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Perspective in Leibniz's invention of *Characteristica Geometrica*: The problem of Desargues' influence

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Abstract

During his whole life, Leibniz attempted to elaborate a new kind of geometry devoted to relations and not to magnitudes, based on space and situation, independent of shapes and quantities, and endowed with a symbolic calculus. Such a "geometric characteristic" shares some elements with the perspective geometry: they both are geometries of situational relations, founded in a transformation preserving some invariants, using infinity, and constituting a general method of knowledge. Hence, the aim of this paper is to determine the nature of the relation between Leibniz's new geometry and the works on perspective, namely Desargues' ones.

Résumé

Durant toute sa vie, Leibniz a cherché à élaborer une nouvelle géométrie, consacrée aux relations et non aux grandeurs : une géométrie de l'espace et de la situation, indépendante des figures et des quantités, et dotée d'un calcul symbolique. Une telle "caractéristique géométrique" partage certains éléments avec la géométrie perspective. En effet, toutes deux sont des géométries des relations situationnelles, fondées dans une transformation qui préserve des invariants, emploie l'infini et constitue une méthode générale de connaissance. Dès lors, le but de cet article est de déterminer la nature de la relation qui existe entre cette nouvelle géométrie leibnizienne et les travaux sur la perspective, notamment ceux de Desargues.

MSC: 01A45; 51-03

Keywords: Leibniz; Desargues; Perspective; Geometric characteristic; 17th century; Invention

1. Introduction

At the end of the 1670s, after he came back from Paris, where between 1672 and 1676 he greatly developed his knowledge of mathematics, Leibniz initiated his work on *Characteristica Geometrica*. His main purpose was to form a geometry of relations, without magnitudes, independent of figures, embedded

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in a complete axiomatic and endowed with an expressive symbolism. It was supposed to be both a calculus and a new geometry which would surpass the Cartesian one and include the Euclidean one. In order to elaborate such an innovative geometry restricted to qualitative relations, Leibniz developed a geometry of "space" and "situation."

Despite appearing to be a fundamental innovation, the Leibnizian geometry exposed in the first texts of *Characteristica Geometrica* seems to be close to the almost contemporary works of Desargues and Pascal concerning geometry in general and perspective in particular. Indeed, as a geometry of quality, anchored in the essentially relational concept of *situs*, in the notion of *extensum*, and in the notion of *congruentia* as a preserving transformation which, as such, is propitious to the knowledge of unvarying things, geometric characteristic resembles the perspective projection of Desargues and the geometry of conics of Pascal. This resemblance, together with explicit references to perspective by Leibniz (1675a, 359; 1677, 62), while he was forming the idea of his innovative geometry of situation, suggests the possibility of a relation between the two approaches to geometry. This possibility has been studied deeply by Javier Echeverría (1982, 1983, 1994), who presented the objective elements of Leibniz's knowledge of Desargues' works on perspective and concluded in favour of the real but limited "influence" of Arguesian[1] perspective on Leibniz's invention of *Characteristica Geometrica*.

In general, it is problematic to affirm that Leibniz discovered in Desargues (or in Pascal) the germ of his invention of *Characteristica Geometrica*. Usually Leibniz willingly admits what he owes to the theoreticians who influenced him in one way or another. But, in this matter, he never says, to my knowledge, that his ideas concerning geometric characteristic or parts of it come from Desargues' works, even though he has no reason whatsoever to hide his continuity with Desargues because of the obvious originality of his own work. Moreover, Leibniz frequently refers to Viète and Descartes whose algebraic method, according to him, needs to be overtaken by a new modern geometry. He also mentions Euclid whose *Elements* offer a criterion to evaluate the efficiency of his geometric characteristic. Within the framework of these references, we may wonder whether it is justified to claim the existence of a 'positive' inspiration of Desargues, rather than the existence of a 'negative' influence of Viète and Descartes, or of Euclid's methodological limitations. Nonetheless, the question of Desargues' influence is still interesting, not only in order to determine the effective degree of such an influence on Leibniz's invention of *Characteristica Geometrica*, but also so as not to miss a possible connection between perspective and geometric characteristic. My purpose is then to read the first texts of geometric characteristic in the light of Desargues' methods, above all to be able to delineate the specific originality of Leibniz and the very nature of his geometry. Indeed, the perspective method is certainly at least a model for Leibniz, his doctrine of science and knowledge, and his conception of space as related to *situs*. But, to find a theory to be a model is not to use it as a foundation nor, even, does it imply influence. As a matter of fact, while Leibniz was aware of the power resulting from the innovations of perspective – such as, for instance, the identification of the points at infinity and the points at a finite distance – and had the intellectual capacity to extrapolate them, including in his own new geometry, yet he did not. Hence, the question becomes: what kind of clues does such a lack give us for our understanding of Leibniz's ingenuity in geometry?

A substantial secondary literature about the relation between Leibniz and Desargues exists, namely in the French language. A first period (covering the 1960s and 1970s), depending on the edition of Desargues' texts, includes the fundamental works of Jean Mesnard (1978) and René Taton (1951, 1978). The important essay by Joseph E. Hofmann, *Leibniz in Paris. 1672–1676* (Hofmann, 1974) should be considered as part of this set of French commentaries for the precision and the completeness of its analysis.[2] A second

[1] In analogy with the term 'Cartesian,' I have chosen to use 'Arguesian' to refer to the ideas or elements coming from Desargues' posterity.

[2] I may also refer to the papers by Hans Freudenthal (1972), Herbert H. Knecht (1974) and Hans Peter Münzenmayer (1979) which offer different studies about the Leibnizian *Analysis situs* itself.

period of secondary literature (at the end of the 1980s and 1990s) is devoted to the invention of geometric characteristic. Javier Echeverría produced a crucial piece of transcription work and edition of the first essays on *Characteristica Geometrica* (1995). Graham Salomon's dissertation (Salomon, 1989) and the papers by G.G. Wallwitz (1991) or E. Giusti (1992) deal with some aspects of its genesis or its posterity.[3] Except for the previously cited papers by J. Echeverría about the relation between Leibniz and Desargues,[4] none of the previous contributions presents a detailed analysis of the relation between Leibniz and Desargues. A third period is currently beginning with the important work of Kirsti Andersen (2007) about perspective and an original reading of the later Leibnizian *Analysis Situs* by Vincenzo De Risi (2007). But, once again, the link between perspective and *Geometria Situs* is not the main topic.

