fluxions, on se propose ici de réaliser une analyse comparative de ces deux livres de Simpson. Cette comparaison place les travaux de Simpson dans le contexte de la transmission et du développement du calcul newtonien au dix-huitième siècle. Pour comprendre les différences entre les deux livres, on examine les influences possibles sur les travaux mathématiques de Simpson, en particulier celles d'Edmund Stone et de Francis Blake. L'analyse comparative des ouvrages de Simpson donne aussi l'occasion de discuter de questions relatives à l'édition des livres mathématiques au dix-huitième siècle.

MSC: 01A50

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1. Introduction

In 1750 Thomas Simpson (1710–1761) published *The Doctrine and Application of Fluxions*, which the author considered to be a new book rather than a second edition of his *A New Treatise of Fluxions* (1737). Although both works contained the direct and the inverse method of fluxions, in the preface of 1750 Simpson stated that this second work was more comprehensive and that the principal matters were handled in a different manner. In the period 1734–1742 British mathematics was dominated by the controversy on the foundations of Newtonian calculus, or the calculus of fluxions, mainly provoked by George Berkeley (1685–1753) in *The Analyst* (Berkeley, 1734). As an immediate consequence, the publication of textbooks on the subject increased dramatically, especially between 1736 and 1758 (Guicciardini, 1989, 55–67). Another feature of this period is that a popular interest in mathematics grew among the British. This also contributed to the abundant production of elementary mathematical treatises and textbooks (Clarke, 1929).

Guicciardini (1989, 58) pointed out that *The Doctrine and Application of Fluxions* (1750) was the most advanced textbook for beginners of the period, and one of the most reprinted. However, little has been

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studied regarding Simpson's two works on fluxions. So far, Clarke (1929) can be regarded as the most comprehensive and thorough book devoted entirely to Simpson and his mathematical works, in general. Over the past thirty years there have been a few short papers on specific aspects of Simpson's mathematical works, which, however, do not focus on the method of fluxions (Farebrother, 1990; Kollerstrom, 1992; Shoesmith, 1985; Stigler, 1984). The study of Simpson's work on fluxions can be found almost exclusively as part of more general studies on the history of fluxions, such as Guicciardini (1989). Given this situation, Simpson's case deserves an in-depth study due to the fact that he published not one but two books on the same subject within a span of thirteen years. What prompted Simpson to write his second work? How did it differ from the first? Focusing on the direct method of fluxions, the main goal of the present paper is to carry out a comparative analysis of Simpson's two works, which illustrates how the development of the method of fluxions was captured in the introductory works of the period. Recent studies, such as Bertomeu Sánchez et al. (2006) and Simon (2009), have convincingly shown the relevance of the writing and reading of introductory works as practices which contribute actively to the development, circulation and consolidation of scientific knowledge.

To understand the differences between *The Doctrine and Application of Fluxions* (1750) and *A New Treatise of Fluxions* (1737), it is necessary to trace the mathematical influence of other authors on Simpson. Among these, I would like to draw special attention to Edmund Stone's *Method of Fluxions* (1730). In *The Mathematical and Philosophical Dictionary* (1787–1795), in the entry corresponding to Thomas Simpson, Charles Hutton regarded Stone (1730) as the first book on fluxions that Simpson read (Hutton, 1795, v. 2, 392). It is worth analysing how accurate this statement is, all the more so because Stone (1730) was one of the first systematic and methodical works on the direct and inverse method of fluxions in English (Guicciardini, 1989, 17–18).

On the other hand, in the preface of 1750 Simpson himself declared:

> I cannot put an end to this preface without acknowledging my obligations to a small tract, entitled *An Explanation of Fluxions in a Short Essay on the Theory*; printed for W. Innys: wrote by a worthy friend of mine (who was too modest to put his name to that, his first attempt).
>
> [Simpson, 1750, preface, x–xi]

This anonymous and "worthy friend" of Simpson's was Francis Blake (1708–1780). The relationship between Simpson and Blake proved to be crucial in the publishing of Simpson's work of 1750. In fact, an immediate outcome of their correspondence was the anonymous tract of 1741 – mentioned in Simpson's preface – *An Explanation of Fluxions in a Short Essay on the Theory*, which was in reality written by Blake. This paper will show that a number of differences between Simpson's works can only be fully understood in connection with the mathematical exchanges between Simpson and Blake.

The comparative analysis of Simpson's works also provides the opportunity to discuss matters concerning the publication of 18th-century mathematical works. A fair number of studies deal with the publishing of scientific works (e.g. Frasca-Spada and Jardine, 2000; Johns, 1998, 2002, 2003; Rousseau, 1982), but only a few focus on the production of mathematical works, mostly in periods other than the 18th century, such as, for instance, Hay (1988). Scientific educational works are the outcome of the interaction of authors, printers, booksellers and publishers. Insofar as the production of introductory works contributes to the circulation and, consequently, development of mathematical knowledge, it should not be excluded from comparative analysis.

The present paper opens by introducing Thomas Simpson, followed by sections on both Stone and Blake. As it happened, Stone, Simpson and Blake were all Fellows of the Royal Society. These opening sections include a brief study of the roles that Stone, Simpson and Blake played in the Society: who proposed them as Fellows, and why, what election procedures they underwent, and how active they were regarding the meetings held by the Society, among other aspects. The fifth section develops the comparative analysis of the two works at a conceptual level, focussing on the direct method of fluxions. This analysis is supported by the study of the likely influences of Stone's treatise and Blake's tract on Simpson's works of 1737 and 1750, respectively. Finally, the sixth section extends the comparative analysis to the production of both works.

2. Thomas Simpson, the weaver mathematician

Thomas Simpson[1] (1710–1761) was born in Nuneaton, the son of a weaver. Impressed by the knowledge shown by a fortune-teller passing by, he developed an interest for arithmetic, algebra and almanac geometry, which led him to teach himself these disciplines. Around 1735–1736 he moved to London, where he set himself up in Spitalfields as a weaver and teacher of mathematics.[2]

At the time, interest in mathematics in Britain was actually widespread and intense, particularly from a utilitarian point of view (Cassels, 1979; Sorrenson, 1996; Stewart, 1999). In keeping with the economic currents of the time, navigation and trade were seen as a means of national progress and therefore boosted interest in the study of mathematics. An evident proof of the growing demand for mathematics can be found in the amount of popular mathematical periodicals that were published in 18th-century Britain, among which *The Ladies' Diary* stood out (1704–1870).[3] Simpson's first mathematical writings were published in *The Ladies' Diary*, of which he became the editor in 1754.[4] He also contributed to the periodical miscellany *The Gentleman's Magazine*[5] in 1736–1738 and to the mathematical periodicals *The Gentleman's Diary*[6] in 1746 and the *Miscellanea Curiosa Mathematica* in 1745–1746.[7] Quite a large number of societies devoted to mathematics existed in 18th-century Britain. One of the best-known examples is the Spitalfields

[1] Here I will just outline some important biographical facts regarding the period on which this paper is centred. See Clarke (1929) for an exhaustive biographical study of Simpson. See also Aikin (1799–1815), Carey (1825, I, 90–94), Chalmers (1812–1817) and Guicciardini (2004a).

[2] More specifically in "Crown-Court, at the End of Long-Alley next Hog Lane", as it was written on the envelopes of the letters addressed to Simpson at the time.

[3] Albree and Brown (2009) contains an outstanding study of the mathematics of *The Ladies' Diary*. For an analysis of gender and polite science concerning this periodical, see Costa (2002).

[4] In 1742–1744 and 1753–1760 Simpson proposed and answered several questions to *The Ladies' Diary* under various pseudonyms, such as Hurlothrumbo, Kubernetes, Anthony Shallow Esq., Timothy Doodle or Patrick O'Cavanah. In his preface, Leybourn (1817) acknowledged the importance of this periodical in fostering British mathematics, both pure and mixed. Also in the preface he spoke highly of Simpson, as a mathematician, as well as the editor of *The Ladies' Diary*. For a thorough account of Simpson's contributions to *The Ladies' Diary* see Leybourn (1817) and Clarke (1929, 24–28).

[5] See, for instance, Simpson's contributions to *The Gentleman's Magazine* in 1736 (v. VI, 739), 1737 (v. VII, 80, 202, 674, 755) and 1738 (v. VIII, 425). *The Gentleman's Magazine* was founded by Edward Cave (1691–1754) in 1731, who conducted it for twenty-three years. It was said to reach up to 10 000 copies in the 1740s, the next compatible periodical being *The London Magazine*, with about 8000 copies (McKenzie and Ross, 1968, 12). On account of his methods to acquire the content of *The Gentleman's Magazine*, Cave was accused of "downright piracy" by Charles Ackers and others (see letter in McKenzie and Ross, 1968, 5). In fact, Cave was a printer established at St. John's Gate in London, in the area around Moorfields. Located outside the city, this area was considered to have a rather dubious reputation, boiling with hacks and illicit printers, as well as Smithfields and Grub Street, not included in the Company of Stationers – a British association of booksellers and printers, created to survive in the competitive market (see Burke, 2000, 166–167). By contrast, others regarded him as the founder of modern periodical miscellany. The truth is that Cave founded an immensely popular magazine, which survived up until 1922. For a thorough account, see Barker (1981), Feather (1988, 59–61), Gorton (1841), Plomer et al. (1932, 47–48).

[6] *The Gentleman's Diary, or the Mathematical Repository* began in 1741, following the success of *The Ladies' Diary*. It contained many useful and entertaining questions written by and addressed to "philomaths" and, in general, to "gentlemen" interested in the study of mathematics. In the preface (1741) the editor claimed that the periodical aimed to promote and encourage the study and practice of mathematics, a useful and necessary discipline for all the various branches of commerce, military discipline, fortification, offensive and defensive and mechanics. Simpson contributed to this periodical in 1746 (no. 2, 24).

[7] See, for instance, Simpson's contributions in *Miscellanea Curiosa Mathematica* (no. III, 92, 96 and 110 and no. IV, 143). The mathematical periodical *Miscellanea Curiosa Mathematica: or, the Literary Correspondence of Some Eminent Mathematicians*

Mathematical Society (1717–1846), of which Thomas Simpson was among the most remarkable members.[8] In 1743 Simpson was appointed second master of mathematics at the Royal School of Artillery at Woolwich. Two years later he was elected Fellow of the Royal Society of London, proposed by Martin Folkes (President of the Royal Society between 1741 and 1752), William Jones, George Graham and John Machin.

In 1730, a new election system for Fellows of the Royal Society had been approved which, unlike the former statute for election, established that every candidate should be recommended by three Fellows at least, before being put to the ballot and approved by the council:

> Every person to be elected Fellow of the Royal Society, shall be propounded and recommended at a meeting of the S(ociety), by three or more members, who shall then deliver to one of the secretaries a paper signed by themselves, signifying the name, addition, profession, occupation, and chief qualifications of the candidate for election, as also notifying the usual place of his habitation; a fair copy of which paper, with the date of the day when delivered, shall be fixed up in the meeting room of the S at 10 several meetings, before the said candidate shall be put to the ballot.
>
> [Weld, 1848, I, 460–461]

Besides this, the candidate should be acquainted with a number of the proposers:

> It was moved and recommended as a good precept, that the certificates for candidates, if natives, ought to signify that the candidate is personally known to the several subscribers, and the president was desired to recommend it to the Society, that members would refrain from soliciting hands to a certificate when the candidate is not personally known to them.
>
> [22 December 1742, Council Minutes, v. 3, 333–334]

Thomas Simpson met this requirement since he was acquainted with Martin Folkes (1690–1754) and William Jones (1675–1749).[9] Yet this change in the system of election evidently meant an obstacle in becoming a Fellow, as Simpson stated in a letter[10] to his friend Francis Blake:

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in Great Britain and Ireland contained a selection of mathematical essays on algebra, trigonometry, the doctrine of chances, astronomy, chronology, geometry, gunnery, infinite series, fluxions, fluent, exponentials, the quadrature of curves, and others, as well as problems, with their solutions. It was edited by Francis Holliday – a friend and pupil of Simpson's –, printed for Edward Cave and sold by John Fuller, who in turn was the editor of *The Gentleman's Diary* from 1741 to 1745. According to the preface, the periodical was issued after the editor of *The Gentleman's Magazine* had been requested by several mathematicians to include more papers on mathematics, then publishing it in a separate pamphlet.

[8] Besides The Spitalfields Mathematical Society, Guicciardini (1989, 65) mentions, among others, the Spalding Gentleman's Society (founded 1717), the Manchester Mathematical Society (founded 1718) and the Northampton Mathematical Society (founded 1721). On The Spitalfields Mathematical Society, see Cassels (1979) and Weld (1848, I, 467, fn. 32). In addition, at the Special Collections of University College London Library, there is a file containing several papers and letters concerning the Spitalfields Mathematical Society (see MS ADD 75).

[9] On the relationship of Simpson with Folkes and Jones, see Clarke (1929, 106–114 and 127–128), respectively.

[10] Clarke (1929) reported that, upon his death, Simpson's papers fell into the hands of one of his pupils, Major Henry Watson (1737–1786), of the Engineers, in the service of the East India Company. They were afterwards acquired for the Phillips collection and sold to Professor R.C. Archibald of Brown University (Providence, R.I.), who gave them to Professor David Eugene Smith (Columbia University), constituting the Smith Historical Collection (henceforth Smith H. Col.), with more than 170 letters. Simpson had correspondents from all over the country, even from Philadelphia (United States of America [USA]). The letters can be put into three groups: (i) letters of acceptance to the Academy of Woolwich or asking for reference to enter it; (ii) letters dealing with mathematical questions, in general, and problems included in *A New Treatise of Fluxions* of 1737, in particular; and (iii) letters on editorial matters and bookselling orders. Of the letters written by Simpson, there are about 28 undated and to unknown recipients. More than likely, these are drafts of the letters that Simpson copied before sending them out, since most of them appear to be crammed with crossing-outs.

> As to the privilege of being admitted a Fellow of ye Royal Society, it is not so easily obtained now; for, then such numbers were entered without any sort of merit, that ye Society (as I am informed) have found it necessary to come to a resolution not to receive any person as a member who has not first distinguished himself by something curious.
>
> [Simpson to Blake, 24 March 1743, Smith H. Coll.]

Therefore his election as Fellow of the Royal Society reveals that he was a man of some "sort of merit".[11] The certificate in favour of his election tells us why Simpson was proposed and elected Fellow:

> Mr Thomas Simpson of the Royal Academy at Wolwich.
>
> Desiring to Offer himself a Candidate for election into this Society: we whose Names are underwritten, do, as well upon our own personal knowledge of him and his abilities, as *upon the reputation of the Several valuable Mathematical Books he is the Author of*; hereby propose and recommend him as well deserving the honour he requests, and as qualified by his Skill in many parts of Philosophical Learning, to be a good and usefull member of the Royal Society.
>
> [Royal Society website, my emphasis]

The fact that he was elected for the value of his mathematical works gives some hints about the social structure of the Royal Society. Simpson's job as a weaver and as a mathematics tutor did not hinder his election as Fellow. Unlike in the period 1660–1699, the Society underwent a modest social expansion from the 1720s onwards.[12] As Sorrenson has noted, "being a schoolteacher who earned a living from mathematics or an instrument maker who worked with his hands was not necessarily an insurmountable obstacle" (Sorrenson, 1996, 35). Yet despite the prestige involved, his election to the Royal Society represented rather a burden to Simpson. In a letter to Blake, Simpson complained of how costly it would be to maintain the fellowship:

> I am not yet a Fellow of the Society, nor am I very fond of that honour, upon account of the expenses, tho' Mr Jones persuades me to it and tells me I shall find my advantage in it.
>
> [Simpson to Blake, 4 July 1745, Smith H. Coll.]

Due to economic difficulties at the time, the council was not willing to allow arrears of the payment of fees.[13] In the draft of the abovementioned new statute there was a proposition "to sue Fellows in arrear of their payments" (Weld, 1848, I, 458–459). Fellows were asked to "liquidate their debts, while others compounded for their past and future subscriptions by the payment of one sum" (Weld, 1848, I, 462). Owing to his reputation, Simpson was exempted from paying the fees upon having requested to be excused[14]:

> The council took into consideration the case of Mr. Thomas Simpson the mathematician, who had applied to the Society to be excused paying his arrears incurred since his first admittance. Upon which the President represented to the council, that Mr. Simpson was undoubtedly a man of great science and had lately been very ill and was in such very low circumstances as render'd him incapable of paying what was due to the

[11] The subject of Simpson's reputation, both personal and as a mathematician, has varied among historians of mathematics. See, for instance, Shoesmith (1985, 352), regarding Simpson's work on probability.

[12] For a detailed description of the social composition of the Royal Society in the 18th century, see Sorrenson (1996).

[13] It was not the first time, nor would it be the last, that the Royal Society encountered financial difficulties since, unlike the Académie des Sciences de Paris, it had to do without government support, to the extent that, in the 1760s, the number of foreign fellows, who paid no fees, had to be cut (Weld, 1848, II, 43–44). In Crosland (2005) there is a comparison between Royal Society of London and the Académie des Sciences de Paris.

[14] Another eminent mathematician, Colin Maclaurin (1698–1746), was also exempted from paying the fees upon his request in 1731, in consideration of his contributions. See the Council Minutes (v. 3, 126).

> Society, and therefore the President proposed that Mr. Simpson should be excused all future payments to the Society and that his bond should be delivered up to him. Which was accordingly put to the ballot and carried in the affirmative.
>
> [4 July 1750, Council Minutes, v. 4, 34]

Although he attended the Society meetings only rarely,[15] the truth is that Simpson was very active in terms of scientific production and this could have made up for his not paying the fees. Between 1737 and 1760 Simpson published a great many mathematical works. His works can be divided into two groups. In the first group we have treatises or books on a particular subject. He wrote not only on algebra, geometry and trigonometry,[16] but also on relatively new subjects, such as the method of fluxions and probability.[17] Meant for beginners, these works provided expositions of the elements of the subject involved. Most of these works were related to "pure mathematics", except one dealing with annuities and life insurance (Simpson, 1742). Aiming mainly at the growing number of insurance companies, this work showed how to compute tables of mortality and expectation of life, which in this case were determined from 10-year observations on the bills of mortality of the city of London.[18]

Simpson regarded the subject of annuities as a very useful one. Usefulness was actually the main trend of the second group of his writings. They were miscellanies, consisting of a collection of papers and essays on a variety of topics, ranging from astronomy, gunnery and mechanics, to analytical topics such as the summation of series, approximating the areas of curves by means of equidistant ordinates or approximating the solution of all kinds of algebraical equations through the method of fluxions.[19] In fact, between 1748 and 1758, Simpson also contributed papers and letters on these topics to the *Philosophical Transactions of the Royal Society* (henceforth *Philosophical Transactions*).[20] In this second group of works I would also include Simpson (1752), the greatest part of which was originally composed for his own teaching at the Royal Academy of Woolwich. Assuming that his papers could be of service to others, especially to those involved in teaching, he decided to collect and publish them in this volume. For instance, in the title pages

15 In a letter to Blake, Simpson acknowledged: "I have been but once to the Society since my admission, but intend to be more constant when weather is better and I can have the pleasure of your company there" (Simpson to Blake, 13 February 1746, Smith H. Coll.).