Thus, my goal is to show that, while inventing his *Characteristica Geometrica*, even if Leibniz sometimes presents the perspective method as a model for his own project of geometric characteristic, or as some anticipatory applications of it, his invention of a new geometry is relative to prior requirements concerning general characteristic rather than to the geometrical innovations and specificities of perspective, although perspective contains some great inventions for the emergence of a qualitative geometry of situation and space. In order to establish the undeniable methodological and conceptual originality of Leibniz's work, not only despite the similarities between his and Desargues' conceptions, but also *thanks to the ever-meaningful incompleteness of these similarities*, I will present, in Section 2, the *Characteristica Geometrica* as it is exposed in the first formulations that Leibniz produced in 1677–1680. I will insist on the double nature of geometric characteristic as 'characteristic' and as 'geometry.' Then, in Section 3, I will elaborate on the nature of the continuity which might be asserted from Desargues to Leibniz. This will require specifying some philological difficulties and, depending on them, drawing as precisely as possible the path that goes from Arguesian perspective to Leibnizian invention. In Section 4, I will establish that this path is not sufficient to unconditionally link Leibniz to Desargues, since their shared ideas are not specifically related to Leibniz's knowledge of previous Arguesian works. They also depend on his own conceptions of geometrical methods and symbolic requirements in his earlier thoughts. Finally, in Section 5, I will draw conclusions about the possibility of others sources for Leibniz's invention of *Characteristica Geometrica*.

2. *Characteristica Geometrica* from 1677 to 1680

2.1. *The project of Characteristica Geometrica*

In January 1677, in Hannover, Leibniz wrote a text entitled *Characteristica Geometrica* (Leibniz, 1677), which begins with a clear explanation for the Leibnizian project of a new geometry:

> Analysis Geometrica nondum habetur absoluta. [...] Cogitavi dudum mederi huic imperfectioni, et efficere, ut in calculo tota figurae ratio situsque appareat, quod alioqui fieri non solet. Contenti enim sunt Analytici magnitudines ad calculum revocare situs vero in figura supponere, unde figuris et linerarum ductu atque imaginationis opera carere non possunt.[5]
>
> [Leibniz, 1677, 50–52]

[3] Around the same time (in the 90s), a new interest appeared in Desargues' work and the geometrical aspects of perspective, thanks to the English translation of the *Brouillon Project* (Field and Gray, 1987), the collective book *Desargues en son temps* (Dhombres and Sakarovitch, 1994), and some papers about the projective method (Bkouche, 1991; Le Goff, 1994). But, in none of these references, was the relation between the invention of *Characteristica Geometrica* and the perspective or projective method examined.

[4] Some details about that subject can also be found in an article on Desargues' posterity (Lanier and Le Goff, 1991).

[5] "A perfect Geometrical Analysis has not so far been achieved [...] I recently thought to amend this imperfection, and accomplish what is besides not habitual, so that all element concerning figure and situation appears in calculus. Indeed, analysts are satisfied with introducing magnitudes in calculus, while subsuming situation to figures, so that they cannot abstain from the drawing figures and lines, and from the labor of imagination."

In fact, Leibniz envisioned the possibility of a new geometrical method without the two main flaws in Cartesian geometry: the lack of completed analysis and the use of figures. He then strove to elaborate a geometry of *new* elements out of which a complete analysis could be elaborated. As such, this new geometry had to become characteristic, that is to say analytically founded, combinatorial, and symbolic. Thus, Leibniz's project appears doubly innovative: it is to include both a new "geometry" and an original "characteristic," as will be presented now.

2.1.1. A characteristic project as a heuristic method

The 1677 text clearly reveals the characteristic dimension of the *Characteristica Geometrica* project. Right from the beginning, the author suggests some rules for the use of characters: a point is designated by a letter A, B, a line is expressed by a formula ${}_1B_2B_3B$, and a straight line by an equation ${}_1B_2B + {}_2B_3B = {}_1B_3B$ (Leibniz, 1677, 52). Leibniz also gives several equations for other *loci*: a circle, an angle, and a right angle. Then, characters must "express" ("*exprimere*") the nature of objects, that is to say, in Leibnizian terms, the composition of the notional content of an object from more elementary notions. For example, the notion of 'line' is broken into "*multorum punctorum locus*" (Leibniz, 1677, 52), so that the notion of 'point' appears more primary than that of 'line.' Thus, in order to choose characters which are the most able to express the nature of geometrical objects, it is required to form the alphabet of the fundamental notions, which enables Leibniz to elaborate the set of corresponding characters. Nonetheless the relation between the characteristic expression and the geometrical object is not only a matter of translation from geometrical concepts into symbolic expressions. It is also and above all a method of knowledge by transposing the *relations* between geometrical objects into *relations* between signs in order to discover, through a simple calculus, something new about signs which corresponds to something intrinsic to objects themselves. For instance, to prove the similarity between two surfaces or sectors, it is sufficient to remark or establish the similarity between their expressions (Leibniz, 1677, 56–58). Indeed, similar objects are supposed to receive similar expressions, if these expressions are well-established and rightly represent the geometrical composition of objects. Reciprocally, any proof of the similarity of expressions is sufficient to prove the similarity of objects.

The heuristic power of characteristic implies a double invariance: the preservation of conceptual composition, i.e. of internal conceptual relations, from the objects into their expressions, and the preservation of external relations between geometrical objects in their corresponding formula. In any case, it suggests the preservation of relations. The question of preservation actually is the core of the characteristic method, since without such an invariance, nothing could be known, but without the incompleteness of preservation, expression and the thing expressed could not be distinguished. Thus, the issue is to delineate what has to be preserved from the object into its expression. That implies geometrically defining objects, in their internal but also relational nature, with regard to the necessity of invariance; establishing the modality of their external relations according to some invariant elements; and setting the rules of calculation, i.e. of the passage from an expression into another, without losing the invariant elements in and between objects. All these elements concern the geometrical part of *Characteristica Geometrica*, as will be developed in the following section.

2.1.2. A new, tentative geometry

As far as the "geometric" aspect is concerned, the 1677 essay is not really completed, or even deeply developed. Nonetheless, it shows some hesitancy which reveals what Leibniz considers to be important.[6] Indeed, he defines the line as the *locus* of many points (Leibniz, 1677, 52), and he later corrects himself, when he mentions the straight line but without any specific consideration of the property of straightness:

[6] The consequences of this point will be developed in Section 4.2.1.

> Optime autem omnia per motum ita explicabimus. Puncti D motu fit recta $_1D_2D_3D$ [...]. Rectae $_1D_2D_3D$ motu fit supercies $_1D_2D_3D$, $_{21}D_{22}D_{23}D$.[7]
>
> [Leibniz, 1677, 56]

Leibniz seems to prefer a definition of lines through motion rather than a definition of lines as the *locus* of many points. Even if it is not an absolute and definite choice, the fact is that Leibniz makes a difference between a static *locus* of points and a dynamic conception of a single point moving and producing a line, which can be extended to the motion of a line generating a surface. The potential of generalization obviously represents the main advantage of such a conception. Another important advantage is implicit into the second definition: the dynamic explanation of the line as the motion of a point which successively occupies several places or punctual *loci* suggests the idea of continuity – which prevents most difficulties pertaining to infinity. Lastly, these definitions are not inconsistent. Indeed, they admit the same characteristic expression: $_1D_2D_3D$. As a set of points, the expression of a line means that one point is $_1D$ at a determined place, another is $_2D$ at another determined place, and a third is $_3D$ at a third determined place, i.e. the various indices of the letter D designate various points, and the uniqueness of the letter reveals the uniqueness of the geometrical nature of objects: they all are points. In the case of the definition through motion, D represents the uniqueness (or invariance) of the point itself, and the indices express the plurality (or variation) of the places occupied by this point. Once again, the consistency of the characteristic expressions is sufficient to assert the equivalence of these two definitions of line. However, choosing a particular definition for the line is not irrelevant, since, and perhaps because of the equivalence between definitions, the choice of such a definition shows some epistemological and conceptual priorities, and gives some crucial clues to understanding the invention of geometric characteristic, as it is presented in the 1679 essays.

Indeed, the relation between the motion of a point and the invariance of some geometrical properties has two implications. The first is the way in which the concepts of motion and preservation are connected in geometry: motion generates a figure and preservation determines the nature of this figure. The second is the nature of the preserved object itself: it deals with a situation, which is not really and explicitly defined in the 1677 text, even if it already signifies some kind of *mutual relation between loci of points* (Leibniz, 1677, 64). Then, in his 1679 essays, Leibniz fills in, or tries to fill in, the gap by structuring his geometric characteristic around the relational notion of *situs*.

2.2. Geometric characteristic as a geometry of situation

2.2.1. Space and point: the problem of the notional alphabet[8]

To specify the conceptual content of geometric characteristic, the main key is the constitution of the notional alphabet, insofar as it sheds light on the novelty of the geometrical theory described by Leibniz. The major texts about this topic are the essay written on August 10th 1679 (Leibniz, 1679a), the Latin draft of the letter to Huygens from September 1679 (Leibniz, 1679b), the letter to Huygens from September 18th 1679 (Leibniz, 1679c) and the text written in 1680, *De primis Geometriae Elementis* (Leibniz, 1680). Leibniz's geometry is then presented as a geometry of "*extensio*" and "*situs*": "*In Geometria duo sunt consideranda in summa, extensio et situs*"[9] (Leibniz, 1680, 276). This assertion can be compared with the first words of the long August 10th 1679 essay:

[7] "But we will best explain everything through motion. The motion of the point D generates/produces the straight line $_1D_2D_3D$ [...]. The motion of the straight line $_1D_2D_3D$ generates/produces the surface $_1D_2D_3D$, $_{21}D_{22}D_{23}D$."

[8] The issue of the definitions of points and space will be further considered in Sections 4.1.2 and 4.2.1.

[9] "In Geometry, only two things have to be considered: *extensio* and *situs*."

> Verum ut omnia ordine tractemus sciendum est primam esse considerationem ipsius Spatii, id est Extensi puri absoluti.[10]
>
> [Leibniz, 1679a, 150]

Thus, the most fundamental element of geometric characteristic seems to be the notion of space as a kind of *extensum* which is not physical, that is to say not defined as containing material objects or as material itself. It is also motionless, so that any motion in it is not a motion of it, a modification of its nature or of its figure (Leibniz, 1679a, 150). It is an absolute *extensum* containing any *extensio*, i.e. any extended object, without being itself a determined extended object. It is therefore unlimited, boundless, infinite: it has no particular figure of its own but contains any given figure. Defined as an absolutely unlimited *extensum* containing any *extensio*, the notion of space leads by analogy to that of point as a specific *extensio*: the simplest, the most limited, so limited that the point is without any magnitude or delineated *extensio* (Leibniz, 1679a, 152). The point is not extended, but neither is it 'nothing' in space. More precisely, its simplicity is defined as a minimum of extension, even as an 'unextension' and, consequently, as the lack of any parts in it. Thus, the only way to determine a point is to consider that it "is in space" without any other spatial quality (quality of extension), that is to say that a point can only be determined by its position in space, by its *situs*.