16 See Simpson (1745, 1747, 1748a and 1760). These works seemed to be very popular at the time, going through four editions or more. Even Simpson (1745) and (1747) were printed in the USA in the 1810s.

17 See Simpson (1737, 1740a and 1750). Simpson (1740a) was essentially based upon De Moivre's *The Doctrine of Chances: or, a Method of Calculating the Probability of Events in Play* (1718), the first textbook on probability theory. As far as the method of fluxions is concerned, before 1737 there were only few introductory books on the subject, the most remarkable being those of Hayes (1704), Ditton (1706), Stone (1730), Hodgson (1736) and Muller (1736), as I will discuss in the following section.

18 Here again he referred to De Moivre as the author of an excellent book on the subject: *Annuities upon Lives; or, the Valuation of Annuities upon any Number of Lives; as also, of Reversions*, to which is added an appendix concerning the expectations of life and probabilities of survivorship (1725).

19 Simpson (1740b, 1743, 1757a). On account of an essay on a general quadrature of hyperbolic curves, contained in his *Essays on Several Curious and Useful Subjects in Speculative and Mixed Mathematics* (1740b), Samuel Klingerstierna proposed Simpson to the Swedish Academy. Klingenstierna (1698–1765) was a renowned Swedish mathematician, with a special interest in optics. One of the first practitioners of calculus in Sweden, he was professor of geometry and experimental physics at Uppsala University. On his travels through Europe he made the acquaintance of the scientific élite of Europe, including Johann Bernoulli, Alexis C. Clairaut and Christian Wolff. Recommended by Martin Folkes, Abraham De Moivre and John Machin, Klingenstierna became a fellow of the Royal Society in 1730. In the following year, Klingenstierna submitted a paper to the *Philosophical Transactions of the Royal Society*, dealing with the quadrature of the circle, which bore some similarities with Simpson's abovementioned work of 1740. One of Klingenstierna's students, Pehr Wargentin (1717–1783), was the permanent secretary to the Royal Swedish Academy of Sciences, to whom Simpson addressed a thank-you letter, dated 24 June 1758 (Smith H. Coll.). On Klingerstierna and the Swedish Academy see http://www.linnaeus.uu.se/online/math/4_0.html. See also Clarke (1929, 43, 189–190).

20 See Simpson (1748b, 1748c, 1751–52, 1755a, 1755b, 1757b, 1758a, 1758b).

of the works in this second group it is not unusual to come across the phrases "Illustrated by a variety of examples", "The whole in a general and perspicuous manner" or "The whole explain'd in a plain and simple manner".

Simpson's interest in useful (mixed) mathematics mirrors the Royal Society's interest in this topic in the 18th century, as Sorrenson (1996) has noted. Even Simpson's recommenders illustrate this trend. While Folkes promoted scientific achievements, observation and experiments, Graham and Jones were instrument makers and Machin was a mathematician. The state of the mathematics carried out within the bounds of the Royal Society of London in the period 1725–1780 contrasts with the period of Joseph Banks's presidency (1778–1820), in which the mathematical sciences were neglected, as Heilbron (1993) has described. As we will see in the next two sections, Edmund Stone and Francis Blake were also inclined towards mixed mathematics.

3. Edmund Stone, the invisible 'Fellow'

A New Treatise of Fluxions (1737) presented Simpson's first approach to the fluxional method. As mentioned earlier, Charles Hutton (1795) reckoned that Stone's translation of L'Hospital's *Analyse des infiniment petits* (1696) was the first book on fluxions read by Simpson:

> ... an acquaintance lent him Mr. Stone's *Fluxions*, which is a translation of the Marquis de L'Hospital's *Analyse des Infiniment Petits*: by the one book, and his own penetrating talents, he was (...) enabled in a very few years to compose a much more accurate treatise on this subject than any that had before appeared in our language.
>
> [Hutton, 1795, v. 2, 392]

Edmund Stone (1695?–1768)[21] was the son of a gardener at Inverary in the service of John Campbell (1680–1743), second Duke of Argyll and Greenwich, and Master-General of the Ordnance from 1725 to 1740. In service of the Duke from an early age, Stone taught himself mathematics and managed to master geometry and analysis, as well as French and Latin, without an instructor or any other guide. At the age of 18 his abilities were discovered by chance by the Duke, who provided Stone with employment that allowed him to commit himself to his studies. He moved to London, where it is not known for sure what he did, although it is not unlikely that he worked as a tutor of mathematics.[22]

In 1725 Stone was elected Fellow of the Royal Society, proposed by John T. Desaguliers (1683–1744).[23] In the Council Minutes of the Society, we are informed that Stone and other candidates "were severally put to the Ballot and elected Fellows" (Council Minutes, v. II, 365). Before the change in the statute of 1730, the recommendation of just one Fellow would suffice to become a Fellow. As Taylor (1966, 27) points out, the Duke's patronage no doubt played a role in Stone's fellowship of the Royal Society.

There is yet another aspect that could explain Stone's fellowship. Stone's scientific contributions were confined mostly to translating and editing. His knowledge of French and Latin enabled him to translate

[21] Hutton stated that Stone's date of birth was imprecise, though it was probably towards the end of the 17th century (Hutton, 1795, v. II, 530). What little is known about Edmund Stone's life is based on a letter from Chevalier Andrew M. Ramsay (1686–1743) to the Jesuit Father Louis B. Castel (1688–1757), later published in the *Journal de Trévoux* (1732, t. 32, 103–113). For general biographical facts see Aikin (1799–1815), Carey (1825, II, 127–128), Chalmers (1812–1817), Guicciardini (1989, 17–18; 2004b).

[22] In the preface to one of his books Stone acknowledged that it was "his friends" who encouraged him to write it (Stone, 1723a). As Taylor (1966, 26–27) has noted, "friends" was the accepted euphemism for "pupils", and it was not rare that an author acknowledged having produced a book under the pressure of such "friends". Therefore it would not be out of place to assume that Stone was a tutor of mathematics while in London.

[23] See the Journal Books of the Royal Society (11 March 1724, v. XIII, 456).

several works into English. These works cannot nevertheless be regarded as mere translations. For, on the one hand, not only did Stone revise and correct the works he translated, but he also enlarged them with additions. On the other hand, there was a clear educational motivation behind his projects (see for instance the prefaces of Stone, 1728, and Stone, 1752).[24] Stone, strongly convinced of the usefulness of Geometry, made it the subject of a first group of translations and revisions (1723b, 1724b, 1728, 1735 and 1752). A second group was concerned with practical mathematics, namely, "sphericks" (1721), the construction and use of mathematical instruments (1723a, 1729), perspective (1724a), the working of ships (1743) and astronomy (1726b, 1766).[25]

Among Stone's few original works *A New Mathematical Dictionary* (Stone, 1726a) stands out. In 1730 Stone issued *The Method of Fluxions, both Direct and Inverse*, which in a way can be said to be a hybrid, half a translation, half an original work. In *The Translator to the Reader* Stone revealed why he had undertaken the work:

> Such a work becomes the more necessary, because there are but two English treatises on the subject, (that I know of) the one being Hays's Introduction to Mathematical Philosophy, and the other, Ditton's Institution of Fluxions.
>
> [Stone, 1730, The Translator to the Reader, xv]

These treatises were, according to Stone, either too lengthy or too short, respectively, and not as simple and clear as L'Hospital's work.[26] The part on the direct method "is but a translation of a well known work"[27]; namely, a translation of L'Hospital's *Analyse des infiniment petits* of 1696, the first printed work on differential calculus, based on the lectures that Johann Bernoulli gave L'Hospital in the 1690s. Stone's translation rendered L'Hospital's work available to the English-speaking world. He converted the Leibnizian language of calculus into the Newtonian language of fluxions, hence attempting to provide a translation between the two systems. In fact, Stone mistook the fluxion of x for the differential, not unusually, since this was a recurrent confusion at the time. To a great extent the second part, on the inverse method of fluxions, is based on Roger Cotes's posthumous work *Harmonia Mensurarum* (1722).[28] In general Stone's work received a positive review in France at the time, especially for being well organised and systematic, though a bit concise.[29]

Yet Johann Bernoulli singled out Stone's work for criticism in his *Remarques sur le livre intitulé Analyse des infinimens petits, comprenant le Calcul Integral, sans toute son étendue, &c. par M. Stone, de la Société Royale de Londres* (Bernoulli, 1742, v. 4, 169–192). His criticisms were not essentially aimed at

[24] How necessary were such translations? Why did Stone choose to translate precisely these works? After the perusal of Stone's works I am convinced that they deserve a deeper examination, along with a survey of their publication and dissemination.

[25] There is an anonymous tract, *The Description, Nature and General Use of the Sector and Plain-Scale* (printed for T. Wright; and sold by J. Coggs Mathematical Instrument Maker, London), first published in 1721, which reached up to four editions (1728, 1736, 1746). The preface is signed by E. S. and subsequently Edmund Stone is credited with it. See, for instance, the entry for Stone in Wallis and Wallis (1986). If such was the case, then this would support Stone's involvement in practical mathematics.

[26] For a general overview of the fluxional works by Charles Hayes (1678–1760) and Humphry Ditton (1675–1715), see Guicciardini (1989, 16–17).

[27] "Qui n'est q'une simple traduction d'un ouvrage assés connu", *Journal de Trévoux* (1732, t. 32, 104). All translations from French are by the author of this paper.

[28] Stone seemed to take a particular interest in Cotes's work on integration. There is a letter from Stone to an unknown recipient, dated 26 July 1727, where Stone asks about the rule of finding the fluent of the fluxion of the cissoid of Diocles according to Mr Cotes's way. See MS Graves 23 (3) Miscellaneous Letters. As we will see, Cotes's *Harmonia mensurarum* would also influence Simpson's work of 1750. On the role played by Cotes on integration as a research field, see Gowing (1983, 34-ff) and Guicciardini (1989, 28–31).

[29] "Quoique rangé, & méthodique, est un peu succinct", *Journal de Trévoux* (1732, t. 32, 106).

the quality of Stone's work, but rather at the French translation of the part on the inverse method, published by M. Rondet in 1735. More precisely, the target of Bernoulli's attack seems to have been the preliminary discourse preceding this translation that appeared in the *Journal de Trévoux* (1736, t. 36, 1118), allegedly authored by the Jesuit Louis B. Castel.[30] It is true that Bernoulli did not agree with Stone's excessive use of series and that he indicated a number of mistakes in Stone's solutions. But, above all, throughout his *remarques* Bernoulli lamented that, in his discourse, Castel praised the results of Newton and Cotes, ignoring Bernoulli's priority in obtaining such results[31] – paradoxically enough, since Castel was considered to be a strong opponent of Newton's philosophy and views on science.

On the death of the Duke of Argyll in 1743, Stone's name was withdrawn from the list of Fellows of the Royal Society.[32] Unlike Simpson, Stone cannot be said to have been a very active contributor to the meetings, nor to the *Philosophical Transactions* either where he published just one paper concerning two cubic curves (Stone, 1739).[33]

The fact that his name was misspelled in the Journal Books of the Royal Society, regarding an account he presented to be read before the Royal Society in 1730, certainly reinforces the idea that, as a Fellow, Stone was rather invisible.[34] No doubt upon his patron's death he might have simply resigned, not being able to afford the fees for fellowship. Apparently he lived poor and neglected until his own death, probably not in personal contact with other Fellows and instrument makers any more (Taylor, 1966, 27). In the latter part of his life he subsisted by giving lessons in mathematics and writing. His works at this period of time were alleged to be of low standard:

> Being obliged to employ himself in writing for a subsistence, he rather injured than increased his reputations by some of his productions.
>
> [Gorton, 1841]

This contrasts clearly with Simpson's situation described in the previous section. The fact that Stone was not exempted from paying the fees, nor even asked to be exempted, seems to indicate that his presence in the Royal Society went rather unnoticed. However, it is important to remember that Stone, as well as Simpson, showed a deep interest in mixed mathematics and its usefulness. Again, this trend should be placed in the context of the Royal Society and its involvement in mixed mathematics in the 18th century.

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30 A French mathematician and teacher of physics and mathematics at the Jesuit school in the rue Saint-Jaques in Paris, Castel belonged to the editorial board of the *Journal de Trévoux* from 1720 to 1745. On Castel, see Desautels (1971).

31 See, for instance, Bernoulli (1742, v. 4, 175). Bernoulli went as far as to accuse Stone of plagiarism on account of Stone's solution of the brachistochrone problem, very similar to the solution given by Jakob Bernoulli some 40 years earlier (Bernoulli, 1742, v. 4, 190).

32 To my knowledge there are no meeting minutes of the Society where Stone's resignation is discussed.

33 Here Stone referred to Newton's *Enumeratio linearum tertii ordinis* (originally printed as an appendix to Newton's *Opticks*) and the *Illustratio Tractatus Domini Newtoni Linearum tertii ordinis*, written by Stirling (1717). In fact the two cubic curves mentioned in the title had already been detected in the 1730s, one by François Nicole (1683–1758) in 1731 and one by Nicolaus Bernoulli (1687–1759) in 1733. See Guicciardini (2009, 111).

34 In the corresponding report it can be read that "Mr Edward (sic) Stone gave the Society an account of a remarkable spot which he saw with his naked eye on the body of the sun" (Journal Books of the Royal Society, XIV, 536). Curiously there was an "Edward Stone", contemporary of "our" Stone and somehow connected with the introduction of aspirin. In Wallis and Wallis (1986) Stone is credited with having written the book *The Whole Doctrine of Parallaxes Explained and Illustrated* (1763). However, from its title page one soon realises that the author is not Edmund, but "the reverend Mr. Edward Stone, A. M. Late fellow of Wadham College, Oxford". Both Stones were involved in a confusion which is the subject of the paper by William S. Pierpoint (1997), 'Edward Stone (1702–1768) and Edmund Stone (1700–1768): Confused Identities Resolved' in Notes and Records of the Royal Society, 51, 211–217.

4. Francis Blake, the "worthy friend"

In 1750 Simpson published his second work on fluxions, *The Doctrine and Application of Fluxions*, in the making of which Francis Blake (1707/8–1780), "Simpson's good friend and patron",[35] was involved significantly. About Blake's life little is known, and what is known is mainly based on his correspondence with Simpson.[36] Educated at Lincoln College, Oxford (1725), Blake was made a baronet in 1774. Although his usual place of residence was Herrington (near Durham), Francis Blake spent some time in London in the period 1724–1756. From their correspondence it is clear that Blake attended Simpson's classes in Lincoln's Inn Square in central London. But the fact that Simpson dedicated his *Essays on Several Curious and Useful Subjects in Speculative and Mixed Mathematics* (1740) to Blake proves that the latter was more than just a pupil to the former.

Between 1738 and 1740 Simpson and Blake exchanged a number of letters on the subject of the foundations of the fluxional method. An immediate outcome of their correspondence was an anonymous tract, *An Explanation of Fluxions in a Short Essay on the Theory*, published in 1741 and written by Blake himself.[37] From the address "to the reader", it is clear that Blake's *Essay* was an attempt to render Newton's method of fluxions more understandable.

> I was induced, from the many disputes concerning Sir Isaac Newton's Method of Fluxions, to try if that most useful and noble kind of investigation might not be establish'd upon more obvious principles. This gave birth to the following essay; which, therefore, you are desired to consider as an explanation of the doctrine itself, and not of Sir Isaac's manner of delivering it. About that I don't mean, nor pretend to take a part in any controversy. It was, doubtless, agreeable to our great author's unbounded invention and discernment: but, I presume, a more familiar demonstration and phrase will neither be unacceptable to you, nor at all derogatory to the merit of his performance, whilst they tend to confirm and elucidate the very same truths.
>
> [Blake, 1741, To the reader]

Such an attempt has to be placed in the context of the discussions of the period about the foundations of the Newtonian method, brought about by Berkeley's criticisms of the calculus (Guicciardini, 1989, 43-ff).

Blake's essay was a very short work, containing a general definition of fluxion, an illustration,[38] a corollary, a remark on notation, a lemma, a scholium, a proposition, three corollaries, the rules to find the fluxion of the product of several quantities, of a fraction and of any power of a variable quantity, and a number of examples of these rules. What is more relevant to the topic of my paper is the process through which the essay was elaborated. As it turned out, its entire contents had been previously discussed in the correspondence exchanged between Simpson and Blake. For example, in his letter of 15 February 1740 Blake sent the introduction, the general definition and the illustration, concluding the letter by asking Simpson's opinion about his illustration:

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35 "Life and Writings of Thomas Simpson", in *The Doctrine and Application of Fluxions*, edited by William Davis (1805, v–xix).

36 Most of the letters in Smith H. Coll. belong to the correspondence between Blake and Simpson (52 from Simpson to Blake, 24 from Blake to Simpson). In addition, there is a thorough account of Blake's connection with Simpson in Clarke (1929). For further biographical details see Goodwin (2004) and Watt (1824).

37 The work was advertised in *The Gentleman's Magazine* in May 1741 (v. XI, 280), with no mention at all about its author. In the preface of his work of 1750 Simpson referred to the anonymity of Blake's work as a symbol of the modesty of his friend. This could also be explained otherwise. In a letter to Simpson, Blake confessed that his work would remain anonymous until he saw the success of it (Blake to Simpson, 27 January 1740, Smith H. Coll.). Either out of modesty or of uncertainty regarding the reception of his tract, or of both, Blake would have felt compelled not to state his authorship openly.

38 "Illustration" here meant an explanation of the previous general definition.

> If this illustration be right, I perfectly understand what I am about, and therefore as soon as you favour me with a line, shall put the finishing hand to the essay.
>
> [Blake to Simpson, 15 February 1740, Smith H. Coll.]

As we have seen, Simpson mentioned Blake's short treatise in his preface of 1750 and acknowledged his indebtedness to it (Simpson, 1750, preface, x–xi). Though little known today, Blake's tract was highly regarded at his time because of its clear notion of fluxions, as Simpson pointed out in a letter to Blake (Simpson to Blake, 5 May 1741, Smith H. Coll.). Besides a reprint of 1763,[39] Blake's tract was reissued as late as 1809, as part of the fourth edition of John Rowe's *An Introduction to the Doctrine of Fluxions* (1751). It was William Davis[40] who revised and corrected this fourth edition and who added Blake's *Essay*, as the advertisement reads:

> ... he [Davis] has to this fourth edition added that short but valuable little essay on the Explanation of the Theory of Fluxions, printed for Wm. Innys, and taken notice of by the late celebrated mathematician Thomas Simpson, in the preface to his excellent Treatise of Fluxions.
>
> [Rowe, 1751, 4th ed. 1809]

In addition, Simpson thought Blake's *Essay* might be of sufficient merit to be elected a Fellow of the Royal Society, as he communicated to Blake in a letter:

> (...) the Society (as I am informed) have found it necessary to come to a resolution not to receive any person as a member who has not first distinguished himself by something curious. But this is no barr to you since ye book you have wrote (or anything you may publish in ye Transactions hereafter) will be sufficient recommendation for you; would your modesty but permit you to own it.
>
> [Simpson to Blake, 24 March 1743, Smith H. Coll.]