This parallelism between space and point certainly shows some hesitancy on Leibniz's part in elaborating the notional alphabet of its new geometry,[11] but most of all some tendency to revisit the two classic notions of space and point. Indeed, in *De Primis Geometriae Elementis* (Leibniz, 1680, 278), Leibniz explains that space and point possess a kind of geometrical simplicity, which also is an analytical elementariness. Indeed, as absolutely extended, space is the least determined extended *locus* and the primary notion required for the definition of any other extended *locus*. As purely situated, a point is no *locus* but the simplest geometrical object and the primary notion required for the definition of any geometrical object considered as a combination of points or of the mutual positions of points (situations). Then, when combination is considered, that is to say when the issue is to use characteristics and to calculate, points then necessarily appear as the elements of geometric characteristic. But such a conception misses the fact that, if geometry is characteristical, characteristics have above all to become geometrical. In other words, the combinatorial nature of characteristics is not only reconcilable with geometry, but essential to it. The idea is, then, to interpret the conceptual parallelism between the simplicity of space as unsituated but extended, and the simplicity of a point as unextended but situated, as a sign of the Leibnizian conception of *Characteristica Geometrica*: a geometry of space, of course, but of a space conceived as combinatorial, that is to say, as relational.

In this context, the various attempts at determining the best alphabet possible are also attempts at correlating the classic conceptions of space and point to the more modern relational concepts of 'extension' and 'situation.' Such attempts require the passage from unextended points to purely extended space, from position to extension, and from discrete punctual elements to continuous combined ones. To solve such a paradox, Leibniz refers to situation as the main notion of *Characteristica Geometrica*.

2.2.2. Situation as the core of geometric characteristic

Space and points can be understood, or defined, via one another. Then, the concept of points considered through the notion of space (as containing any situation) means that a point is the geometric object which is nothing but what "is in space," what has a situation. But this is not so easy to understand the other way

[10] "Truly, in order to treat everything in order, one should know that the first thing is the consideration of Space itself, i.e. pure absolute *Extensum*."

[11] In 1679, some texts begin with points (Leibniz, 1679d, 72–74; 1679e, 246; 1679f, 82), whereas others firstly consider space (Leibniz, 1679g, 94; 1679h, 138).

round in terms of how space can, sometimes, be considered from unextended points. There, the notion of congruence is crucial.

Any point is necessarily similar to another, in other words, points are all congruent in space (Leibniz, 1679a, 152). Congruence is defined as the possibility of coincidence, i.e. of geometrical identity, and space appears as the *locus* of all these necessarily congruent points (Leibniz, 1679b, 240). The consequence is that points do not have any situation as such, in the sense that they are certainly endowed with a situation, but this situation does not belong to them: as congruent, they are also spatially indifferent. The only way to determine a point as ‘this’ point and not any other is to consider or perceive simultaneously its many relations to other points. Its situation is then always mutual or “relational:” “*relatio loci vel situs*” (Leibniz, 1679a, 152). Then, even if Leibniz considers that it is helpful to conceive space as the *locus* of all points (Leibniz, 1679a, 162), he also more or less explicitly admits that space is not reducible or subsumed to the notion of point: space is rather considered as an unlimited *continuum*, containing in itself the possibility of ‘co-perceived’ objects, identical to one another, and distinguished only by their mutual situation (Leibniz, 1680, 276). This integrates the modality of distinctness between identical objects, “congruence,” in its relation to situation (Leibniz, 1679a, 184). Indeed, the notions of situation and congruence are correlated: without congruent objects, differences between geometrical objects could be entirely determined by means of figures or quantities, without any consideration for their mutual positions. But, as they are identical in form and magnitude, congruent objects reveal the very nature of geometrical objects as purely spatial objects, that is to say as situated objects in space. Consequently, space can be conceived in itself as the geometric object which contains an infinity of congruent objects, themselves congruent to an infinity of objects (Leibniz, 1679b, 204).

Furthermore, in his essay from August 1679, Leibniz explicitly links congruence, motion, and mutual situation, in the framework of his claim that space contains an infinity of objects all congruent to a given one (Leibniz, 1679a, 204–206). Mutual situation is presented as what is preserved in motion: the traditional possibility of the superposition of congruent objects is then replaced by the possibility of the motion of a determined, mutual situation of several points from one *locus* in space to another *locus* “*servata forma*” (Leibniz, 1679a, 204). And such a possibility always exists thanks to the nature of space as an absolute *extensum* containing all the possible *extensa* and, for any given one, all its congruent *extensa*, only distinguished from one another by co-perception. Thus, thanks to congruence, any situation, punctual or mutual, can move without any intrinsic change, so that the motion produces a trajectory (“*via*”) which is itself a geometrical object, determined only by congruence and situation. Indeed, Leibniz defines trajectory as “*locus continuus succesivus*” (Leibniz, 1679a, 162), that is to say as an extended (“*continuus succesivus*”) geometrical object (“*locus*”). The motion of a point produces a line, that of a line produces a surface, that of a surface a solid, because these motions are continuous. And they are continuous because anywhere in space a geometrical object meets a congruent object (Leibniz, 1679a, 228–230).

Hence, space equates to the *locus* of all points, of all continuous trajectories of these points, and of all the continuous trajectories of these trajectories. It is also the locus of any punctual situation, of any mutual situation, and of any of their combinations. Through the notion of congruence, it is then possible to reconcile both conceptions of trajectories and situations, that is to say to finally reconcile the completeness of space and the infinity of discrete points. Indeed, by determining a certain invariance of situations in motion, congruence is both the constitutive element of geometrical objects as constituted by mutually situated congruent points, and the element founding the possibility of a relation between geometrical objects. It implies that situation is simultaneously the fixed situational position of points and the successive incidence of points with several continuous places. It is, above all, the conceptual element which enables one to think the relation between space and situation, although Leibniz does not explicitly define space as the “order” of these situations,[12] and only relates the concept of geometrical space to the notions of motion, invariance, and situational relations. Lastly, the concept of congruence implies identity and plurality, continuity and multiplicity, invariance and transformation, incidence and motion: all these elements can also be found in perspective. This now allows me to address the place of Leibniz's reading of the works on central projection in his discovery of geometric characteristic.

3. From Desargues to Leibniz

In his paper "*Leibniz, interprète de Desargues*," J. Echeverría prepares the ground for analyzing the signs of a continuity from Desargues to Leibniz.[13] The author is cautious, since he defines this continuity as an "interpretation" of Desargues by Leibniz, more than an "influence." Several reasons, mostly philological, justify such a circumspection, but there certainly is a perspective element that Leibniz recognizes as a fundamental one, as the letter to Gallois written in 1675 shows:

> Car Messieurs des Argues et Pascal ont fort bien fait de prendre les ordonnées generalement par des lignes convergentes ou paralleles, d'autant plus que les paralleles peuvent estre prises pour une espece de convergentes, dont le point de concours est éloigné infiniment.[14]
>
> [Leibniz, 1675a, 359]

In this letter, Leibniz's main purpose is his own method of metamorphosis in mathematics, which he used to calculate the quadrature of the circle in 1673. He obtained this quadrature by dividing the area under the circle, not into contiguous infinitesimal rectangles, but into contiguous infinitesimal triangles, all having the same apex.[15] His aim is to show Gallois that a new kind of ordinate system can be more advantageous than the Cartesian system of parallel ordinates. Thus, the reference to Desargues and Pascal designates two different elements. First, Leibniz is interested in the idea of a new way of ordering space, even if it is not yet explicit that something like a spatial order appears here. Secondly, the quotation reveals how easily and spontaneously Leibniz understands the identification of parallel lines with those that intersect, by means of an infinitely distant point. At the least, it is obvious here that Leibniz regards the integration of the notions of infinitely big and infinitely small into mathematics as progress, just as Desargues and Pascal did before him.

Nevertheless, these elements are not sufficient to prove that Leibniz is influenced by Desargues, or even simply interprets him. They suggest rather that Leibniz has his own mathematical ideas and considers Desargues' works as some particularly interesting illustrations of his geometrical project. Thus, I will now present Desargues' ideas, not by beginning with the *Brouillon Project*, but rather just like Leibniz himself, by reading his disciples. Then, I will examine the *Brouillon Project*, in order to determine whether Leibniz was interested in the most Arguesian elements or only in some particular aspects of the conceptions of Desargues' disciples.

3.1. The conditions of Leibniz's discovery of Arguesian methods

Although it has not been proven that Leibniz directly read Desargues' main text, *Brouillon Project d'une atteinte aux événements des rencontres du cône avec un plan* (Desargues, 1639), one may claim that Leibniz

[12] Leibniz will later more explicitly write: "*Spatium est ordo coexistendi seu ordo existendi inter ea quae sunt simul*" (Leibniz, 1714–1715, 18). ("Space is the order of coexistence, that is to say, the order of existence between things which are simultaneously.")
[13] According to J. Echeverría (1994, 283), Leibniz is mostly interested in perspective scales.
[14] "For Messieurs des Argues and Pascal have done very well to generally take ordinates by means of convergent or parallel lines, all the more so as parallels can be considered as a kind of convergent lines, the point of intersection of which is infinitely distant."
[15] See Debuiche (2009, 97).

knew about Desargues, his book, his concepts and, most of all, his methods. Indeed, during his Parisian stay, Leibniz was connected to Desargues: he knew Carcavy, who was a friend of Pascal's and defended the perspective methods (Taton, 1978, 109), and he read the book by Philippe de La Hire, *Nouvelle méthode en géométrie*... (La Hire, 1673), inspired by Desargues. Besides, he later annotated the beginning of *Maniere universelle de Mr Desargues* by Abraham Bosse (1648), in which some texts of Desargues himself can be found: some passages from the *Livret de perspective adressé aux Théoriciens* (1643, nowadays lost), a reprint of an eleven-page essay from 1636 entitled *Exemple de l'une des manieres universelles du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autres nature, qui soit hors du champ de l'ouvrage* (Desargues, 1636), and three geometrical propositions from 1648 (Taton, 1951, 206–212). All this shows that Leibniz was rather well informed about the network of perspectivists,[16] but also that his access to Arguesian concepts was, perhaps entirely, indirect, determined by the reception of Desargues' ideas, namely by La Hire, Bosse, and also Pascal (1640, 1654, 1657). Hence, if Desargues' perspective had any influence on Leibniz's invention of *Characteristica Geometrica*, it is probably mostly by means of his knowledge of the posterity of Desargues' works, but not by a personal and complete reading of them.

Leibniz however had Desargues in mind, when he wrote his essay in 1677 and linked, explicitly, his work about a geometry as "geometrical analysis" and the method of Desargues in his *Brouillon Project*: "*Et ut fecit Desargesius nova aptaque nomina excogitare utile erit, quo facilius sit accurate sine figuris ratiocinari. Et hic verus est quoque technicorum nominum usus*"[17] (Leibniz, 1677, 62). Such a reference to Desargues' neologisms could give the impression that Leibniz had read the *Brouillon Project*, but this is certainly not sufficient to prove that he did. Moreover, the previous quotation reveals a lack of knowledge of the *Brouillon Project*, rather than a precise interpretation of it. Indeed, Leibniz seems to think that Desargues' lexicon is a good example for his own demands of notional analysis and characteristical terminology. In that perspective, primary terms always have to be judiciously chosen and, according to Leibniz, the same should hold as far as the neologisms of Desargues' *Brouillon Project* are concerned. Yet, the 1677 passage reveals rather that Leibniz does not really know the *Brouillon Project* and its core, because Desargues devotes most of the first pages of his book to the relation of involution. Thus, by taking the Desargues' method as a model for a new method of invention in geometry, while Desargues' book is focused on the proportions of magnitudes, Leibniz does not make such a good choice, since his geometry could be considered less innovative than it claims to be. Besides, Desargues' lexical innovations are not correlated to the thinking of the concept of an *ars inveniendi* founded in a geometrical analysis, while Leibniz seems to consider that Desargues' production of neologisms is similar to, or reconcilable with, his own approach. Finally, Tschirnhaus's remark to Leibniz, in his letter of April 1678, corroborates the idea of Leibniz's partial, imperfect or superficial knowledge of Desargues' work in 1677:

> Tandem ut ad ea revertar quae loqueris de lingua Philosophica, ac alijs similibus non utique haec percipio; nec quoque quod dicis de lingua quadam Geometrica, qua Dom. Desargues subtilissimas ratiocinationes

[16] For example, in 1675, he annotated the letter from Oldenburg from April 1673 (Oldenburg, 1673), and he summarized: "*Desargues et Pascalii conica. Censura Huretii contra Desargues*" (Leibniz, 1675b, 75). That proves that Leibniz had heard something about the criticism by Grégoire Huret, the author of *Optique de portraiture et peinture* (Huret, 1670), and about the methods of Desargues and Bosse. He also knew the polemical texts of Dubreuil and Aleaume, which he has annotated (Echeverría, 1994, 283–285). Lastly, in a crossed-out passage of his letter to Étienne Périer from August 1676, he mentioned Desargues, Pascal, Bosse, and implicitly La Hire (Leibniz, 1676a, 591).