Blake was indeed elected Fellow of the Royal Society in 1746, being a member of the Council during 1751. As in Simpson's case, Folkes, Jones and Machin were among the Fellows who supported Blake's election owing to his mathematical skills. Yet his essay was not mentioned in the certificate of admittance for Fellow:

> Desiring to offer himself a Candidate for Election into this Royal Society, We do Accordingly propose him, and upon our own knowledge hereby recommend him, *as a Gentn of merit and learning, well versed in Mathematical and Philosophical knowledge* and well qualified to be a valuable and usefull member of this Society.
>
> [Royal Society website, my emphasis]

As we will see in the following section, changes that are found in Simpson's work of 1750 regarding the general definition of fluxion and of higher-order fluxions, and the rule of the product, appeared in Blake's essay and in the correspondence between Blake and Simpson.

[39] Indicated in Wallis and Wallis (1986, entry 738 BLA, 246) and Guicciardini (1989, 186) In a letter dated 2 March 1741 (Smith H. Coll.), Blake informed Simpson that he was thinking about the possibility of reprinting his tract with some alterations, including, among others, the fluxion of logarithms and exponentials, along with some examples of quadratures, rectifications and physical problems. Yet, as Clarke (1929, 93) pointed out, such revised edition of his text did not come out eventually.

[40] William Davis (1771–1807) was a mathematician and an editor of mathematical books. A member of the Mathematical and Philosophical Society in London, he was also the editor of the periodical *The Gentleman's Mathematical Companion*, the first part of the republication of *The Gentleman's Diary or the Mathematical repository*. *The Gentleman's Mathematical Companion* was allegedly associated with the Spitalfields Mathematical Society from 1793 to 1827. See Cassels (1979, 241). For further details about Davis, see Albree and Brown (2009) and Watt (1824).

Before concluding this section it is worth mentioning that Blake's interests covered experimental philosophy and mechanics. In particular, he contributed several papers on compressibility of water and design of steam engines to the *Philosophical Transactions of the Royal Society* (Blake, 1751–52a, 1751–52b and 1759). Together with Simpson's and Stone's scientific production, Blake's penchant appears to be in keeping with the views of the Royal Society on mixed mathematics and utility (Sorrenson, 1996).

5. From Simpson (1750) back to Simpson (1737): a comparative analysis

As a consequence of Berkeley's attack on Newtonian calculus, the number of treatises on fluxions soared in the period 1736–1758. It was right at the beginning of this period, in 1737, when Simpson published *A New Treatise of Fluxions: wherein the Direct and Inverse Method are Demonstrated after a New, Clear, and Concise Manner, with their Application to Physics and Astronomy: also the Doctrine of Infinite Series and Reverting Series Universally, are Amply Explained, Fluxionary and Exponential Equations Solved: Together with a Variety of New and Curious Problems* (in quarto). Not only was this his first work on fluxions, but it was his first mathematical work at all. His intention in publishing such a work was to show the true principles of fluxions along with several useful applications, given the absence of English works on the subject:

> When I first proposed my intentions of publishing a work of this nature, *there was not, that I know of, any book in the English tongue, founded upon the true principles of fluxions*, that contained any thing very material, especially in the practical part; and tho' there had been some very curious things done by several learned and ingenious gentlemen, who had treated upon that subject, *the principles were nevertheless left obscure and defective*, and all that had been done by any of them in infinite series very inconsiderable.
> [Simpson, 1737, preface, i, my emphasis]

This long sentence strikes me as intriguing, given that Stone referred to the treatises of Hayes (1704) and Ditton (1706) in his book. In addition, Simpson failed to refer to other introductory works published before his own treatise, namely, *The Doctrine of Fluxions* (1736) by James Hodgson (1672–1755) and *A Mathematical Treatise* (1736) by John Muller (1699–1784).[41] Simpson stated that there was no English treatise on fluxions at that moment, apart from "some very curious things done by several learned and ingenious gentlemen" (Simpson, 1737, preface) that were nevertheless not clear and not based upon the real principles. Still, it would be odd if he had not acquainted himself with other works on fluxions. For instance, in an entry in the British Biographical Archive I came across the following reference:

> ... he [Simpson] was entirely at a loss to discover any English author who had written on the subject, except Mr Hayes; and that gentleman's work being a folio, and then pretty scarce, he was unable to purchase it.
> [Aikin, 1799–1815 and Chalmers, 1812–1817]

Therefore Simpson could have written his treatise mainly for want of a work that introduced the "true" principles of fluxions, not acknowledging the works published on the topic up to that moment. This seems to be a recurrent discourse at the time. We have seen earlier that Stone regarded the elaboration of plain and easy introductions to the fluxional method as necessary. The need of introductory works on fluxions encouraged a number of authors in this period to prepare works on the subject. For instance, James Hodgson's main intention in publishing his work on fluxions was "to introduce the true method of fluxions, most of the books that have hitherto appeared upon that subject having in them little more than the name" (Hodgson, 1736, preface). Some years later, John Rowe's *Introduction to the Doctrine of Fluxions* intended to remove "those

[41] On the works written by Hodgson and Muller, see Guicciardini (1989, 56–57 and 62).

clouds of darkness and obscurity, in which the treatises on this subject are too often involved" (Rowe, 1751, preface). Simpson explicitly agreed on the necessity of clear and plain works on fluxions, as he would write to Blake in 1742. Following the publication of Maclaurin's *A Treatise of Fluxions* (1742), Blake was concerned about the similarity of some results of this work to his own tract.[42] In his reply, Simpson gave his view on what an introduction to fluxions should be, comforting Blake in the following terms:

> I cannot think you have the least cause to be under any concern at what y[e] world may suggest with regard to your essay, because it was not only published long enough before Mr. MacLaurin's book appeared, but because *what you have done therein is vastly more clear and satisfactory* than any thing to be met with in that author; and this is not my bare opinion, but y[e] opinion of all that I have heard speak about y[e] two performances. And if I might freely declare my mind I should rather think he made that alteration in his plan in consequence of having seen your essay than that you had borrowed any thing from him. But be this as it will, I am confident that *a much plainer book may be wrote on y[e] subject than his, and that an easy explication of y[e] theory illustrated by a sufficient number of well chosen examples, will be of much greater use to a beginner* than both those large volumes...
>
> [Simpson to Blake, 7 August 1742, Smith H. Coll., my emphasis]

It is worth highlighting that Simpson regarded Maclaurin's treatise as not entirely useful for "beginners", all the more so as, according to Blake, Maclaurin addressed it to "great mathematicians":

> I remember to have seen several sheets of it [Maclaurin's manuscript], long ago, and if he has not altered his method, I dare say he addresses himself only to great mathematicians.
>
> [Blake to Simpson, 8 February 1740, Smith H. Coll.]

Such views stress the interaction between research and teaching in the context of the emergence of the fluxional calculus as a new branch of mathematics. The fact that the foundations of it were not yet clear enough would naturally represent an obstacle in the teaching of the method of fluxions.[43] Consequently, the search of efficient ways to teach the new method could have led to the production of works addressed to beginners.

But there is still another factor that could have given rise to this kind of discourse. Owing to the large amount of works on the subject, the authors could have felt the need to advertise their own as a means to survive in the competitive market of fluxional works. They understandably sought to produce books that could explain the true principles of fluxions better than their rivals, in an attempt to reach a wider audience, to the extent that Simpson could have made the most of his popularity as mathematics tutor in the area of Spitalfields by connecting his book with his lectures, as can be inferred from the title page:

> All parts of the mathematics are taught abroad by the author; who may be heard of at Mr. Cave's, at St. John's Gate; the Angel in Bunhill-Fields, or at his Lodgings in Crown-Court, near upper Moor-Fields.
>
> [Simpson, 1737]

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[42] As an answer to Berkeley's *The Analyst* (1734) Maclaurin published his two-volume *A Treatise of Fluxions* (1742), which was an attempt to provide Newton's calculus with systematic rigorous foundations. In his first book Maclaurin relied largely on Archimedean geometry and kinematical approach to fulfil his aim. Only in the second book did Maclaurin introduce the notation and algorithms of fluxions and applied them to solve a number of physico-mathematical problems. On Maclaurin, see Bruneau (2011), Grabiner (1997, 2002 and 2004) and Guicciardini (1989, 47–51).

[43] This fact could also have resulted in the lack of agreement in classifying works of fluxions. Hence, Simpson's work of 1750 was advertised in *The Gentleman's Magazine* under the category "History, Philosophy and Surgery" (v. XX, 384). Similarly, Blake's *Essay* was advertised under "History and Philology" in May 1741 (v. XI, 280), whereas Maclaurin's *Treatise of Fluxions* appeared under "Mathematicks and Physick" in May 1742 (v. XII, 280).

In 1750 Simpson published *The Doctrine and Application of Fluxions: Containing* (*Besides What is Common on the Subject*), *a Number of New Improvements in the Theory, and the Solution of a Variety of New and Very Interesting Problems in Different Branches of the Mathematicks* (in 2 volumes, in octavo), a new book rather than a second edition of his *A New Treatise of Fluxions* (1737).[44] In the preface Simpson stated what prompted him to improve his former work on fluxions and to write this new work:

> Having, in the year 1737, published a piece, of this same subject, under the title *A Treatise of Fluxions* (...) it may be proper here, first of all, to assign the reasons why this book is sent abroad into the world as a new book, rather than a second edition of the said treatise. Which, in short, are these two: first, because *the present work is vastly more full and comprehensive*; and, secondly, because *the principal matters* in it, which are also to be met in that treatise, *are handled in a different manner*.
>
> [Simpson, 1750, preface, v, my emphasis]

Actually, concerning the order and treatment of the several parts of the work, it can be regarded as a two-level work, because "the lower, as well as the more experienced, class of readers" (Simpson, 1750, preface, vi) were taken into consideration. Regarding the first principles, that is, the first part of the work, further down in the preface Simpson claimed that:

> ... the ease and benefit of the young beginner have been particularly consulted: to load such an one with a multitude of rules and precepts, before giving him any taste of their use and application, would, certainly, be very discouraging (...) I have therefore, after demonstrating the first principles, proceeded immediately to exemplify their utility in several entertaining enquiries, before touching at all upon the inverse method, or the more difficult parts of the direct.
>
> [Simpson, 1750, preface, ix–x]

The most difficult parts of the inverse and the direct method were transferred to the second part of the work, together with such matters that could be "too tedious or hard to a learner at first setting out" (Simpson, 1750, preface, x).

Following Simpson's arguments as to why his work of 1750 was different from that of 1737, this section of the present paper is divided into two parts. In order to understand why Simpson considered his later work to be "vastly more full and comprehensive" (Simpson, 1750, preface, v), in the first part there is a comparison of the contents and structure of both works. The second part carries out a conceptual comparative analysis to grasp which matters were "handled in a different manner" (Simpson, 1750, preface, v), and why. Here it is worth discussing the mathematical influences that could have had an impact on the elaboration of Simpson's works on fluxions, especially those of Stone and Blake.

5.1. Simpson (1737) versus Simpson (1750): on contents and structure

To begin with, *The Doctrine and Application of Fluxions* (1750) was much lengthier than *A New Treatise of Fluxions* (1737). Simpson's work of 1737 was divided into four parts and a supplement (230 articles altogether) and that of 1750 consisted of two parts (a total of 485 articles and a table of hyperbolical

[44] Some years later he would proceed similarly with regard to his work on geometry. He first published his *Elements of Plane Geometry* in 1747, and then the *Elements of Geometry*, "the Second Edition, with large Alterations and Additions", came out in 1760. It would be illuminating to analyse whether this was characteristic of Simpson's mathematical work, or whether this trend was shared by other British mathematical authors.

Table 1
Content comparison of Simpson (1737) and Simpson (1750), Part I.

1737		1750 (Part I)
Part I	I	I
	II	II
	III	III
	IV	IV
	V	V
Part II	I	VI
	II	Some results connected with section II (Part II).
Part III	I	VI
	II	VII+VIII
	III	IX+X
Part IV	I	XII
Supplement		Some problems connected with sections I, IX and XI (Part II).

logarithms).[45] The supplement of 1737 was a collection of miscellaneous problems, which had to be left out and printed as an independent supplement.[46] In the preface it is said to contain:

> an investigation of the areas of spherical triangles; the curve of pursuit; the paths of shadows; the motion of projectiles in a medium; and the manner of finding the attractive force of bodies in different forms, acting according to a given law; together with a general method for solving exponential equations; and (at the desire of several gentlemen in the excise) some easy and very useful problems in stereometry, wherein the practical rules for reducing all the varieties of casks into equal cylinders are deduced.
> [Simpson, 1737, preface, iii]

From Table 1 it is clear that the topics included in the first part of Simpson (1750) are mainly the same as those covered by his work of 1737.

In Ditton (1706, 165), there is a list of the typical uses and applications of the direct method (tangents, extreme values, points of inflexion, evolutes and caustics) and of the inverse method (rectification, quadrature, cubature, centres of gravity and oscillation). Therefore not surprisingly Simpson included most of these topics in his work of 1737. However, at a time when the method of fluxions was not yet solidly articulated, this structure could not be taken for granted. Indeed, despite his list of common topics, Ditton presented no systematic study of them in his book, which consisted essentially of a discussion on the nature of fluxions, along with a collection of problems concerning isochronal curves and curves of quickest descent. In contrast, Hayes (1704) certainly covered all these topics, from an elementary point of view, though as in Ditton the inverse method was not treated systematically. As Guicciardini (1989, 17) pointed out, Stone (1730) can be considered as the first systematic treatise in English, with a methodical treatment of the direct and inverse methods.[47] It is true that Simpson (1737) did not include any chapter on caustics by reflexion and by refraction, unlike Stone (1730, sections VI and VII, part I), Muller (1736, section IV,

45 A detailed list of the contents of both works is gathered in the Annex (Tables A.1 and A.2).

46 See the preface in Simpson (1737). This is why, in *The Gentleman's Magazine*, the supplement was advertised in July 1737, a few months later than the book itself (May 1737). Then, in August 1737, both the book and the supplement were advertised together, the supplement being able to be purchased separately. See *The Gentleman's Magazine* (v. VII, 320, 454 and 518).

47 On the contents of Stone (1730), see the Annex (Table A.3).

Table 2
Content comparison of Simpson (1737) and Stone (1730).

Simpson (1737)		Stone
Part I	I	I (Part I)
	II	III (Part I)
	III	II (Part I)
	IV	IV (Part I)
	V	V (Part I)
Part II	I	I (Appendix)
	II	
Part III	I	II (Appendix)
	II	III + IV (Appendix)
	III	V (Appendix)
Part IV	I	Some Problems in VIII (Appendix)
Supplement		Some Problems in VIII (Appendix)

book II) and Hodgson (1736, section V, part II). Yet it incorporated sections centred on the study of tangents and of points of inflexion, topics that were not addressed by Muller (1736).[48]

A distinguishing feature of the introductory works of this period was the extent of the applications they incorporated. While Hodgson (1736) determined centres of gravity, percussion and oscillation of geometrical figures, Stone (1730), Muller (1736) and Simpson (1737) supplemented the typical uses and applications with sections that gathered more practical and advanced problems. In his last section, Muller showed the use of fluxions to solve a collection of physico-mathematical problems, mainly concerning mechanics. Only a small number of these problems could also be found in the works of Stone (e.g. Stone, 1730, §127; Muller, 1736, §317) and of Simpson (e.g. Muller, 1736, §308; Simpson, 1737, §207). Beyond mechanics, Simpson's supplement extended the applications of the fluxional method to geodesy, stereometry and practical astronomy. Simpson's supplement shared a few of the "miscellaneous problems" proposed by Stone in section VIII (e.g. Simpson, 1737, §193 and Stone, 1730, §121 on geodesy; Simpson, 1737, §186 and Stone, 1730, §123 on stereometry). These common problems, however few, seem to point to an interest in broadening the scope of the uses and applications of fluxions to fields other than mechanics.

So after all, it is reasonable to conclude that, in general, Simpson (1737) and Stone (1730) bear clear similarities regarding their scope and structure, as Table 2 indicates.

In addition, it is not unusual to find similar problems and examples in both works, although their wording might differ, however slightly. In particular, Simpson tended to contextualise the problems and examples in his work. For instance, while Stone proposed the following example, in an abstract and decontextualised way:

> The circle *AEB* being given in position, as also the points *C* and *F* without the same: to find the point *E* in the periphery being such, that the sum of the right lines *CE*, *EF* may be a minimum.
> [Stone, 1730, example X, §58]

[48] Actually, Muller merely defined the subtangent in terms of the fluxions of the ordinate and the abscissa (Muller, 1736, §164) but he supplied no example of how to determine the tangent to a given curve. It is worth mentioning here that in 1741 John Muller was appointed headmaster of the new Royal Military Academy of Woolwich, where he also held the position of professor of fortification and artillery, fields on which he wrote several standard books. On the relationship of Simpson and Muller, in particular during the period when they were both teaching at Woolwich, see Clarke (1929, 191–199). Clarke hinted that, while Simpson's treatise of 1737 was in progress, Muller's *Mathematical Treatise* (1736) was brought to Simpson's attention. Consequently, one could wonder about the hypothetical extent of Muller's influence on Simpson's work. However, as we will see, Simpson (1737) did not share many conceptual similarities with Muller (1736).

Simpson formulated a similar example in his book as follows:

> Having given the situation, and distances of three towns A, B, C, from each other; to find a place where a tradesman may fix his habitations, so as to keep the three markets of those towns with riding the fewest miles?
>
> [Simpson, 1737, example XIV, §39]

Points and right lines in Stone (1730) have become towns and their distances for the sake of a certain tradesman in Simpson (1737). Again, while Stone began the problem XV (Stone, 1730, section VIII, Appendix, §124) with the words "If AC be a horizontal line upon the point C", Simpson preferred to use "a piece of wood of an indefinite length" instead (Simpson, 1737, Problem XXIV, Supplement, §207). Simpson's interest in putting problems into "real" context can be seen as an effort to render the method of fluxions plainer and easier to understand, and to illustrate its usefulness.

In short, the work of Stone could be regarded as a source of inspiration for Simpson's first work on fluxions as far as content and structure are concerned, especially the part on the direct method. This, in turn, illustrates how the shaping of contents and structure of introductory works evolved, not yet fully agreed in the emergence of the fluxional method as a new branch of mathematics.