[17] "And, as Desargues did, it will be useful to contrive new and appropriate names, so that it will be easier to reason accurately without figures. And such is also the true use of technical names."

> instituit sine figuris et calculo, nunquam sanè haec vidi, nisi quae de Sectionibus habet conicis perpulchra, sed quae aliquo modò imaginationem fatigant.[18]
>
> [Tschirnhaus, 1678, 406]

Besides, some *marginalia* in his specimen of Bosse's *Maniere universelle de Mr Desargues* show that Leibniz changed his mind, and no longer considered Desargues' method as a good model for an *ars inveniendi*:

> Cette méthode n'est pas assés propre à éclairer l'esprit, parce qu'elle ne nous fait connoistre qu'à la fin les raisons pourquoy l'auteur nous mene comme cela. Elle n'est pas si propre à l'invention mais elle a l'avantage de surprendre les lecteurs quand ils se trouuent menés à quelque chose sans y penser: et on retient mieux les choses qu'on admire.[19]
>
> [Leibniz, 1673–1676, 210]

Indeed, by reading Bosse's text which is almost totally inspired by Desargues, and even partially supervised by him, Leibniz recognizes that the projective method does not fulfill the promises that he hoped to find in it.

Therefore, in order to delineate the relation between Leibniz and Desargues as thoroughly as possible and to suggest a justified exegesis of the signs of this relation, one must analyse Bosse's book.[20] One should also consider Desargues' *Brouillon Project* itself, even if Leibniz had not read it, but in order to have an accurate knowledge of the conceptual background of Bosse's perspective. In fact, such a background is also shared by Leibniz in his discovery of the perspective projection, because of the deep influence of the master on his disciples.

3.2. Manière Universelle de Monsieur Desargues by Abraham Bosse (1648)

3.2.1. Bosse's essay: A partially interesting reading for Leibniz

Bosse's essay is composed of a series of sections: a first text written by Desargues himself and entitled "*Reconnaissance de Monsieur Desargues*," the "*Avant-Propos*," the "*Advertissement*," the introduction entitled "*Du particulier de ce traité*," and a series of fourteen chapters constituting the first part. There follow 124 commentated plates which take up the unillustrated discourse of the chapters. The second part is devoted to the theory of shadows. Leibniz annotated the first hundred pages of Bosse's essay (up to the page numbered 86), that is to say the preliminary texts, the first part and about forty plates, but not the second part (Leibniz, 1673–1676). As Bosse's essay is accessible, it is possible to read it to grasp the elements

[18] "Finally, in order to turn to what you say about the philosophical language and other similar things, I do not understand it; and I also do not understand concerning what you say about a certain geometrical language that M. Desargues instituted by means of a very subtle reasoning without using any figure or calculus; really, I have never seen that, except for the very beautiful things he has concerning conic sections, which however weary somewhat the imagination."

[19] "This method is not suitable enough to enlighten the mind, because only at the end does it makes one aware of the reasons why the author leads us in such a way. It is not that suitable for invention but has the advantage to surprise readers when they are led to find something without thinking about it: and one remembers better the things one admires."

[20] The presented elements will be helpful in Sections 3.3.1 and 4.2.1 in order to compare the transformation methods in perspective and Leibniz's geometric characteristic.

Besides, I have chosen not to present details about La Hire's *Nouvelle méthode en géométrie*... (La Hire, 1673), for I do not think that this essay could have inspired Leibniz. The reading of La Hire probably did not arouse such enthusiasm as was typical of Leibniz when he discovered some new and fruitful ideas. In fact, Desargues' innovation about finite and infinite is not present in La Hire's essay, nor is a new conception of space or geometry. Except for the possibility and the modality of a generalization of simple demonstrations about circles to the less simple conics, there is no great idea in this text. But a similar idea was already very well known by Leibniz, thanks to a letter from Oldenburg (1673).

likely to interest Leibniz for his own invention of a calculus of situation and his rather innovative use of the concept of space.

In the introduction "*Du particulier de ce traité*," Bosse claims that one of Desargues' most relevant inventions is that of perspective scales ("*échelles de perspective*"), the first of the "*quatre choses qui sont purement la decouverte ou Invention de l'Autheur*" (Bosse, 1648, 18).[21] The main idea of the Arguesian work consists in unifying, first the objects of perspective art by proposing a unique method for the art of lines and the art of shadows (Bosse, 1648, 17), secondly the methods used in the art of representing a three-dimensional object: the method of perspective representation and the method of "geometral" representation, that is to say graphical representation as used in architecture.[22] The goal of Bosse's book is then to show how it is possible to use these two methods as one (Bosse, 1648, 26). But Bosse does not give any demonstration of such a universal method for representation and, above all, he does not intend to elaborate a treatise of geometry which is Desargues' prerogative (Bosse, 1648, 5). More precisely, in these first hundred pages, a great variety of details contain some aspects likely to stimulate Leibniz. For instance, in the "*Reconnaissance of Monsieur Desargues*," written on October 1st 1647, Desargues himself presents the two goals of his geometry: first, the development of a general method applicable to several arts such as portraiture, sundials or the cut of stones in architecture, and secondly the foundation of the certainty and ease of such a method on its geometrical nature. These two elements resonate with Leibniz's project of a "general science," of which any other discipline would be a specimen, and with his postulate of the necessity for a method to be elegant (as a mark of its rationality) and easy (as a condition of its reliability) (Leibniz, 1679a, 148–150).

More elements common to Leibniz and Bosse (and Desargues through him) can be found in the "*Avant-Propos*," referring to methodological aspects, such as the necessity of fundamental theoretical rules and of the clarity of terms. But Leibniz had already thought of those elements when he read Bosse, so that it is difficult to assert that his reading of Bosse influenced him. In that perspective, the fact that Leibniz read Bosse's text incompletely could be justified. If Leibniz can only approve of Desargues' method when it purges language of its ambiguity, he cannot agree with the use of shapes as Bosse presents it in the "*Advertissement*" (Bosse, 1648, 12). Bosse's goal is to make its discourse accessible to theoretical geometers as much as to practical ones and to workers. Then he chooses to alternate between shortened and developed discourses, complete and successive figures, in order to offer any keys that the reader might need to apply the new method. Thus, shapes belong entirely to the process of understanding of the rules of the method, and it is quite likely that Leibniz immediately disapproved of such a use and, consequently, began to despise Bosse's book and through it, Desargues' method, for his own geometry has to be conducted without drawn figures.[23]

For these reasons, if there is an influence of Bosse and Desargues on Leibniz, it cannot pertain to the role of shapes or the use of terms for reasoning. Nonetheless, in chapter 4 (Bosse, 1648, 27–28) and also in the followings sections, Bosse presents the use of scales as a new method, in which Leibniz seems very interested (Leibniz, 1673–1676, 222–225) and which seems similar to the Leibnizian thought concerning the geometric knowledge of things in space, of their mutual situations and their figures.

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21 "[...] four things which are purely the discovery or the invention of the author."

22 Because of the lack of a suitable translation of the French term, I have chosen to remain "geometral" which will always refer to this kind of graphical and architectural representation in breadth, length, and depth.

23 Leibniz underlines in Bosse's text (Bosse, 1648, 37): "[...] *que quand vous aurez apris les regles de la perspectiue*, [...] *si vous ne voulez, de la regle, et du compas*" ("[...] when you have learnt the rules of perspective [...] you will not have to use a straightedge or a compass any longer, if you do not want to") (Leibniz, 1673–1676, 210).

3.2.2. The main role of the method of scales in perspective representation

Bosse clarifies the differences between the "geometral" representation of an object and its perspective representation. The first consists in the determination of the three standard figures of the object, corresponding to breadth, length, and depth, or, in other words, to the three planes of "*assiette*," "*profil*," and "*eslevation*." Synthesizing these three graphical figures brings out a representation in relief which does not depend on the point of view, that is to say on the position of the eye. In contrast, perspective representation depends on vision and, more precisely, on the position of the eye in relation to the object, but also to the representation of the object (that is to say to the plane of projection).[24] Among perspective scales, a first one, called "*échelle des éloignements*" ("scale of removals"), gives the unit of measure in relation to the object distance from the picture (determined by a perpendicular direction). It represents the depth of perception and gives the impression of a third dimension. The second one, called "*échelle des mesures*" ("scale of measures") gives the unit of measure of segments parallel to the landline but separated from it. The "geometral" scale is also determined by two series of ordinates but only one unit of measure. Thus, the "*maniere*" of Desargues lies in the correspondence between these two representations: the geometral one which takes into consideration the relief of the object and parts of it, and the perspective one which paradoxically reproduces this relief on a two-dimensional plane. Desargues' idea, taken up by Bosse, consists in giving advantage to the graphical method, called the "*petit pied geometral*" method, over the perspective method:

> Il est absolument impossible de venir jamais à bien posseder la pratique du traict de la perspective, si l'on ne possede bien cette sorte de pratique de traict du geometral.[25]
>
> [Bosse, 1648, no page]

Such a preference can be analyzed as the affirmation of the preeminence of relief on plane, or of the deeply spatial nature of geometrical objects on their two-dimensional representation. Indeed, the "*petit pied géométral*" method does not deal with the objects in themselves, but with any object as determined by its mutual relations to the other objects or/and by the mutual relations of its parts to one another in the three dimensions of "*assiette*," "*profil*," and "*eslevation*." This method, essentially based on relative configurations, necessarily supposes the concept of a three-dimensional space in which objects are connected to one another through their positions.[26] "Geometral" scale is then what determines the size of representation: it consists in a ratio between objects and their representations, so that geometral representations are similar to objects, since they have the same shape, but increased or reduced according to a unit of measure (which is not necessarily quantified). As such, "geometral" scale can be thought of as what maintains a kind of order: the dispositional order of the spatial relations of objects. On the other hand, perspective representation seems quite different. In fact, perspective scale is unproportional and the representing figure is not (always) similar to the figure of the represented object: "*une mesme grandeur ne sert pas à la mesurer*

[24] The complete perspective representation is constituted thanks to the correspondence between the "geometral" scale and the two perspective scales. See Andersen (2007, 427–447), Bkouche (1991), and Le Goff (1994).

[25] "It is not possible to ever master the practice of tracing in perspective if one has not mastered this kind of practice of tracing in the 'geometral'."

[26] Bosse points out that painters easily apply perspective rules to objects such as a pavement or a building of which they intuitively have "geometral" knowledge, whereas they abandon the perspective demands for representing portraits (Bosse, 1648, 40). Thus, if painters renounce perspective representation in the cases of objects whose "geometral" dimensions cannot be determined, that strengthens the prevalence of "geometral" considerations over merely perspective ones. As a result, perspective representation seems to be strongly linked to the idea of the spatial nature of objects, since an object cannot be represented by perspective if it cannot be grasped through the "geometral" relations of its own parts.

semblablement en tout sens d'un bout à l'autre" (Bosse, 1648, 48).[27] But in some respects the perspective representation and the "geometral" one are seen to have a common nature.