So far the comparative analysis has been centred on the contents of Simpson (1737) and the first part of Simpson (1750). The second part of Simpson (1750) is essentially new compared with Simpson (1737). As mentioned above, Simpson (1750) could be regarded as a two-level work. We have already seen that the second part contained the most advanced uses and applications of fluxions, specially involving the inverse method. Given its difficulty, this second part was clearly aimed at proficient readers, rather than absolute beginners. A detailed discussion of the inverse method is beyond the scope of this paper. But seeing as the inverse method is given so much importance in Simpson (1750), it is nevertheless worth providing here a brief overview of the main sections on this topic.

The second part of Simpson (1750) opens with a section on the study of fluxions involving exponential quantities and spherical triangles, the latter being a very useful topic for practical astronomy. The second section focuses on the resolution of fluxionary equations, in particular separation of variables, and first-order linear differential equations. Sections III through VI show how to find fluents by means of different techniques: transformation of fluxions, namely, integration by substitution (sections III and IV); determination of the fluents of rational fractions according to Cotes's forms (section V); determination of the fluents when quantities, their logarithms, arcs and sines are involved together, including problems of what today we would call integration by parts (section VI). Regarding section V, it is worth noting that Simpson introduced the trigonometric abbreviations *Sin* and *Cos*, which were not usual in Britain at that time (Cajori, 1928–1929, II, 165–166). In section VII, Simpson addressed the convergence of infinite series, or, more precisely, the manner of making fluents converge, by means of infinite series.[49] Four sections concerned with several uses of fluxions complete the book: the motion of bodies in resisting mediums (section VIII),

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[49] That is, Simpson used the sum of series to compute fluents, much as Maclaurin did in his *Treatise* of 1742. In particular, they discussed the formula for the remainder term, in order to make a series converge much faster, but they computed the remainder in different ways. For instance, they both compute the value of the series:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots$$

which is equal to $1/8$ of the periphery of a circle with radius 1 (Simpson, 1750, part II, §352; Maclaurin, 1742, book II, §845). Simpson computed the remainder as follows, using the first eight terms:

$$\frac{1}{2\cdot 13}\left(1+\frac{1}{15}+\frac{2}{15\cdot 17}+\cdots+\frac{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}{15\cdot 17\cdot 19\cdot 21\cdot 23\cdot 25\cdot 27}\right)$$

the attraction of bodies of different forms (section IX), isoperimetrical problems (section X) and problems of various kinds (section XI).

A closer look reveals two notable aspects of this second part. First, it contains the solution of a number of problems that Simpson had published in earlier works. For instance, section XI is largely based on the Supplement of Simpson (1737) (e.g. problems regarding exponential quantities, spherical triangles and isochronal curves). Likewise, two essays dealing with the attraction of spheroids (or ellipsoids of revolution), which first appeared in Simpson (1743), were reworked in section IX (Simpson, 1743, 1–37; Simpson, 1750, book II, §§374–405).[50] Guicciardini pointed out that some of the best examples of applications of the method of fluxions could be found in Simpson (1743) (Guicciardini, 1989, 68). Indeed, the so-called Simpson rule for numerical integration was presented in Simpson (1743, 109–111). So it seems that Simpson gathered part of his former work on fluxions in the second part of his work of 1750, in a more organised and systematic way.

The second aspect is that Simpson (1750) incorporated, as the title declared, a number of new results in the theory. This applied especially to the second part, all the more so because some of its contents were closely connected with Simpson's research topics, such as series (Simpson, 1748c, 1751–52 and 1758b) and isoperimetrical problems (Simpson, 1755a and 1757a); here lies probably one of the most powerful reasons that drove Simpson to write a second book on the subject of fluxions. In 1750 Simpson broadened the scope of his former work by incorporating the most advanced and latest uses and applications of fluxions. Not only this, but he also intended to present them in a systematic way, accessible to beginners.

5.2. *Simpson (1737) versus Simpson (1750): a conceptual comparison*

The most remarkable differences between Simpson (1737) and Simpson (1750) can be detected in the first section (first part) of both works. For the comparative analysis, I will focus on the topics of this section that were presented somehow differently in 1737 and in 1750 – namely, the definitions of fluxion and higher-order fluxions, and the rule of the product. Given that all these topics were included in the works of Stone and Blake, it will be illuminating to trace the mathematical influences that Simpson might have received from them. Therefore, the comparative analysis is peppered with references to either Blake (1741) or Stone (1730), although not exclusively to them. In addition, despite not being included in Blake's essay, in my analysis I also compare the study of maxima and minima, for Simpson's treatments in 1737 and 1750 differ significantly.

5.2.1. *On the nature of fluxions*

In both *The Doctrine and Application* (1750) and *A New Treatise* (1737) Simpson considered magnitudes to be generated kinematically. Yet his definition of 1750 was a bit more sophisticated, for Simpson pointed out, though not explicitly, that a curve was generated by two motions:

> Conceive a right-line mg to be carried along uniformly, parallel to itself, from A towards Q, and let, at the same time, a point p so move in that line, as to describe, or trace out, the given curve ACG...
>
> [Simpson, 1750, part I, §48]

which, he claimed, produced a more exact result than the sum of 100 000 terms of the original series. Meanwhile, if we apply Maclaurin's formula, we will get the same result (to the sixth decimal position) by summing the first five terms this time:

$$\frac{1}{2\cdot 13}+\frac{1}{2\cdot 13^2}-\frac{1}{13^4}+\frac{8}{13^6}-\frac{34}{13^8}.$$

50 Simpson solved the problem on the attraction of spheroids by means of infinitesimal techniques, in a non-kinematic way. For a thorough account of his solution, see Guicciardini (1989, 73–79).

Stone held an ambivalent view on this matter, to say the least. On the one hand, he claimed that L'Hospital's work was much easier than those of Hayes and Ditton, "not deterring his readers at first from proceeding, by dwelling long on the explication of an intricate definition, but comes immediately to the algorithm, or arithmetic of the Art" (Stone, 1730, the Translator to the Reader, xvj). Hence in his work, as in L'Hospital's, a curve is considered to be a polygon:

> Postulate II. Grant that a curve line may be consider'd, as the assemblage of an infinite number of infinitely small right lines: or (which is the same thing) as a polygon of an infinite number of sides, each of an infinitely small length, which determine the curvature of the line by the angles they make with each other.
>
> [Stone, 1730, part I, §3]

On the other hand, in *The Translator to the Reader* Stone defined a curve in a different way, this time in terms of the kinematic generation of magnitudes:

> We are to consider quantities not as made up of very small parts, but as described by a continued motion... A line is described, not by the apposition of little lines or parts, but by the continual motion of a point...
>
> [Stone, 1730, The Translator to the Reader, xvj]

The kinematic definition of curves was fundamental for Newtonians, as opposed to the infinite-angled polygon advocated by the differential calculus. Yet, as Guicciardini has pointed out, four different approaches to fluxional calculus may be distinguished in Newton's work: (1) differentials or infinitesimal components of finite magnitudes; (2) moments, or infinitesimal components generated by motion; (3) rates of change of variable quantities; (4) fluxions or velocities of flowing quantities (Guicciardini, 1989, 6). It is, therefore, not hard to imagine that the introductory works of the period reflected the contemporary confusion around the nature of fluxions. Even adherents of the same approach could express it in different ways. For instance, Stone adopted again a double standard. In the text, the definition of fluxion matches up with that of dx, according to the Leibnizian approach: "The infinitely small part whereby a variable quantity is continually increased or decreased, is called the fluxion of that quantity" (Stone, 1730, part I, Definition II).[51] Meanwhile, in *The Translator to the Reader* Stone defines fluxions kinematically as:

> *the velocities of the increases of increments of magnitudes* thus moving in very small equal particles of time, *at the first instant of the generation of those increments are called fluxions*, and those magnitudes fluent, or flowing quantities.
>
> These fluxions are nearly proportional to the increments of the fluent or flowing quantities, generated in very small equal parts of time; but accurately as the velocities wherewith they arise and begin to be generated; that is, they are those very velocities; as was said before.
>
> [Stone, 1730, The Translator to the Reader, xvij, my emphasis]

They are "nearly proportional", and not exactly proportional, because "velocities are mutable, or accelerated continually" (Stone, 1730, The Translator to the Reader, xvij). Hodgson presented a similar approach, which relied on Newton's introduction to the *Tractatus de Quadratura Curvarum* (1676, first published in 1704), as he acknowledged in the preface (Hodgson, 1736, preface, xj). Accordingly, fluxions were not:

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[51] This misinterpretation, rather usual at the time, was obvious in his *A New Mathematical Dictionary* (1726a). In particular, in the entry for differential calculus in the second edition (1743), Stone mistook the infinitely small differences between variable quantities for fluxions. On the contrary, Hodgson warned against such confusion in his preface (Hodgson, 1736, preface, v–viij).

> … the velocities of the increments considered as actually generated, but as arising or beginning to be generated, and are accurately and exactly, in the first ratio of the increments, consider'd as arising, or in the first moment of their generation.
>
> [Hodgson, 1736, 49]

Later, Hodgson added that fluxions were "in the first ratio of their *nascent augments*, or in the last ratio of their *evanescent decrements*" (Hodgson, 1736, 50).

By contrast, in Simpson (1737) a fluxion was not exactly a velocity:

> Definition II: The fluxions of variable quantities are always measured by their relation to each other; and are ever expressed by the finite spaces that would be uniformly described in equal times, with the velocities by which those quantities are generated.
>
> [Simpson, 1737, part I, §2]

Finally, Muller simply defined a fluxion as the velocity with which the point *a* arrives at *M* (Muller, 1736, §165). If Simpson happened to have read Muller's manuscript, as Clarke (1929) hinted, apparently it did not have any impact on his definition of fluxion of 1737.

As is evident from the quote above, the relation between fluxions played a relevant role in Simpson (1737), unlike in 1750. By way of illustration, while in Simpson (1737, part I, §6) the stress was placed on finding the ratio of fluxions of two quantities, in Simpson (1750, part I, §6) the same proposition turned into finding the fluxion of a given quantity.

On this particular point Simpson (1737) could have been influenced by the *Tractatus de Quadratura Curvarum*, either directly through Newton's work or indirectly through Hodgson (1736), as the following result seems to prove:

> For example, suppose the curve-lined space *Amv* and the parallelogram *As* to be generated like the former, by the continued motion of the line *rs*, moving uniformly, then will the fluxions of these two spaces be to each other, as the parallelograms *ma*, *Sa*, or as the generating lines *mv*, *vs*; and because *vs* is invariable, the fluxion of the curve-lined space *Amv*, will be always as the ordinate *vm*, as evidently appears from the definition…
>
> [Simpson, 1737, §4]

This result could be found in the introduction to Newton's *Tractatus de Quadratura Curvarum*, which, in turn, was later included in Hodgson (1736, 50). Yet, in 1750, Simpson abandoned this approach and refined his definition of fluxion as follows:

> Every quantity so generated is called a variable, or flowing quantity: and the magnitude by which any flowing quantity WOULD BE uniformly increased in a given portion of time, with the generating celerity at any proposed position, or instant (was it from thence to continue invariable) is the fluxion of the said quantity at that position, or instant.
>
> [Simpson, 1750, part I, §2, original emphasis]

Given the influence of Maclaurin's interpretation of Newtonian calculus in Great Britain, one could wonder how relevant it was to Simpson (1750). Maclaurin's definition of fluxion also involved velocities and distances:

> The velocity with which a quantity flows, at any term of the time while it is supposed to be generated, is called its Fluxion which is therefore always measured by the increment or decrement that would be generated in a given time by this motion, if it was continued uniformly from that term without any acceleration

> or retardation: or it may be measured by the quantity that is generated in a given time by an uniform motion which is equal to the generating motion at that term.
>
> [Maclaurin, 1742, I, §11]

There is actually one difference between the two definitions. Maclaurin defined the fluxion as the velocity of the flowing quantity, and this flowing quantity could be measured by the product of the velocity and the increment of time. By contrast, already in the preface Simpson stressed the fact that fluxions could not be considered as "meer velocities":

> The consideration of time, which I have introduced into the general definition, will, perhaps, be disliked by those who would have fluxions to be meer velocities: but the advantage of considering them otherwise (*not as the velocities themselves, but the magnitudes they would, uniformly, generate in a given finite time*) appear to me sufficient to obviate any objection on that head. (...) Besides, tho' Isaac Newton defines fluxions to be the velocities of motions, yet he hath recourse to the increments, or moments, generated in equal particles of time, in order to determine those velocities.
>
> [Simpson, 1750, preface, vi, my emphasis]

Simpson's fluxion could be *interpreted* as the velocity multiplied by the increment of time. This way Simpson dodged the circularity involved in Maclaurin's definition (Guicciardini, 1989, 49). If we now compare Simpson's definition with that in Blake's *Essay*, the similarities are striking:

> General DEFINITION. The word FLUXION properly apply'd always supposes the generation of some quantity (term'd fluent of flowing quantity) with an equable, accelerated, or retarded velocity, and is ITSELF the quantity which MIGHT BE UNIFORMLY GENERATED, in A CONSTANT PORTION OF TIME, with the amount or remainder of that velocity, at the instant of finding SUCH FLUXION.
>
> [Blake, 1741, 2, original emphasis]

The relationship between Simpson and Blake turned out to be crucial in the approach adopted by Simpson in 1750. The evolution from Simpson's definition of fluxions of 1737 to his definition of 1750 can be easily traced back from their correspondence.[52] In fact, the definitions, propositions, illustrations and rules included in Blake's *Essay* had been discussed in detail previously in the letters Blake and Simpson exchanged and subsequently appeared in Simpson's work of 1750. Their correspondence illustrates a case of transition from private knowledge to public knowledge and should be understood in the perspective of the shaping of the teaching of fluxional methods.

5.2.2. Higher-order fluxions

The new definition of fluxion in Simpson (1750) brought about changes in the definition of higher-order fluxions and the rule of the product. Blake played an even more relevant role in this discussion, as Simpson acknowledged in his preface of 1750: "whose manner [Blake's] of determining the fluxion of a rectangle and illustrating the higher orders of fluxions I have, in particular, follow'ed, with little or no variation".

In 1737 Simpson defined higher-order fluxions as follows:

> Definition. As the first fluxion of a variable quantity shews the distance or space that would be uniformly described in any equal part of time, with the velocity by which it is generated; so the second fluxion of that quantity, shews the space that would be described in the same time, with the velocity that would be uniformly generated in that time; and is ever proportional to the celerity of the increase or decrease of the

[52] See, for instance, the letters from Blake to Simpson dated 8 May 1739, 27 January 1740 and 15 February 1740 (Smith H. Coll.) and Clarke (1929, 56–58).

> first fluxion, which is its flowing quantity. In the same manner the third fluxion shews the variation of that increase, and the fourth fluxion, the variation of that variation, &c. *ad infinitum*.
>
> [Simpson, 1737, part I, §18]

Then illustrated this general definition with a particular example:

> Now to make this appear as plain as possible, let us again suppose the point m to move uniformly from A along AB, with a velocity that is capable to carry it over any invariable distance $gr(=\dot{x})$ in any equal part of time; and let the point n be also moved along the line CD, so that any distance CS described thereby, shall be always equal to the cube of the distance AR, described in the same time by the other point m; then by calling AR, x, and CS, x^3, we shall have $3x^2\dot{x}$ equal to the distance that would be passed over uniformly by the point n, with a velocity that it has at the point S, in the above-named time; (as is evident from Prop. II) but as this distance is continually increasing, let the same be represented by the distance of the moveable point v from the immoveable point H; then will the fluxion of the line $HV(=6\dot{x}^2x)$ which is always as the velocity of the said moveable point in any cotempory position (E) with the points m, n, at R and S, truly express the second fluxion of the line CS, or quantity x^3; or, which is to the same effect, the velocity that would be uniformly generated by the point S, in the above-named time, will be capable to carry it over a space $= 6x\dot{x}\dot{x}$ in the same time: but this distance is also continually increasing, and therefore its fluxion $= 6\dot{x}\dot{x}\dot{x}$, which shews the celerity of that increase, is the third fluxion of the quantity x^3, or of the line CS.
>
> [Simpson, 1737, part I, §18]

In the example above the fluxion $\dot{x}$ is considered to be invariable. In this regard Simpson added the following note:

> NB. It is *absolutely necessary* to make the first fluxion of some one of the simple quantities invariable in every expression whatsoever, not only for the greater ease in the operation, but that it may (as ever being the same) serve as a standard to measure the various mutations of the other quantities.
>
> [Simpson, 1737, part I, §20, my emphasis]

which is reminiscent of the Corollary II of Stone (1730, part I, §64), where the fluxion of one of the variables was assumed to be necessarily invariable.

In 1750, Simpson opened the general definition of higher-order fluxions stating right away that:

> The second fluxion of a quantity is the fluxion of the variable or algebraic quantity expressing the first fluxion already defined. By the third fluxion is meant the fluxion of the variable quantity expressing the second...
>
> [Simpson, 1750, part I, §18]

This time the illustration is not as tedious as the one given in 1737:

> Thus, for example, let the line AB represent a variable quantity, generated by the motion of the point B, and let the (first) fluxion thereof (or the space that *might be* uniformly described in a given time, with the celerity of B) be always expressed by the distance of the point D from a given, or fixed point C: then if the celerity of B be not everywhere the same; the distance CD, expressing the measure of that celerity, must also vary, by the motion of D, from, or towards C, according as the celerity of B is an increasing or a decreasing one: and the fluxion of the line CD, so varying (or the space (EF) that *might be* uniformly described in the aforesaid given time, with the celerity of D) is the second fluxion of AB. Again, if the motion of B be such that neither it, nor that of D, (which depends upon it) be equable, then EF, expressing the celerity of D, will also have its fluxion GH; which is the third fluxion of AB, and the second fluxion of CD.

> And thus are the fluxions of every other order to be considered, *being the measures of the velocities by which their respective flowing quantities, the fluxions of the preceding order, are generated*.
>
> [Simpson, 1750, part I, §18]

While in 1737 Simpson illustrated the explanation with moving diagrams for x (a quantity flowing uniformly), for x^3, for the fluxion of x^3, and the combination of these, in 1750 there were diagrams for the flowing quantity x and its subsequent fluxions, which rendered the explanation a bit simpler and clearer.

After the definition, Simpson proceeded to show how to compute the second and third fluxions of x^3, applying the rule to find the fluxion of any given power of a variable quantity and considering $\dot{x}$ to be constant. Then, in contrast to 1737, Simpson computed the fluxion of x^3 when x is generated with a variable celerity; that is to say, when the fluxion of x is not constant:

> In the preceding example the root x is supposed to be generated with an equable celerity: but, if the celerity be an increasing or a decreasing one, then $\dot{x}$, expressing the measure thereof, being variable, will also have its fluxion; which is usually denoted by $\ddot{x}$: whose fluxion, according to the same method of notation, is again designed by $\dddot{x}$; and so on, with respect to the higher orders.
>
> [Simpson, 1750, part I, §20]

While in 1737 it was "absolutely necessary" to consider the first fluxion of any of the quantities invariable, in 1750 it would only be "convenient" (Simpson, 1750, part I, §21). By supposing one of the quantities to flow uniformly, the fluxionary equation would be reduced to a more concise expression and "it may serve as a standard to which the variable fluxions of the other quantities, depending thereon, may be always referred" (Simpson, 1750, part I, §21). The selection of a quantity flowing uniformly, or with constant differential, is equivalent to what would be the choice of the independent variable. It was standard procedure at the time, as explained in detail by Bos (1974, 25-ff) and Guicciardini (1989, 4–5) for the Leibnizian calculus and the calculus of moments in Britain, respectively. For instance, Bos illustrates how the choice of the *independent* variable affected the formulas for the radius of curvature, even when the specification was avoided and no variable quantity was supposed to have constant differential (Bos, 1974, 35–42).

The explanation given by Simpson can be found in Blake's *Essay*, as an Illustration, where the velocity could be equable, accelerated or retarded. Blake's Illustration was in fact the subject of Simpson's reply to a letter from Blake dated 9 January 1739. Likewise, the Corollary in Blake's *Essay* can be read in a letter from Simpson to Blake in 1740. Namely, the subject of higher-order fluxions was discussed in the correspondence between Blake and Simpson in 1739–1740 and the results of this discussion were first gathered in Blake's *Essay*.[53] Again Simpson made his indebtedness to Blake in this matter quite clear:

> for I scruple not, and think myself in justice, obliged, to acknowledge that whatever hints I may have advanced with regard to these higher orders of fluxions were chiefly derived from a former letter of yours, to which I stand indebted for a more general and less complex idea on the subject than I before entertained.
>
> [Simpson to Blake, 29 January, no year, Smith H. Coll.]

In short, although Simpson (1737) and Simpson (1750) shared basically the same definition of higher-order fluxions, in 1750 Simpson was convinced that "the higher orders of fluxions are render'd much more easy and intelligible" (Simpson, 1750, preface), thanks to the new definition of fluxion. The comparison of both works suggests that the nature of higher-order fluxions, together with its teaching, was a matter of concern at the time.

[53] For a summary of their correspondence on the subject of higher-order fluxions, see Clarke (1929, 65–70).

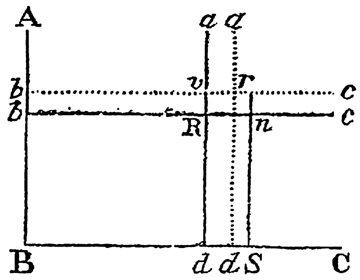

Figure 1. From Simpson (1737), Section I, part I.

5.2.3. The rule of the product

The differentiation of the product was a key issue for the question of how to avoid the vanishing infinitesimal quantities. In his *Principia Mathematica*, Newton considered A, B to be increased and decreased by $\frac{1}{2}a$ and $\frac{1}{2}b$ respectively, where a and b were the corresponding moments. The product of $A - \frac{1}{2}a$ and $B - \frac{1}{2}b$ yielded:

$$AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab \tag{1}$$

Likewise, the product of $A + \frac{1}{2}a$ and $B + \frac{1}{2}b$ yielded:

$$AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab \tag{2}$$

When subtracting (1) from (2) one obtained the moment of the original rectangle AB, namely $aB + bA$ (Newton, 1687, book II, section II, lemma II). The consideration of half the increments of A and B, instead of their whole increments, was strongly critised by Berkeley in 1734 as based on false reasoning (Berkeley, 1734, §9). Using the whole increments the fluxion of the rectangle AB would really be $aB + bA + ab$, from which ab should be removed to obtain Newton's result. But the elimination of this quantity was not based upon legitimate reasoning.[54]

At a time marked by Berkeley's criticisms of the calculus, it is important to see not only how Simpson introduced this topic in his two works on fluxions, but also Simpson's view on the vanishing quantities. In 1737 Simpson showed how *to find the fluxion of the rectangle of any two variable quantities whatsoever* (Simpson, 1737, part I, §11). In his proof, Simpson did not use the characteristic dot notation, describing the process in a discursive way, not even using the letters x, y for the variable quantities. Given the rectangle RB, generated by the motion of two perpendicular lines, ad, bc, the fluxions of the sides Bb and Bd are Rv and Rn, respectively (see Figure 1). Simpson concluded that the fluxion of the rectangle BR was the sum of the fluxions of the rectangles on the sides Rd and Rb – that is, the sum of the rectangles $Rb \times Rv$ and $Rd \times Rn$. This approach was evidently deficient, as Simpson considered the sides of the rectangle as two quantities working independently, and not as being linked to each other by the product. In other words, in Simpson's proof the original rectangle, when flowing, did not become a new rectangle, without any mention of the vanishing rectangle $Rv \times Rn$. In this way Simpson had apparently dodged the question of the vanishing terms of second order in this particular case, through an inadequate approach.

54 Thirty years before Berkeley's attack, an approach similar to Newton's could be found in Hayes's *Treatise of Fluxions* (1704). The relevance of the rule of the product in Berkeley's criticism of the calculus has been discussed at length in Blay (1986) and Bruneau (2011, 224–228).

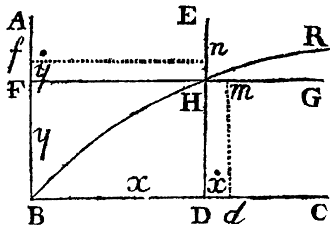

Figure 2. From Simpson (1750), Section I, part I.

In contrast to Simpson, Stone showed that the fluxion of xy, first being $y\dot{x} + x\dot{y} + \dot{x}\dot{y}$, became $y\dot{x} + x\dot{y}$ because:

> $\dot{x}\dot{y}$ is a quantity infinitely small, in respect of the other terms $y\dot{x}$ and $x\dot{y}$: For if, for example, you divide $y\dot{x}$ and $\dot{x}\dot{y}$ by $\dot{x}$, we shall have the quotients y and $\dot{y}$, the latter of which if infinitely less than the former.
> [Stone, 1730, Section I, part I, §5]

No wonder that Stone referred to infinitely small quantities in his proof, given that his direct method was nothing but a translation of de L'Hospital (1696). Meanwhile, Hodgson based his reasoning on kinematical terms (Hodgson, 1736, preface, xiij–xv). The flowing quantities x and y became $x + \dot{x}o$, $y + \dot{y}o$, respectively, where $\dot{x}$, $\dot{y}$ were the velocities of the increments of x and y, and o a very small quantity. After multiplying the quantities $x + \dot{x}o$ and $y + \dot{y}o$, and taking away xy to get the fluxion, the rectangular space $\dot{x}\dot{y}oo$ could be rejected because when the rectangle flowed back to its original state this space would vanish. Concerning the vanishing quantities, in the preface Hodgson had already pointed out the differences between the method of fluxions and the differential calculus:

> In the former method quantities are rejected, because they really vanish; in the latter they are rejected, because they are infinitely small; which cannot but leave the mind in some ambiguity or confusion.
> [Hodgson, 1736, preface, vij]

The vanishing rectangle that resulted from the fluxion of the product was indeed a thorny issue at the time. For instance, in Muller (1736, §168) we find a totally different approach, based on the fact that a secant line would eventually coincide with a tangent line. From similarity of triangles and the equation of the subtangent, he had allegedly found the fluxion of a rectangle using exclusively finite quantities and "after a new manner, without rejecting any quantity for its smallness" (Eames, 1737–1738, 88). However, by considering a secant line turning into a tangent line, he was somehow masking the point in question. In 1750, Simpson introduced the fluxion of the product of two quantities otherwise. To begin with, he used the dot notation for fluxions and the treatment was not as verbal as in 1737. This time the rectangle *DF* was not regarded as a plain rectangle, but as the sum of two spaces, or areas, separated by the line *BHR*, which was the path of the intersection of the motions of the right lines *BA* and *BC* (Figure 2). The fluxion of the rectangle, specifically, the sum of the fluxions of each space, was computed as follows: "The fluxion of the space or area *BDH* is truly expressed by the rectangle Dm... and that of the area, or space *BFH*, by the rectangle Fn... " (Simpson, 1750, part I, §10). Why did Simpson use the word "truly" in this sentence? To answer this question we have to turn again to the correspondence between Blake and Simpson. As we will see, the line *BHR* would prove useful to define the fluxion of each area.

The fluxion of the rectangle was the subject of a number of letters they exchanged in the summer and autumn of 1740. In fact, Blake began working with the fluxion of a triangular area, which later inspired Simpson's approach to the fluxion of the rectangle. The outcome of their exchanges was first gathered as the Lemma, Scholium and Proposition in Blake's *Essay* (Blake, 1741, 8–11; Clarke, 1929, 61–63). In particular, in the Scholium, Blake explained why the fluxion of any of the areas was truly a rectangle, and not the curvilinear trapezium, following the definition of fluxion:

> SCHOLIUM
> It has been commonly objected to the accuracy of fluxions, that the trapezium or curvilineal space *BCdeD*, not the rectangle $\dot{x}y$, is the fluxion geometrically exact. But, this objection is built, I apprehend, upon a false idea of the thing. It supposes a fluxion a COMPLETE part of a flowing quantity, and an INFINITY of fluxions to constitute the flowing quantity, which are mistakes (per definition and lemma). The area *BCdeD* is the increment; the space that WOULD have been generated in TIME *mn* with y variable; and indeed if $\dot{x}$ be imagined infinitely little, an infinity of increments may constitute the area *ABC*. But, in fluxions, our reasoning is quite different: a fluxion can no more be called a part of the fluent, than an effect a part of the cause. For instance; from the fluxion given we know the fluent, and vice versa, just as when a cause is known to produce a certain effect, we can infer the ONE from a knowledge of the OTHER. (...)
>
> [Blake, 1741, 9–10, original emphasis]

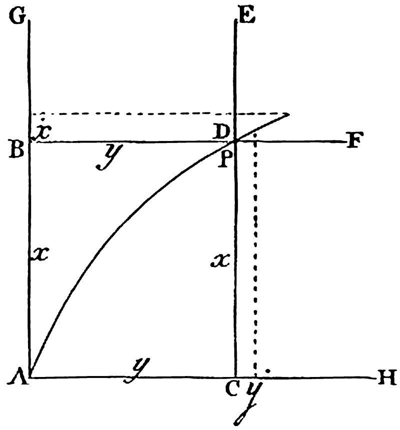

Figure 3. From Blake (1741).

This Scholium was the immediate consequence of considering magnitudes as being uniformly generated at the instant of finding the fluxion, and not as made up of an infinite number of infinitesimal parts put together.[55] Accordingly, in 1750 Simpson wrote:

> ... the rectangle *RrsS* will, here also, be the fluxion of the generated space *Amn*: Because, if the length and velocity of the generating line *mn* were to continue invariable from the position *RS*, the rectangle *RrsS* would then be uniformly generated, the very celerity wherewith it begins to be generated, or with which the space *Amn* is increased in that position.
>
> [Simpson, 1750, part I, §4]

Going beyond the definition of fluxion, Maclaurin proved geometrically by *reductio ad absurdum* that the incremental curvilinear trapezium was equal to the rectangle, that is, the fluxion of the given area (Maclaurin, 1742, book I, §§107–108). The equality of these two figures is also discussed by Stone. By definition, the curvilinear trapezium was the fluxion of a given area (Stone, 1730, part I, Definition II). But, at the same time, from Postulate I this trapezium could be considered to be equal to the small rectangle, because these two figures differed in an infinitely small triangle (Stone, 1730, part I, §2).

55 To support his view, Blake concluded his Scholium by stating that "a fluxion can no more be called a part of the fluent, than an effect a part of the cause". As we will see, the relationship between fluent and fluxion in terms of cause and effect would be the target of negative criticisms on account of the publication of Simpson (1750).

That Simpson disagreed fundamentally with the nature of the infinitesimal method is evident from a letter he wrote to Blake on account of the fluxion of the triangular area, wherein he declared to be:

> Abundantly well satisfied both with regard to y^e method itself and y^e manner in which you deliver it; and am extreamly glad that you are now able to effect your desires without borrowing aught form y^e arithmetic of infinites, which, you know, I never was fond of, being well assured that to render it accurately true and, at y^e same time, sufficiently plain and intelligible, would be a difficulty vastly greater than any in fluxions considered in y^e light in which you now have put them.
>
> [Simpson to Blake, no place, no date, Smith H. Coll.]

Later in the same letter, considering the fluxion of the triangle, Simpson proceeded to show Blake how to compute the fluxion of the rectangle. Finally, this result was included in Blake's short *Essay* as the proposition mentioned above and, subsequently, in Simpson's work of 1750, both illustrating this result with the same figure (Figs. 2 and 3).

Although Simpson did not explicitly discuss the elimination of vanishing quantities in the case of the fluxion of the product, he did so when finding the fluxion of a power, in particular, x^2 (Simpson, 1737, part I, §6; Simpson (1750), part I, §6):

> *Case* 1. Let $\dot{x}$ express the fluxion of x, (according to the foregoing notation) and let the fluxion of x^2 be required.
>
> Conceive two points m and n to proceed, at the same time, from two other points A and C, along the right-lines AB and CD, in such sort, that the measure of the distance $CS(y)$, described by the latter, may be, *always*, equal to the square of that $AR(x)$, described by the former moving uniformly.
>
> Furthermore, let r, s, and R, S, be any contemporary positions of the generating points, and let the lines $\dot{x}$ and $\dot{y}$ represent the respective distances that *would be* uniformly described, in the same time, with the celerities of those points at R and S, then those lines will express the fluxions of Am and Cn in this position.
>
> Moreover, since $Cs = Ar^2$ and $CS = AR^2$, if Rr be denoted by v, we shall have $CS(y) = x^2$, and $Cs(= (x-b)^2) = x^2 - 2xv + v^2$, and consequently $Ss(= CS - Cs) = 2xv - v^2$; from whence we gather that, while the point m moves over the distance v, the point n moves over the distance $2xv - v^2$. But this last distance (since the square of any quantity is known to increase faster, in proportion, than the root) is not described with an uniform motion (like the former), but an accelerated one; and therefore is equal to, and may be taken to express, the uniform space that might be described with the mean celerity at some intermediate point e, in the same time. Therefore, seeing the distances that might be described, in equal times, with the uniform celerity of m, and the mean celerity at e, are to each other as v to $2xv - v^2$, or as 1 to $2x - v$, or, lastly, as $\dot{x}$ to $2x\dot{x} - v\dot{x}$, (all which are in the same proportion) it is evident, that, in the time the point m would move uniformly over the distance $\dot{x}$, the other point n, with its celerity at e, would move uniformly over the distance $2x\dot{x} - v\dot{x}$. This being the case, let r, R, and s, S, be now supposed to coincide, by the arrival of the generating points at R and S, then e (being always between s and S, will likewise coincide with S; and the distance $2x\dot{x} - x\dot{x}$, which might be uniformly described in the aforesaid time, with the velocity at e, (now at S), will become barely equal to $2x\dot{x}$; which (*by the Defin.*) is equal to ($\dot{y}$), the true fluxion of Cn or x^2.
>
> [Simpson, 1750, part I, §6]

The key point in Simpson's proof is that the distance $2x\dot{x} - v\dot{x}$ becomes $2x\dot{x}$ when the point e coincides with S. Pleased with the soundness of his explanation, Simpson believed his way of presenting the fluxion of a power was not:

> ... liable to the common objection brought against the Doctrine of Ultimate Ratios, for as the quantities to be compared (or the velocities of m & n) do not vanish at R & S, when their ratio is to be taken, there is no necessity conjuring up the ghosts of departed quantities to supply their places.
>
> [Simpson to Blake, 1 October 1750, Smith H. Coll.]

Here Simpson was clearly referring to Berkeley's complaints about shifting the hypothesis, namely, treating the increment first as non-zero and then letting it vanish.

In short, while in 1737 Simpson presented the fluxion of the rectangle verbally, without referring to the vanishing rectangle, in 1750 he abandoned the discursive approach and, from his new definition of fluxion, he elaborated more on the proof, in an attempt to render the question unambiguous.

5.2.4. The study of maxima and minima

The comparison of Simpson's sections on maxima and minima of 1737 and 1750 reveals that Simpson worked mainly with the same problems (17 examples in 1737, 22 examples in 1750), even illustrating them with the same figures. Yet both texts differ in their definition and characterisation of maxima and minima. In 1737 Simpson defined the least and the greatest possible ordinates by means of a curve generated by two motions, one uniform and the other one variable (Figure 4):

> *General Illustration*: Suppose the right line *AB* to move on continually, with an uniform motion parallel to itself, over the immoveable lines *nbd*, *mrd*; it is evident that as soon as the end of the line *AB* has moved over the point n, and has arrived to any position rb, there will be a space *brmn*, bounded on every side; which is what we call a space generated by the motion of the line *AB*: now whilst the said line *AB* moves continually forward, the lines nb and mr will be augmented: but the ordinate or line rb (which boundeth the space on that side) is no longer increased or decreased, than till it arrives to line vb [sic](vc); at which time it is consequently the least or the greatest that it can possibly be. Now if while the line *ABab*, keeps moving on uniformly, there be supposed a point r, to move so in the said line as to be always in the curve-line *mcd*, then will the velocity with which that point is continually carried in the direction of the said line *AB*, *ab*, always be, exactly, as the fluxion of the ordinate br; which when the said ordinate is the least or greatest possible, will become equal to nothing, that is, the point r will then have no velocity at all in respect of the direction ba: for supposing the contrary was asserted. I shall then ask whether r be moved upwards or downwards? If upwards, I answer that the ordinate br is still increasing, and therefore is neither the least nor the greatest; if downwards, it has been greater will be less than it now is, which is equally absurd; therefore the fluxion of the said ordinate br, though expressed by any kind of an equation whatsoever, will then become equal to nothing, and seeing all equations or expressions whatsoever may be signified by the ordinates of such like curves, consequently the fluxion of any quantity expressing the value of a maximum, or a minimum must be equal to nothing.
>
> [Simpson, 1737, part I, §23]

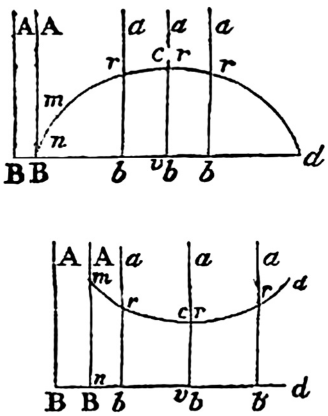

Figure 4. From Simpson (1737), Section II, part I.

This approach, based upon the nature of the curve and the connection between the abscissa and the ordinate, was the standard at the time. It can be found not only in Stone (1730), but also in Hayes (1704), Hodgson (1736) and Muller (1736). However, unlike Stone, Simpson did not include the case of cusps in his study of maxima and minima.

The approach of 1750 diverged from the one of 1737 in that Simpson abandoned the figure of the curve in defining maxima and minima. Over a straight line he combined the motion of two points, one moving uniformly (m) and the other one (n) moving towards the former with variable speed (Figure 5). If the motion of the point n is accelerated, the distance between n and m will at first increase until the point acquires the same velocity as m. After this, the distance between them will start to decrease:

> *General Illustration*: Let a point *m* move uniformly in a right line, from A towards B, and let another point *n* move after it, with a velocity either increasing, or decreasing, but so that it may, at a certain position, *D*, become equal to that of the former point *m*, moving uniformly.

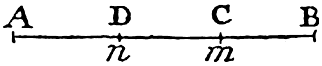

Figure 5. From Simpson (1750), Section II, part I.

> This being premised, let the motion of *n* be first considered as an increasing one; in which case the distance of *n* behind *m* will continually increase, 'till the two points arrive at the contemporary positions *C* and *D*; but afterwards it will, again, decrease; for the motion of *n*, 'till then, being slower than at *D*, it is also slower than that of the preceding point *m* (by hypothesis) but becoming quicker, afterwards, than that of *m*, the distance *mn* (as has been already said) will again decrease: and therefore is a maximum, or the greatest of all, when the celerities of the two points are equal to each other.
>
> [Simpson, 1750, part I, §22]

Similarly, Simpson defined a minimum when the motion of n is retarded and, obviously, its velocity at the starting situation is greater than that of the point m. And at the end of this "general illustration" he discussed why the fluxion at a maximum or minimum had to be equal to zero:

> Since then the distance *mn* is a maximum or a minimum, when the velocities of *m* and *n* are equal, or when that distance increases as fast through the motion of *m*, as it decreases by that of *n*, its fluxion at that instant is evidently equal to nothing. Therefore, as the motion of the points *m* and *n* may be conceived such that their distance *mn* may express the measure of any variable quantity whatever, it follows, that the fluxion of any variable quantity whatever, when a Maximum or Minimum, is equal to nothing.
>
> [Simpson, 1750, part I, §22]

Without further explanation on the theory of extreme values, a collection of examples followed the "general illustration". Simpson did not apply any algorithm to determine whether the point was a maximum or a minimum, inferring its nature from the context of the problem. It is worth reminding here that in 1742 Maclaurin had presented the first specific rule for characterising maxima and minima (Maclaurin, 1742, book II, §858). From his series expansion, based in turn on Taylor's series, Maclaurin developed his rule by means of the successive fluxions of a quantity. Apparently this result had no impact on Simpson's section on maxima and minima. What is more, Simpson seemed to confine the use of series to the inverse method exclusively.

Going back to the "general illustration", one could wonder why Simpson explained maxima and minima in terms of velocities in 1750, without any reference to a curve. It is certainly a very unusual procedure, which I have not found elsewhere. This topic was not discussed in the correspondence between Blake and Simpson, nor was included in Blake's *Essay*. Apparently, this time this change was not inspired by the exchanges between Blake and Simpson. Could Simpson have tried to approach the topic in fluxional terms exclusively? In using such a procedure, without taking a curve into account, Simpson could have undertaken the characterisation of maxima and minima in terms of speed, without turning to increasing and decreasing ordinates, as Leibnizians did.

6. On publishing matters and reception

Of all the authors of treatises on fluxions published between 1736 and 1758, Simpson was the only one who managed to issue two – clearly different – works on the subject. The differences between the production of his works of 1737 and 1750 are nothing if not significant.

A New Treatise of Fluxions (1737) was regarded as a "valuable treatise" at the time, as it was described in *The Gentleman's Diary* in 1741 (v. I, 44). However, Simpson's *The Doctrine and Application of Fluxions* (1750) doubtless attained a greater success than his first book. While there were no subsequent editions for the work of 1737, *The Doctrine and Application of Fluxions* was reprinted in 1776 (second edition), 1805 (third edition), and as late as 1823 (fourth edition), being one of the most reprinted treatises at the time, as Guicciardini (1989, 58) has pointed out. In addition, it soon reached Continental Europe. In France there was a first advertisement in the *Journal Britannique* as early as 1750 (*Journal Britannique*, 1750, t. 3, 223–224). In the thorough review that appeared in this journal subsequently, Simpson's work of 1737 was considered to be an "ébauche", a preliminary draft of his subsequent work of 1750, which was considered "rather a new treatise than a second edition of the first".[56] A few years later *The Doctrine and Application of Fluxions* got to Spain, through the French Jesuit network, where it was translated into Spanish and adapted by the Jesuit Tomàs Cerdà (1715–1791).[57]

Nonetheless, Simpson's work was not well received everywhere. With Berkeley's attack on Newton in the background, the principles of fluxions were the source of several disputes raised by Robert Heath (1720–1779) by the mid-18th century. Both of Simpson's works were the target of Heath's attacks in *The Ladies' Diary* and other publications.[58] Around 1738–1739, Heath accused Simpson of piracy on account of his *A New Treatise of Fluxions* (1737). A renewed dispute began about the time when Heath became the editor of *The Ladies' Diary*.[59] In an article in this periodical, Heath defended the notion of increments as inherent to the nature of fluxions. In this regard, he recommended the *Doctrine of Fluxions* by William Emerson (1743), over those writers who tried to ignore increments:

> Those who desire further satisfaction as to the nature of fluxions, of their noble use and transcendant excellence, may consult Mr. Emerson's *Doctrine* of the *whole art*, which is… the best of any… Those writers will find themselves mistaken, who pretended to derive the finite ratios of motion, or fluxions producing magnitudes, without the previous consideration of increments, which include the very notion of what a fluxion is. This *some* have attempted by multiplying quantities into their velocity, and some by other means, the result of which originally depends on *incremental principles*, if they would consider the matter as far as it will go.
>
> [Heath, 1746, as quoted by Cajori, 1919, 209]

[56] "Plûtôt un traité nouveau qu'une seconde Edition du premier", *Journal Britannique* (1751, t. 4, 142).

[57] Cerdà came to know Simpson's work when he was studying in Marseille around 1755 under the supervision of the Jesuit Esprit Pézénas (1662–1776), who happened to be the translator of Maclaurin's *Treatise of Fluxions*. On the reception of Simpson's work in Spain, in connection with Cerdà, see Ausejo and Medrano (2010) and Berenguer (2011).

[58] According to Clarke (1929, 150), this criticism might have led Simpson to consider the writing of a new text on the subject. On the disputes between Heath and Simpson, see Albree and Brown (2009, 18–19), Cajori (1919, 207–224) and Clarke (1929, 28–30 and 155–161).

[59] Robert Heath was the editor of *The Ladies' Diary* between 1745 and 1753. In 1753 he was dismissed on account of his bad manners and Simpson succeeded him as editor, holding the position until 1760. See Albree and Brown (2009).

Though covertly, Simpson could be counted among those writers who had tackled the notion of fluxion by "multiplying quantities into their velocity".

More explicit criticisms of Simpson's work were to come. In 1750, a negative review of *The Doctrine and Application of Fluxions* appeared in *The Monthly Review*.[60] The anonymous author of this review, under the pseudonym of *Cantabrigiensis*, regarded Simpson's definition of fluxion as being very odd, in that it referred not to a real, but to an imaginary thing. According to *Cantabrigiensis*, Simpson mistook the effect for the cause:

> ... for the thing generated must owe its existence to something, and this can only be the velocity of its motion; but it can never be the *cause of itself*, as his definition would erroneously suggest...
>
> [*The Monthly Review*, 1750, iv, 130, original emphasis]

Besides, although Simpson claimed to have avoided the method used by the foreigners (i.e., the infinitesimals), he had turned to it after all in the opinion of *Cantabrigiensis*. In general, *Cantabrigiensis* disapproved of Simpson's book for being too large, with neither order nor method, and without any, or with very few, new things in it. In this review, Simpson was also accused of having appropriated Emerson's title for his new work, namely, *Doctrine*, "to give his book a character, and make it sell" (*The Monthly Review*, 1750, iv, 131). This review made John Turner (editor of *Mathematical Exercises*, 1750–1752) publish a defence of Simpson in the *Mathematical Exercises* (1751, III, 34), which, in turn, led to the publication of the *Truth Triumphant: or Fluxions for the Ladies* (1752), authored by Heath, with which the debate attained its peak. Written as a dialogue carried out by supporters of Emerson, this short book openly attacked Simpson's *The Doctrine and Application of Fluxions*.[61] The substance of Heath's arguments can be found in the review mentioned above. All the more so given that this review was reprinted and included in the *Truth Triumphant*. One can reasonably assume that *Cantabrigiensis* could possibly be Heath himself, or any of his supporters in his crusade against Simpson.

More essential were the differences between *A New Treatise of Fluxions* (1737) and *The Doctrine and Application of Fluxions* (1750) concerning their publication and sale. As far as *A New Treatise of Fluxions* is concerned, both publication and sale could be regarded as rather unremarkable. Thomas Gardner, a printer and publisher in London between 1735 and 1756, with a rather dubious reputation,[62] printed Simpson's treatise of 1737. From Simpson's preface it is clear that there were subscribers:

> Having given this short view of the material things contained in this treatise, I think my self in justice obliged, to prevent all suspicion of fraudulent or equivocal behavior towards those gentlemen who have favoured me with their *subscriptions*, to acquaint them with my reasons for ending the book here: and I doubt not that they will candidly excuse the necessity I was under, of not keeping up strictly to every part of my *proposals*.
>
> The twenty sheets proposed are here delivered exclusive of the preface, &c. (and *much cheaper* than any thing of this kind hitherto published); but in order to be perspicuous, and to prevent breaking the connexion of the parts, and to shew their dependence on each other, I found my self obliged for want of room here to leave out the whole fifth part, which being a collection of miscellaneous problems, independent of the first four parts, will be printed by way of supplement, in the same manner, and so as to be bound up with them, by all those that have a mind to buy it, and compleat their book; but if any gentleman shall think himself imposed on by this method of proceeding, he is at liberty to *receive back his subscription money*.
>
> [Simpson, 1737, preface, iii, my emphasis]

60 See *The Monthly Review* (1750, iv, 129–131, article xxv).

61 For a thorough discussion on this publication, see Cajori (1919, 212–219).

62 On Gardner and his connection with literary property matters, see Boswell's *Life of Johnson* (1791, 518), as quoted by Plomer (1922).

Publication by subscription was a system of advance payment by the purchaser that covered the costs of production. This system was used quite often in the 18th century, especially for works that did not appear commercially attractive.[63] Gardner, not unusually, was prepared to receive subscriptions for proposals and works. Therefore we can infer that Gardner accepted Simpson's treatise of fluxions for publication on the basis of subscription.

A common practice at the time was to include a list of subscribers in the book.[64] Although there was no such list in Simpson's *A New Treatise of Fluxions*, we have evidence of its existence.[65] In the ledger of the printer Charles Ackers[66] (1702/03–1782), McKenzie and Ross (1968) found, under Simpson's entry, that 750 "Proposals for a Treatise of Fluxions" were printed at £1 in 1735.[67] These could be the proposals to which Simpson referred in the extract of the preface above. Such proposals for printing by subscription were often advertised in periodicals like *The Gentleman's Magazine* and *The Gentleman's Diary*, including a brief description of the volume to be printed, its price and where subscriptions could be received.[68] I have not found any advertisement for proposals for Simpson's treatise on fluxions, though the book was advertised a month before being issued in *The Gentleman's Magazine* (v. VII, 194), as most of Simpson's works were.

Frustratingly, there are no more details and no copy of the proposals for Simpson's treatise is known to exist. Could the people who bought these proposals be the subscribers of the work? In the title page we are informed that Simpson's treatise was printed by Thomas Gardner:

> For, and are to be had of, the Author in Crown-Court Long-Alley, near upper Moorfields; Geo. Powell in Shrewsbury-Court, White-Cross-Street; Rob. Shirtcliffe at a School-House in Wimple-Street, near Oxford-Chapel; Dan. Eagland at the Alienation-Office, in the Temple.
>
> [Simpson, 1737]

My guess here is that, once the work was printed, the people in charge of selling the book – Simpson himself, George Powell, Daniel Eagland and Robert Shirtcliffe – were somehow connected with the subscriptions, although the list of those willing to sell the work grew in subsequent printings.[69] Little is known about George Powell, apart from the fact that he seemed to have been interested in fluxions, since his name appears in the list of subscribers of John Rowe's *An Introduction to the Doctrine of Fluxions* (1751). Yet

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63 On the system of subscriptions see Atto (1938) and Feather (1980, 1–3; 1985, 58).

64 For instance, in the second volume of John Harris's *Lexicon Technicum* (1710) a 12-folio list of subscribers was given at the end: "A Catalogue of the Names of as many of the subscribers to *Lexicon Technicum* as came to our hands". Among the subscribers, there were also booksellers. It is worth commenting that in the first volume (1704) the list was eight-folio long, which means that the number of subscribers was not static and could increase at different points of the production. See Wallis and Wallis (1986, entry 701HAR10).

65 According to Feather (1980, 3), the case of books published without a list of subscribers is an area in which more work is needed.

66 A printer in London, Ackers was particularly known for being the printer of the *London Magazine*, as well as one of its proprietors. He was in charge of the printing of the *Elements of Plane Geometry* for the author, Thomas Simpson, and for Farrer and Turner in 1747 (1000 copies) and then for John Nourse (1000 copies, in quarto sheet). See McKenzie and Ross (1968, 225 and 296).

67 Ledger 23 Dec. 1735; 750; £1 (not seen), in McKenzie and Ross (1968, 296).

68 See, for instance, *The Gentleman's Magazine* (1736, VI, 492), and *The Gentleman's Diary*, at the end of the volume "The British Telescope", by Edmund Weaver (1746).

69 For instance, besides Eagland, Shirtcliffe and Powell, in May 1737 Francis Morland (at the Golden Ball near Dover-Street, Picadilly, or at S. John's Gate) was also included in the list (see *The Gentleman's Magazine*, VIII, 320). Then, in August 1737, the list of "sellers" also included Mess Innys and Manby in Paul's Church Yard, A. Bettesworth in Paternoster Row, J. Brindley in New Bond-Street and E. Cave St. John's Gate (see *The Gentleman's Magazine*, VIII, 517).

in the ledger of Ackers I have found evidence supporting my guess, for Powell was said to have paid £1 to Ackers for the 750 proposals for Simpson's work:

> Due from Mr Simpson (23 Dec. 1735). To paper and printing 750 proposals for a Treatise of Fluxions. Pd by Mr Powell April 1737. £1.
>
> [McKenzie and Ross, 1968, 73]

With regard to Daniel Eagland, in the list of deaths in *The Gentleman's Magazine* for the year 1764 he was said to be a well-known mathematician (v. 34, 148). Acquainted with Thomas Simpson,[70] Eagland took Simpson's side in the aforementioned dispute with Heath (Clarke, 1929, 28–30). In addition, a dispute was raised between Heath and John Holmes, an innovative schoolmaster, on account of the Greek grammar written by the latter in 1735. From the title of a pamphlet published by Heath in 1738, and which was meant against Holmes, it is clear that both Simpson and Eagland supported Holmes in this controversy about a reform of the English grammar school system.[71]

Also acquainted with Simpson, Robert Shirtcliffe (1720–1762) was a teacher of mathematics and an excise officer between 1737 and 1740 (Wallis and Wallis, 1986, entry 740SHI, 262). It was precisely for the use of the excise officers that he published *The Theory and Practice of Gauging, Demonstrated in a Short and Easy Method* (1740), wherein on several occasions he referred to Simpson as his "ingenious friend" in connection with the development and summing of series of powers (Shirtcliffe, 1740, 88 and 109).

Simpson's first treatise, which was supported by subscribers and sold by acquaintances, had a different history than that of John Muller's *A Mathematical Treatise: Containing a System of Conic-Sections; with the Doctrine of Fluxions and Fluents, Applied to Various Subjects* (1736). Despite being his first book, printed by Gardner as well, it was sold by Innys and Manby, and Nourse, leading booksellers and publishers in London at the time (Feather, 1981; Plomer, 1922, 137). This could be explained by the fact that Muller was somehow connected with the Duke of Argyll. While producing his *Mathematical Treatise*, Muller held a teaching position in the Tower of London, where the Board of Ordnance had its headquarters. Muller dedicated his book to the Duke of Argyll who, as we have seen, happened to be Master General of the Ordnance. Again, one could assume that Muller's connection with the Duke of Argyll could have granted him the exclusive production of his very first work.

Meanwhile, Simpson, a rather unknown author, with no institutional bonds so far, published his first work by subscription, rather than turning to important booksellers. However, we have noted earlier that Innys and Manby took part in the sale of subsequent printings of Simpson's work. It seems very likely that, once his work achieved some success, Innys and Manby considered it worth selling.

The production of *The Doctrine and Application of Fluxions* turned out to be radically different. This time it was a prominent London bookseller, John Nourse (*bap*. 1705–1780), who undertook the publishing and selling of this work and of its second edition (1776). Nourse primarily traded in language books, contemporary foreign literature, and in scientific books. With a special interest in mathematical works, Nourse produced the works of authors such as Roger Cotes, William Emerson, John Landen, Colin Maclaurin, Isaac Newton, Benjamin Robins, Robert Simson or Brook Taylor, among others.[72] He was appointed as

[70] See for instance the letters from William Piper to Simpson (15 May 1739) and from Henry Beighton to Simpson (7 July 1741), in Smith H. Coll.

[71] *A battle fought with the boasters: or, Patroclus's weak defence / by force defeated; and H-lm-s, S-mp-n, E-gl-d, and all their vaunting host, cast headlong into the sea of ignorance. By Philomathematicus's army of arguments*, where Patroclus was Holmes's pseudonym during the controversy and Philomathematicus the name chosen by Heath for himself. On the dispute Heath–Holmes see Stoker (1995).

[72] See, for instance, the list of books printed for John Nourse annexed at the end of the second edition of Simpson's *The Doctrine and Application of Fluxions*.

bookseller by the Society for the Encouragement of Learning[73] and held the title of "Bookseller of His Majesty" from 1762 to 1780.[74]

From 1740 onwards, most of Simpson's works were printed for Nourse who, in some cases, was also in charge of subsequent editions.[75] In Feather (1981) there is a study of the surviving agreements regarding the publication of several works produced by John Nourse, in particular, the books of John Landen (1719–1790), the only mathematical work mentioned in Feather's paper. Although there is no trace of the agreements and dealings between Nourse and Simpson, from Feather's paper one can speculate about this point.[76] According to Feather, Nourse worked mainly on two levels. On the one hand, he purchased the copyrights both of books he had commissioned and of unsolicited works (either of completed manuscripts or of already printed editions). On the other hand, Nourse was entitled to various rights in the future publication of the books of a certain author. From what we know about the aims and scope of Simpson's second work on fluxions, it was very likely an unsolicited work. At the same time, it could also fit in the second category, given that Nourse was in charge of the second edition of *The Doctrine and Application of Fluxions*, more than 10 years after Simpson had passed away.

To picture how the dealings concerning the publishing of Simpson's work could have gone, it is worth having a look at the correspondence of John Nourse kept in the file MS Graves 23 (2), at the University College London (UCL), Library. In this file, there is one letter from Francis Holliday (1717–1787) to Nourse, in which the former showed interest in having his latest work published by the latter and stated why his book was worth publishing:

> I have a small MS in Fluxions which has some time ago been revised by my friend Emerson and which he has advised me to print it, and indeed I have drawn it up on purpose as an easy Introduction to Fluxions; I have laid down things so plain, that a smatterer in Algebra may easily comprehend it, it contains about 240 pages in folio MS and should be glad you would purchase it of me, because what Mathematics you print are well corrected. I shall be glad to be inform'd in your answer whether you have yet rec.[d] Mr Emerson's Doctrine of Chances and Annuities which he has ready for the press.
>
> [6 May 1775, MS Graves 23 (2)]

Holliday was here referring to his *An Introduction to Fluxions, Designed for the Use, and Adapted to the Capacities of Beginners* (1777), printed for Nourse. In the preface, he admitted having written his book "for beginners, and not for those who have made some proficiency in fluxions" (Holliday, 1777, preface). Therefore his view is clearly in keeping with the concern for the elaboration of plain and easy introductions to fluxions that we have discussed earlier.

Turning again to Simpson's works, *The Doctrine and Application of Fluxions* was much more expensive than *A New Treatise*. While the latter could be purchased at 5s, in sheets, with an additional 1s 6d for the supplement, the former was sold at 10s 6d, in sheets, in its first edition, going up to 12s in the second edition. It was surely a high price to pay even at that time, as can be inferred from Davis's initiative to

[73] The Society for the Encouragement of Learning was founded with the object of securing writers' rights regarding their works. This kind of societies flourished in the 18th century when the system of subscriptions proved to encourage plagiarism and imitation. See Atto (1938).

[74] For further biographical details on Nourse, see Barber (2004).

[75] In both editions of *The Doctrine and Application of Fluxions* there is an updated list of books written by Simpson and printed for Nourse, with their corresponding size and price. See also Wallis and Wallis (1986, entry 735SIM, 221–224).

[76] To my knowledge there is just one really short letter from Simpson to Nourse, preserved in Smith H. Coll., in which the former wrote that he had already sent the preface, title and other parts of one of his works to Mr Bettenham, one of the very few "topping" printers in 18th-century London (Handover, 1960, 198). He added that he had "not yet heard a word from him. Which I have thought proper to let you know, in order that the present delay may not be placed to my account" (Simpson to Nourse, 13 March 1760, Smith H. Coll.).

reduce Simpson's work to one volume (in octavo) in his edition of 1805, thus rendering it much cheaper.[77] It seems that authors had a word in establishing the size, kind of paper and even the price they intended for their books, as is evident from the following letter from John Landen to Nourse:

> I have written a discourse concerning my Residual Analysis, which I design shall be published as soon as it can be printed. I would have it very elegantly *printed in quarto*, (with wood ro tin cuts) *upon such paper*, and with such letter as my Lucubrations. If you chuse to purchase th copy, you shall have it for less than I would take of any other person. *It will make about 5 or 6 sheets, which shall be sold for 1.6 or 2*, as you may think proper. *The subject is new and very interesting*, and I (...) *a considerable number will be speedily sold*; therefore, expect you will not give me less than two guineas per sheet. However I shall leave it to you to pay me for it according as it shall sell. Part of the copy I purpose to send you by the Peterborough coach tomorrow; and you shall have the remainder before the compositor shall want it.
>
> [Landen to Nourse, 28 August 1758, MS Graves 23 (2), my emphasis]

Here again we find the figure of the author performing the tasks of a literary agent, trying to convince the bookseller why his work was worth publishing.[78] In short, although there is no surviving letter with details on the agreements between Nourse and Simpson, it is not too difficult to imagine Simpson acting as an agent for himself, advertising his own work and providing some tips and recommendations about publishing details.

In this section we have spotted some differences in the production of *A New Treatise of Fluxions* (1737) and *The Doctrine and Application of fluxions* (1750). The publication and sale of *A New Treatise of Fluxions* were undertaken by subscribers and acquaintances of Simpson's, still a rather unknown author. Thereafter, in 1750 Simpson managed to get a second work on fluxions produced by one of the most exclusive booksellers in London, most likely thanks to the success achieved not only by his first book, but also by his subsequent mathematical works.

7. Final remarks

In this paper I have carried out a comparative analysis of Simpson's *A New Treatise of Fluxions* (1737) and *The Doctrine and Application of Fluxions* (1750) from two points of view. First, I have tackled the comparative analysis of the concepts, discussing in turn the mathematical influences that could have had an impact in the shaping and reshaping of Simpson's treatises on fluxions. With regard to the hypothetical impact of Stone (1730) on Simpson (1737), my study shows that the scope and structure of Simpson's first treatise could have been inspired by Stone's work. It also illustrates the search for suitable ways to teach the fluxional calculus at a time dominated by the debates on its foundations. From the analysis of their works and correspondence it is clear that Simpson and Blake were not foreign to these debates. In fact, the changes that the concepts and definitions underwent in Simpson (1750), as compared with Simpson (1737), doubtless resulted from the discussions that Simpson and Blake held on this matter. Hence, for instance, their discussions around the fluxion of the rectangle or the nature of the vanishing quantities could have well arisen as a consequence of Berkeley's criticisms of Newtonian calculus. The fact that Maclaurin's *A Treatise of Fluxions* was the most relevant answer to Berkeley's attack, led us to wonder about its influence on Simpson's *The Doctrine and Application of Fluxions*. Of course, Maclaurin's *Treatise* was addressed to a more sophisticated audience than Simpson's work. Leaving this difference aside, in my study I singled out

77 See the advertisement in the third edition of Simpson (1750).

78 The book mentioned in the letter corresponds to the agreement no. 147, dated 22 January 1760, that appears in Feather (1981, 15).

a number of aspects in which Simpson's work of 1750 differed from Maclaurin's, such as the definition of fluxion or the treatment of maxima and minima.

From the comparative analysis of Simpson's works on fluxions, it is worth highlighting the role of the correspondence between Blake and Simpson in polishing the definitions in Simpson's work of 1750 (e.g. the definition of fluxion, the rule of the product). In short, Simpson's work of 1750 was nothing if not the outcome of his earliest readings, plus his work of 1737 and his interactions with Blake. The connection between Blake and Simpson illustrates the making of knowledge from "correspondence", and the subsequent transition from private knowledge to public knowledge.

The development and communication of knowledge also involved the production of books. This leads us to the second line of analysis – namely, the comparative analysis of the production of these two works of Simpson. As we have seen, the production of *The Doctrine and Application of Fluxions* (1750) was far more exclusive than that of *A New Treatise of Fluxions* (1737). Not only was the earlier work published by subscriptions, but Simpson also made the most of his connections to sell and produce his first book. The circle of men engaged in the production of Simpson's fluxions – from Cave, a printer and editor of *The Gentleman's Magazine*, to pupils and friends, such as Eagland, Powell and Shirtcliffe – allows us to have a glimpse of those people who could have been interested in fluxions in the 1730s, especially in the area around Spitalfields.

While Simpson carried out a wide range of authorial performances in 1737, the production of *The Doctrine and Application of Fluxions* (1750) was, in contrast, undertaken by Nourse, a prominent London bookseller. Unlike Stone and Muller, Simpson had no patron who could have supported the production of his first book. After its success, Nourse could have been inclined to undertake the production of Simpson's works from 1740 onwards and, in particular, *The Doctrine and Application of Fluxions*.

No doubt this shift in Simpson's reputation as a scientific author was a factor in his election as Fellow of the Royal Society of London in 1745, to the extent of being exempted from paying the fees. Simpson's deep interest in mixed mathematics, in keeping with the penchant of the Royal Society towards this topic at the time, could by all means contribute to his reputation.

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Annex

Table A.1
Simpson (1737). Table of contents.

PART THE FIRST
Section I: Of the nature and manner of determining the fluxions of variable quantities.
Section II: Of the solution of problems de maximis & minimis.
Section III: Of drawing tangents to curves.
Section IV: Of curves of retrogression.
Section V: Of finding the evolutes of curves.

(*continued on next page*)

Table A.1 (*Continued*)

PART II
S. I: Of infinite series in general; with the various methods of investigating the series of surd and compound fractional quantities.
S. II: Of the use of infinite series in the reduction of equations.
PART III
S. I: Of fluent in general.
S. II: Of the rectification and quadrature of curves.
S. III: Of the contents of solids, and their convex superficies.
PART IV
S. I: Of the descent of heavy bodies, and the doctrine of pendulums.
Supplement

Table A.2
Simpson (1750). Table of contents.

PART I
S. I: On the nature, and investigation, of fluxions
S. II: Of the application of fluxions to the solution of problems de maximis et minimis
S. III: The use of fluxions in drawing tangents to curves
S. IV: Of the use of fluxions in determining the points of retrogression, or contrary flexure in curves
S. V: The use of fluxions in determining the radii of curvature, and the evolutes of curves
S. VI: Of the inverse method, or the manner of determining the fluent of given fluxions
S. VII: Of the use of fluxions in finding the areas of curves
S. VIII: The use of fluxions in the rectification, or finding the lengths, of curves
S. IX: The application of fluxions in investigating the contents of solids
S. X: The use of fluxions in finding the superficies of solid bodies
S. XI: Of the use of fluxions in finding the centers of gravity, percussion, and oscillation of bodies
S. XII: Of the use of fluxions in determining the motion of bodies affected by centripetal forces
PART SECOND
S. I: The manner of investigating the fluxions of exponentials, with those of the sides and angles of spherical triangles
S. II: Of the resolution of fluxional equations, or the manner of finding the relation of the flowing quantities from that of the fluxions
S. III: Of the comparison of fluents, or the manner of finding one fluent from another
S. IV: Of the transformation of fluxions
S. V: The investigation of fluent of rational fractions, of several dimensions, according to the forms in Cotes's Harmonia Mensurarum
S. VI: The manner of investigating fluents, when quantities, and their logarithms; arcs and their sines, &c. are involved together: with other cases of the like nature
S. VII: Showing how fluents, found by means of infinite series, are made to converge
S. VIII: The use of fluxions in determining the motion of bodies in resisting mediums
S. IX: The use of fluxions in determining the attraction of bodies under different forms
S. X: Of the application of fluxions to the resolution of such kinds of problems de maximis et minimis, as depend upon a particular curve, whose nature is to be determined

Table A.3
Stone (1730). Table of contents.

Section I. On finding the fluxions of quantities
Section II. The use of fluxions in drawing tangents to all sorts of curves
Section III. Of the use of fluxions in finding the greatest and least ordinates in a curve, to which the solution of problems de maximis and minimis may be reduced
Section IV. Of the use of fluxions in finding of the points of inflexion and retrogression of curves
Section V. The use of fluxions in the doctrine of evolute and involute curves
Section VI. The use of fluxions in finding of causticks by reflexion
Section VII. The use of fluxions in finding of causticks by refraction
Section VIII. The use of fluxions in finding the points of curves touching an infinite number of curves, or right lines given in position
Section IX. The solution of some problems depending upon the methods aforegoing
Section X. The use of fluxions in geometrical curves after a new manner, from whence is deduced the method of Descartes and Hudde.
In the Appendix
Section I. Of the reduction of fractional expressions and surd quantities to infinite series
Section II. Of finding the fluents or flowing quantities of fluxionary expressions
Section III. Use of the inverse method in the quadrature of curve-lined spaces
Section IV. Use of fluxions in the rectification of curves
Section V. Of the use of fluxions in the cubation of solids, and in the quadrature of their surfaces
Section VI. Of the use of fluxions in finding the centres of gravity of figures
Section VII. Of the use of fluxions in finding the centres of percussion of figures
Section VIII. Of the resolution of some miscellaneous problems by fluxions

References

Primary sources

Berkeley, G., 1734. The Analyst, or, a discourse addressed to an infidel mathematician. In: Fraser, A.C. (Ed.), The works of George Berkeley, 1901, vol. III. Oxford Clarendon Press, Oxford.

Bernoulli, J., 1742. Opera Omnia. Tomus Quartus. Marci-Michaelis Bousquet & Sociourum. Lausanne & Genevae.

Blake, F., 1741. An Explanation of Fluxions in a Short Essay on the Theory. Printed for W. Innys, London.

Blake, F., 1751–52a. The best proportions for steam-engine cylinders, of a given content, consider'd. Philosophical Transactions of the Royal Society 47, 197–201.

Blake, F., 1751–52b. Spherical trigonometry reduced to plane. Philosophical Transactions of the Royal Society 47, 441–444.

Blake, F., 1759. The greatest effect of engines with uniformly accelerated motions considered. Philosophical Transactions of the Royal Society 51 (1), 1–6. Read in 1756 but delayed in printing.

Ditton, H., 1706. An Institution of Fluxions: Containing the First Principles, the Operations with some of the Uses and Applications of that Admirable Method. According to the Scheme Perfix'd to his of Quadratures, by (its First Inventor) the Incomparable Sir Isaac Newton. Printed by W. Botham, for J. Knapton, London.

Eames, J., 1737–1738. An Account by Mr. John Eames, F. R. S. of a Book entituled, a Mathematical Treatise, containing a System of Conic-Sections, with the Doctrine of Fluxions and Fluents, applied to various subjects. By John Muller. Philosophical Transactions of the Royal Society 40, 87–89.

Emerson, W., 1743. The Doctrine of Fluxions: not only Explaining the Elements thereof, but also its Application and Use in the Several Parts of Mathematics and Natural Philosophy. Printed by J. Bettenham, sold by W. Innys, London.

Hayes, C., 1704. A Treatise of Fluxions: or, an Introduction to Mathematical Philosophy; Containing a Full Explication of that Method by which the most celebrated Geometers of the Present Age have made such Vast Advances in Mechanical Philosophy. Printed by E. Midwinter, for D. Midwinter and T. Leigh, London.

Heath, R., 1752. Truth Triumphant: or, Fluxions for the Ladies. Shewing the Cause to be Befor the Effect, and Different from it; That Space is not Speed, nor Magnitude Motion. With a Philosophic Vision …. Printed for W. Owen, London.
Hodgson, J., 1736. The Doctrine of Fluxions, Founded on Sir Isaac Newton's Method, Published by Himself in his Tract upon the Quadrature of Curves. Printed by T. Wood, for the author, London.
Holliday, F., 1777. An Introduction to Fluxions, Designed for the Use, and Adapted to the Capacities of Beginners. Printed for J. Nourse, London.
L'Hospital, G.F.A. de, 1696. Analyse des Infiniment Petits. Pour l'intelligence des lignes courbes. Imprimerie Royale, Paris.
Maclaurin, C., 1742. A Treatise of Fluxions, in Two Books. T. W. and T. Ruddimans, Edinburgh.
Muller, J., 1736. A Mathematical Treatise: Containing a System of Conic-Sections; with the Doctrine of Fluxions and Fluents, Applied to Various Subjects; …. Printed by T. Gardner for W. Innys and R. Manby, J. Nourse, London.
Newton, I., 1687. Philosophiae Naturalis Principia Mathematica. Jussu Societatis Regiae, London.
Newton, I., 1704. Tractatus de Quadratura Curvarum. In: Newton, I., 1704. Opticks: or, a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light. Also two Treatises of the Species and Magnitude of Curvilinear Figures. For Smith and Wallford, London.
Rowe, J., 1751. An Introduction to the Doctrine of Fluxions. By W. Owen for J. Noon, London. Fourth edition, with additions, 1809. To which is added, an Essay on the theory. Revised, carefully corrected, and prepared for the press, by the late W. Davis.
Shirtcliffe, R., 1740. The Theory and Practice of Gauging. Demonstrated in a Short and Easy Method. Printed by H. Woodfall for the author, London.
Simpson, T., 1737. A New Treatise of Fluxions: wherein the Direct and Inverse Method are Demonstrated after a New, Clear, and Concise Manner, with their Application to Physics and Astronomy: also the Doctrine of Infinite Series and Reverting Series Universally, are Amply Explained, Fluxionary and Exponential Equations Solved: Together with a Variety of New a Curious Problems. Printed by T. Gardner, for the Author, G. Powell, R. Shirtcliffe, D. Eagland, London.
Simpson, T., 1740a. The Nature and Laws of Chance. Printed by Edward Cave, London.
Simpson, T., 1740b. Essays on Several Curious and Useful Subjects in Speculative and Mixed Mathematics. Printed by H. Woodfall for J. Nourse, London.
Simpson, T., 1742. The Doctrine of Annuities and Reversions: Deduced from General and Evident Principles, with Useful Tables Shewing the Values of Single and Joint Lives, &c. … . Printed for J. Nourse, London.
Simpson, T., 1743. Mathematical Dissertations on a Variety of Physical and Analytical Subjects. … . For T. Woodward, London.
Simpson, T., 1745. A Treatise of Algebra: wherein the Principles are Demonstrated and Applied, in Many Useful and Interesting Enquiries, and in the Resolution of a Great Variety of Problems of Different Kinds. … . Printed for J. Nourse, London.
Simpson, T., 1747. Elements of Plane Geometry. To which are Added, an Essay on the Maxima and Minima of Geometrical Quantities, and a Brief Treatise of Regular Solids;. … . Printed for the Author; Farrer; Turner, London.
Simpson, T., 1748a. Trigonometry, Plane and Spherical, with the Construction and Application of Logarithms. J. Nourse, London.
Simpson, T., 1748b. The motion of projectiles near the Earth's surface consider'd. Philosophical Transactions of the Royal Society 45 (486), 137–147.
Simpson, T., 1748c. Of the fluent of multinomials, and series affected by radical signs, which do not begin to converge till after the second term. Philosophical Transactions of the Royal Society 45 (487), 328–335.
Simpson, T., 1750. The Doctrine and Application of Fluxions: Containing (Besides What is Common on the Subject), a Number of New Improvements in the Theory, and the Solution of a Variety of New and Very Interesting Problems in Different Branches of the Mathematicks (2 vols.). For J. Nourse, London (2nd ed. 1776; 3rd ed. revised and corrected by W. Davis, 1805; 4th ed. revised and adapted, 1823).
Simpson, T., 1751–52. A general method for exhibiting the value of an algebraic expression involving several radical quantities in an infinite series: wherein Sir Isaac Newton's theorem for involving a binomial, with another of the same author, relating to the roots of equations, are demonstrated. Philosophical Transactions of the Royal Society 47, 20–27.

Simpson, T., 1752. Select Exercises for Young Proficients in the Mathematics. Containing.... For J. Nourse, London.
Simpson, T., 1755a. An investigation of a general rule for the resolution of isoperimetrical problems of all orders. Philosophical Transactions of the Royal Society 49 (1), 4–15.
Simpson, T., 1755b. A letter to the Right Honourable George Earl of Macclesfield, President of the Royal Society, on the advantage of taking the mean of a number of observations, in practical astronomy. Philosophical Transactions of the Royal Society 49 (1), 82–93.
Simpson, T., 1757a. Miscellaneous Tracts on Some Curious, and Very Interesting Subjects in Mechanics, Physical-Astronomy, and Speculative Mathematics.; .... For J. Nourse, London.
Simpson, T., 1757b. The resolution of a general proposition for determining the horary alteration of the position of the terrestrial equator, from the sun and the moon. Philosophical Transactions of the Royal Society 50 (1), 416–427.
Simpson, T., 1758a. A further attempt to facilitate the resolution of isoperimetrical problems. Philosophical Transactions of the Royal Society 50 (2), 623–631.
Simpson, T., 1758b. The invention of a general method for determining the sum of every 2d, 3d, 4th, or 5th, &c term of a series. Philosophical Transactions of the Royal Society 50 (2), 757–769.
Simpson, T., 1760. The Elements of Geometry, with their Application to the Mensuration of Superficies and Solids, to the Determination of the Maxima and Minima of Geometrical Quantities, and to the Construction of a Great Variety of Geometrical Problems. For J. Nourse, London.
Stone, E., 1721. Clavius's Commentary on the Sphericks of Theodosius Tripolite: or, Spherical Elements, Necessary in all Parts of Mathematicks, wherein the Nature of the Sphere is Considered. Made English by Edmund Stone. Printed for J. Senex, W. Taylor and J. Sifson, London.
Stone, E., 1723a. The Construction and Principal Uses of Mathematical Instruments. Translated from the French of M. Bion, Chief Instrument-Maker to the French King. To which are added, the construction and uses of such instruments as are omitted by M. Bion; particularly of those invented or improved by the English. Printed by H. W. for J. Senex and W. Taylor, London (2nd ed. 1758).
Stone, E., 1723b. An Analytick Treatise of Conick Sections, and their Use for Resolving of Equations in Determinate and Indeterminate Problems. Being the posthumous work of the Marquis de L'Hospital, Honorary Fellow of the Academy Royal of Sciences. Made English by E. Stone. Printed for J. Senex, W. Taylor, W. and J. Innys and J. Osborn, London.
Stone, E., 1724a. An Essay on Perspective. Written in French by William-James 's Gravesande... And now translated into English. Printed for J. Senex, W. Taylor; W. and J. Innys; J. Osborn; and E. Symon, London.
Stone, E., 1724b. Mathesis Enucleata: or, The Elements of the Mathematicks. To which is annexed, an introduction to specious analysis, or, algebra. The second edition, corrected and very much amended by E. Stone, etc. Midwinter and Taylor, London.
Stone, E., 1726a. A New Mathematical Dictionary: wherein is Contain'd, not only the Explanation of the Bare Terms, but likewise an History of the Rise, Progress, State, Properties, etc. of Things, both in Pure Mathematics and Natural Philosophy, so far as these last Come under a Mathematical Consideration. Printed for J. Senex, W. and J. Innys, J. Osborn, T. Longman, and T. Woodward, London. With a second edition with large additions, in 1743, printed for W. Innys, T. Woodward, T. Longman and M. Senex, London.
Stone, E., 1726b. The Elements of Physical and Geometrical Astronomy. By David Gregory. Done into English, with Additions and Corrections. The Second Edition. To which is annex'd Dr Halley's Synopsis of the Astronomy of Comets. The whole newly revised, and compared with the Latin, and corrected throughout, by Edmund Stone. In two volumes. Printed for D. Midwinter, London.
Stone, E., 1728. Euclid's Elements of Geometry, Briefly, yet Plainly Demonstrated by Edmund Stone, FRS. Printed for D. Midwinter and J. Osborn and T. Longman, London. A second volume in 1731; a second edition in 1765.
Stone, E., 1729. A New Treatise of the Construction and Use of the Sector. Containing, the Solution of the Principal Problems by that Admirable Instrument in the Chief Branches of Mathematics, viz. Arithmetick, Mensuration, Plain Trigonometry, Spherick Geometry, Projection for the Sphere, Geography, Astronomy, Dialling, &c. Being a work of the late Mr Samuel Cunn's teacher of mathematicks and now carefully revised by Edmund Stone. Printed for J. Wilcox and T. Heath, London.
Stone, E., 1730. The Method of Fluxions both Direct and Inverse. The Former being a Translation from the Celebrated Marquis de L'Hospital's Analyse des Infiniment Petits: and the Latter Supply'd by the Translator. For W. Innys, London. French translation by M. Rondet, 1735.

Stone, E., 1735. Geometrical Lectures: Explaining the Generation, Nature and Properties of Curve Lines. Read in the University of Cambridge, by Isaac Barrow, D. D. Translated from the Latin Edition, revised, corrected and amended by the late Sir Isaac Newton. Printed for S. Austen, London.
Stone, E., 1739. A Letter from Edmund Stone, F. R. S. to – concerning Two Species of Lines of the Third Order, Not Mentioned by Sir Isaac Newton, Nor Mr. Sterling. Philosophical Transactions of the Royal Society 41, 318–320.
Stone, E., 1743. The Theory of the Working of Ships, Applied to Practice. Containing the Principles and Rules for Sailing with the Greatest Advantage Possible. By Mons. Pitot, of the Royal Academy of Sciences at Paris. Translated from the French by Edmund Stone. Printed for C. Davis and P. Vaillant, London.
Stone, E., 1752. Euclid's Elements of Geometry, the first six, the eleventh and twelfth books. Translated into English, from Dr. Gregory's edition, with notes and additions. Printed for and sold by T. Payne, London.
Stone, E., 1766. Some Reflections on the Uncertainty of Many Astronomical and Geographical Positions, with Regard to the Figure and Magnitude of the Earth, the Finding the Longitude at Sea by Watches, and other Assertions of the Eminent Astronomers. . . . Printed for J. Marks, London.

Secondary literature

Aikin, J., 1799–1815. General Biography. In: Baillie and Sieveking (Eds.) (1984).
Albree, J., Brown, S.H., 2009. A valuable monument of mathematical genius: The Ladies' Diary (1704–1840). Historia Mathematica 36, 10–47.
Atto, C.H., 1938. The Society for the Encouragement of Learning. The Library. The Transactions of the Bibliographical Society XIX (3), 263–288.
Ausejo, E., Medrano, F.J., 2010. Construyendo la modernidad: nuevos datos y enfoques sobre la introducción del cálculo infinitesimal en España (1717–1787). Llull 33, 25–56.
Baillie, L., Sieveking, P. (Eds.), 1984. British Biographical Archive. A one-alphabet cumulation of 324 of the most important English-language biographical reference works originally published between 1601 and 1929 [microform]. K.G. Saur, London and New York. Accompanied by printed guide in 4 v. entitled: British biographical index (1990).
Barber, G., 2004. Nourse, John. In: Oxford Dictionary of National Biography. Oxford University Press, Oxford. http://www.oxforddnb.com/index/101050997/John-Nourse, accessed 22 March 2010.
Barker, A.D., 1981. Edward Cave, Samuel Johnson and the Gentleman's Magazine. University of Oxford, Oxford.
Berenguer, J., 2011. L'aportació de Tomàs Cerdà en la introducció del càlcul diferencial i integral a l'Espanya del segle XVIII. Master Thesis. Supervisor: M. Rosa Massa-Esteve. Available at http://www.recercat.net/handle/2072/203635.
Bertomeu Sánchez, J.R., García Belmar, A., Lundgren, A., Patiniotis, M., 2006. Introduction: Scientific and technological textbooks in the European periphery. Science & Education 15 (7–8), 657–665.
Blay, M., 1986. Deux moments de la critique du calcul infinitésimal. Michel Rolle et George Berkeley. Revue d'histoire des sciences 39, 223–253.
Bos, H., 1974. Differentials, higher-order differentials and the derivative in the Leibnizian calculus. Archive for the History of Exact Sciences 14, 1–90.
Bruneau, O., 2011. Colin Maclaurin ou l'obstination mathématicienne d'un newtonien. Presses Universitaires de Nancy, Nancy.
Burke, P., 2000. A Social History of Knowledge. From Gutenberg to Diderot. Blackwell, Oxford.
Cajori, F., 1919. A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse. Printed in Great Britain by Neill and Co., Ltd., Edinburgh. Reprinted: Classic Reprint Series. Forgotten Books. Amazon UK.
Cajori, F., 1928–1929. A History of Mathematical Notations, in two volumes. Open Court, Chicago. Reprinted: Dover Publications, New York (1993).
Carey, G.C., 1825. The Artisan, or Mechanic's Instructor. Printed for W. Cole, London.
Cassels, J.W.S., 1979. The Spitalfields Mathematical Society. Bulletin of the London Mathematical Society 11, 241–258.
Chalmers, A., 1812–1817. The General Biographical Dictionary. In: Baillie and Sieveking (Eds.) (1984).
Clarke, F.M., 1929. Thomas Simpson and his Times. Waverley Press (Baltimore), New York.

Costa, S., 2002. The "Ladies' Diary": Gender, Mathematics, and Civil Society in Early-Eighteenth-Century England. Osiris, 2nd Series 17, 49–73.

Crosland, M., 2005. Relationships between the Royal Society and the Académie des Sciences in the late eighteenth century. Notes and Records of the Royal Society of London 59, 25–34.

Davis, W., 1805. Life and Writings of Thomas Simpson. In: Simpson (1750, 3rd edition).

Desautels, A.R., 1971. Castel, Louis-Bertrand. In: Gillispie, C.C. (Ed.), Dictionary of Scientific Biography, 1970–1990, vol. 3. Charles Scribner's Sons, New York, pp. 114–115.

Feather, J.P., 1980. Book Prospectuses before 1801 in the John Johnson Collection, Bodleian Library, Oxford. A catalogue with microfiches. Oxford Microform Publications.

Feather, J., 1981. John Nourse and his authors. Studies in Bibliography 34, 206–227.

Feather, J., 1985. The Provincial Book Trade in Eighteenth-Century England. Cambridge University Press, Cambridge.

Feather, J., 1988. A History of British Publishing. Croom Helm, London (2nd ed. 2006).

Farebrother, R.W., 1990. Studies in the history of probability and statistics XLII. Further details of contacts between Boscovich and Simpson in June 1760. Biometrika 77 (2), 397–400.

Frasca-Spada, M., Jardine, N., 2000. Books and the Sciences in History. Cambridge University Press, Cambridge and New York.

Goodwin, G., 2004. Blake, Francis. Revised by J. Cross. In: Oxford Dictionary of National Biography. Oxford University Press, Oxford, http://www.oxforddnb.com/index/101002577/Francis-Blake, accessed 22 March 2010.

Gorton, J., 1841. A General Biographical Dictionary, new ed. In: Baillie and Sieveking (Eds.) (1984).

Gowing, R., 1983. Roger Cotes. Natural Philosopher. Cambridge University Press, Cambridge.

Grabiner, J.V., 1997. Was Newton's calculus a dead end? The Continental influence of Maclaurin's Treatise of Fluxions. American Mathematical Monthly 104 (5), 393–410.

Grabiner, J.V., 2002. Maclaurin and Newton: the Newtonian style and the authority of mathematics. In: Withers, C.W.J., Wood, P. (Eds.), Science and Medicine in the Scottish Enlightenment. Tuckwell Press, East Linton.

Grabiner, J.V., 2004. Newton, Maclaurin and the authority of mathematics. American Mathematical Monthly 111 (10), 841–852.

Guicciardini, N., 1989. The Development of Newtonian Calculus in Britain, 1700–1800. Cambridge University Press, Cambridge.

Guicciardini, N., 2004a. Simpson, Thomas. In: Oxford Dictionary of National Biography. Oxford University Press, Oxford, http://www.oxforddnb.com/index/101025594/Thomas-Simpson, accessed 22 March 2010.

Guicciardini, N., 2004b. Stone, Edmund. In: Oxford Dictionary of National Biography. Oxford University Press, Oxford, http://www.oxforddnb.com/index/101026567/Edmund-Stone, accessed 22 March 2010.

Guicciardini, N., 2009. Isaac Newton on Mathematical Certainty and Method. MIT Press, Cambridge, Massachussets.

Handover, P.M., 1960. Printing in London from 1476 to Modern Times. Ruskin House (George Allen & Unwin Ltd.), London.

Hay, C., 1988. Mathematics from Manuscript to Print 1300–1600. Clarendon Press, Oxford.

Heilbron, J.L., 1993. A mathematician's mutiny, with morals. In: Horwich, P. (Ed.), World Changes. Thomas Kuhn and the Nature of Science. MIT Press, Cambridge, pp. 81–129.

Hutton, C., 1795. A Mathematical and Philosophical Dictionary: Containing an Explanation of the Terms, and an Account of the Several Subjects, Comprized under the Heads of Mathematics, Astronomy, and Philosophy both Natural and Experimental: with an Historical Account of the Rise, Progress, and Prsent State of these Sciences... (2 vols.). By J. Davis for J. Johnson and G. G. and J. Robinson, London.

Johns, A., 1998. The Nature of the Book: Print and Knowledge in the Making. University of Chicago Press, Chicago and London.

Johns, A., 2002. Science and the book. In: Barnard, J., McKenzie, D.F., Bell, M. (Eds.), The Cambridge History of the Book in Britain (7 vols.), vol. IV. Cambridge University Press, Cambridge, pp. 274–303.

Johns, A., 2003. The ambivalence of authorship in early modern natural philosophy. In: Biagioli, M., Galison, P. (Eds.), Scientific Authorship: Credit and Intellectual Property in Science. Routledge, New York, pp. 67–90.

Kollerstrom, N., 1992. Thomas Simpson and 'Newton's method of approximation': an enduring myth. The British Journal for the History of Science 25 (3), 347–354.

Leybourn, T., 1817. The Mathematical Questions, Proposed in the Ladies' Diary, and their Original Answers, Together with some new Solutions, from its Commencement in the year 1704 to 1816 (4 vols.). Printed by W. Glendinning, London; and published by J. Mawman, J. Deighton and son, Cambridge; and J. Parker, Oxford.

McKenzie, D.F., Ross, J.C. (Eds.), 1968. A ledger of Charles Ackers, printer of the London Magazine. Oxford University Press for the Oxford Bibliographical Society, Oxford. New Series, vol. XV.

Plomer, H.R., 1922. A Dictionary of the Printers and Booksellers who were at Work in England, Scotland and Ireland from 1668 to 1725. In: Baillie and Sieveking (Eds.) (1984).

Plomer, H.R., Bushnell, G.H., Dix, E.R., 1932. A Dictionary of the Printers and Booksellers who were at Work in England, Scotland and Ireland from 1726 to 1775. Printed for the Bibliographical Society at the Oxford University Press, Oxford, 1932 (for 1930) (The Bibliographical Society, London, 1968, Facsim. ed.).

Rousseau, G.S., 1982. Science books and their readers in the eighteenth century. In: Rivers, I. (Ed.), Books and their Readers in Eighteenth-Century England. Leicester University Press, Leicester, pp. 197–255.

Shoesmith, E., 1985. Thomas Simpson and the arithmetic mean. Historia Mathematica 12, 352–355.

Simon, J., 2009. Circumventing the 'elusive quarries' of popular science: the communication and appropriation of Ganot's physics in nineteenth-century Britain. In: Papanelopoulou, F., Nieto-Galan, A., Perdiguero, E. (Eds.), Popularising Science and Technology in the European Periphery. Ashgate, Aldershot, pp. 1800–2000.

Sorrenson, R., 1996. Towards a history of the Royal Society in the eighteenth century. Notes and Records of the Royal Society of London 50, 29–46.

Stewart, L., 1999. Other centres of calculation, or, where the Royal Society didn't count: commerce, coffee-houses and natural philosophy in early modern London. The British Journal for the History of Science 32, 133–153.

Stigler, S.M., 1984. Studies in the history of probability and statistics. XL. Boscovich, Simpson and a 1760 manuscript note on fitting a linear relation. Biometrika 71 (3), 615–620.

Stoker, D., 1995. The "grammarians' battleground": controversies surrounding the publication of John Holmes' *Greek Grammar*, http://faculty.ed.uiuc.edu/westbury/Paradigm/STOKER.html, accessed 7 May 2012.

Taylor, E.G.R., 1966. The Mathematical Practitioners of Hanoverian England 1714–1840. For the Institute of Navigation at the University Press, Cambridge.

Wallis, R.V., Wallis, P.J., 1986. Biobibliography of British Mathematics and its Applications. Part II. 1701–1760. Epsilon Press, Newcastle upon Tyne.

Watt, R., 1824. Bibliotheca Britannica. In: Baillie and Sieveking (Eds.) (1984).

Weaver, E., 1746. The British telescope: being an ephemeris of the cœlestial motions. With an almanack for the year of our Lord 1746. Printed by T. Parker, for the Company of Stationers, London.

Weld, C.R., 1848. A History of the Royal Society with Memoirs of the Presidents (2 vols.). J.W. Parker, London (Thoemmes Press, Bristol, 2000, Facsim. ed.).

Manuscripts

Council Minutes (*Copy*) *of the Royal Society* (1663–1822), ref. no. CMC. Repository GB 117 The Royal Society.

Graves Scientific Papers, MS Graves 23 (2, 3), University College London (UCL) Special Collections.

Journal Book (*Copy*) *of the Royal Society* (1660–1826), ref. no. JBC. Repository GB 117 The Royal Society.

Mathematical Society Scrapbook, MS ADD 75, University College London (UCL) Special Collections.

Smith Historical Collection, Thomas Simpson, Catalogued Manuscripts and Papers, Rare Book & Manuscript Library, Columbia University.

Journals and periodicals

Journal Britannique. Edited by M. Maty. Chez H. Scheurleer, La Haye (1750–1757), http://gallica.bnf.fr/, accessed 21 May 2012.

Journal de Trévoux, ou Mémoires pour l'histoire des sciences et des beaux-arts. Imprimerie de S.A.S., Trévoux (1701–1767), http://gallica.bnf.fr/, accessed 21 May 2012.

Miscellanea curiosa mathematica: or, The literary correspondence of some eminent mathematicians in Great Britain and Ireland (vol. I, no. I–IX). Edited by F. Holliday. Printed for E. Cave, London (1749).

The Gentleman's Diary, or The Mathematical Repository. Printed for the Company of Stationers, London (1741–1840).
The Gentleman's Magazine: and historical chronicle, by Sylvanus Urban. Printed by E. Cave, London (1731–1833).
The Ladies' Diary: or, Woman's Almanack. Printed for the Company of Stationers, London (1704–1841).
The Monthly Review, or New Literary Journal. Printed for R. Griffiths, London (1749–1844).