First, perspective representation also takes into consideration all spatial dimensions, even if it is a two-dimensional representation. Indeed, perspective scale is determined by the three magnitudes of the eye position in relation to the plane of representation (its height, its distance to the plane and its "*décalage*" or gap in relation to the perpendicular to the plane).[28] Thus, the failure of preservation operating in perspective scale depends on the eye position in relation to the projection plane and to the object: it is determined by a certain direction in relation to the eye position (Bosse, 1648, 55–56). As such, it recalls Desargues' work on conics considered as perspective representation of a circle on a projection plane. In this work a geometrical analogy between parameters of the eye (the vertex of the cone), of the projection (the position of the plane), and of the object itself (the conic), expressed by the rules of perspective projection, is established. This leads to the second common point between "geometral" and perspective methods: despite the inequality, a correspondence which cannot be rigorously considered as similarity exists between the object and its perspective representation. Bosse writes:

> Ce n'est pas qu'une telle representation en perspective ne tienne aussi quelque chose de la mesme forme du sujet qu'elle represente, & qu'elle ne se mesure de mesme que la sorte qu'on nomme geometrale; ainsi qu'il sera montré dans son lieu.[29]
>
> [Bosse, 1648, 48]

Having the "*mesme forme*" does not mean of course that there is only a size difference, which is similarity. It reveals something more theoretical, not only methodological or simply representational, connected to the correspondence between "geometral" and perspective representations, by means of a relation between "geometral" and perspective scales (Bosse, 1648, 41–42). Indeed, if the parameters of both scales are given, knowing perspective representation enables one to trace back to "geometral" representation. Thus, plane perspective representation is sufficient to express the spatial relations or figures of the represented objects. But, in that case, perspective representation is sufficient only because "geometral" representation is contained in it, so that the latter appears both as the most fundamental representation (for it is the criterion for the quality of perspective representation which has to enable the geometer to find it again), and as a representation which shares some common nature with perspective representation. In fact, the correspondence between the two scales expresses a deep identity between the "geometral" method and perspective representation, and therefore a deep identity of nature between three-dimensional and two-dimensional representation: "*ce sont deux especes d'un seul & mesme genre, ou si vous voulez deux cas divers d'une seule proposition de geometrie*" (Bosse, 1648, 52–53).[30]

Two conclusions may be drawn. First, the "geometral" method seems to be the most general case of perspective: the case where the parameters of the distance between the eye and the plane of projection, and of its gap in relation to the perpendicular are not taken into account. Such a process would not be scenographic, but an ichnography in three dimensions. It is totally conceivable, for some aspects of objects in perspective representation are preserved in their shape, whereas the "geometral" method would be the case where all the aspects would be preserved, except the object size. Secondly, it appears that the quantitative difference between the representation of an object by the perspective method and its representation

[27] "[...] a single magnitude does not serve to measure it similarly in every direction from one end to the other [...]."

[28] Nonetheless, with the identification of cones with cylinders by means of the identification of the parallel lines and the secant ones, the position of the eye leaves room for the direct relation of the object with the projection plane.

[29] "It is not that such a representation in perspective does not retain something of the very form of subject it represents, and that it is not measured similarly to the one named 'geometral;' as will be showed in its proper place."

[30] "These are two species of a one and the same genus or, if you will, two different cases of one geometrical proposition."

by "geometral" method is not sufficient to distinguish these two methods through their nature. This implies then that the quantitative aspect is not essential to them, so that the nature of the "*forme*" which remains the same ("*la mesme*") is neither quantitative nor proportional. All this suggests that Desargues' "*manière universelle*" is not merely based on quantitative or proportional preservation, but on a preservation of "*formes*" which allows a plurality of scales and the correspondence between them. It is then a general perspective method, anchored in a spatialization whose core is not magnitudes, but a way to transform the relations between magnitudes by means of scales. It is also a method by which it is possible to go either from plane to space, or from space to plane.

Conceived as such, it is possible, but not necessary, that Desargues' work stimulated Leibniz and perhaps even highlighted some of his intuitions on geometry. The reading of Desargues himself could yet furnish some tracks to delineate the Arguesian origin of Leibniz's ideas about a geometry of the preservation of forms.

3.3. The Arguesian method and its possible influence on Leibniz

3.3.1. From the method of scales to the projective method

The main invention made by Desargues, according to Bosse, is the method of corresponding scales. This method presents a threefold interest. The first interest consists in the innovative geometrization of space, regardless of the bodies or objects in it: a space thought out by means of a reticulation through a system of ordinates. Nonetheless, this system is designed to place objects, that is to say to represent their mutual positions and their own configuration, which implies, at least partially, preserving their mutual proportions. As such, space is defined as a space of the many relations between objects and the parts of them.

The second point of interest lies in the identification of points at infinity and points at a finite distance, and in the preservation by perspective of the involution relation which founds the construction of the system of ordinates. As a matter of fact, Desargues uses the invariance of a four-point involution relation by perspective in order to determine the scale of removals. By considering the point at an infinite distance as the point where parallel lines converge, as well as the point at a finite distance as the point where secant lines converge, Desargues can consider the involution relation in any case, including the case where one of the conjugate points is pushed to infinity. In that case, the second conjugate point is the midpoint of the two given points. This method calls for three remarks. First, there is an invariance, not of magnitudes, not even of ratios of magnitudes, but of a certain ratio of magnitudes determined for two couples of points through a set of converging lines, and preserved by a central projection: the involution relation. Thus, the invariant element is not totally quantitative but also relational and, consequently, qualitative. Secondly, the use of infinity allows one to put the more geometrical notion of "convergence" ("*concours*") of lines or planes, instead of the quantitative notion of distance, at the core of perspective considerations. Indeed, "convergence" is the theoretical element through which secant lines and parallel ones, or secant planes and parallel ones, are considered both as sharing the same nature determined by "convergence," and, however, as distinguished by the property, finite or infinite, of that convergence. Thirdly, as explained by Kirsti Andersen (2007, 428), such a method accurately accomplishes the passage from three-dimension spatial ordinates into the two-dimensional plane, since the function of the scale of "removals" ("*éloignements*") consists in representing the spatial depth in the planarity of picture.

This leads me to the third and last point of interest: through his works on perspective, Desargues prepares his major text, *Brouillon project d'une atteinte aux événements des rencontres du cône avec un plan* (1639). The essay's title explicitly holds central the notion "convergence," since conics are defined as the "events of convergence of circle with plane." More precisely, it consists in the generalized study of conics as sections of a cone projected on a plane through a transformation preserving the involution relation. Hence, any examination of the conic properties can be reduced to the case of the convergence of lines with the circle, since any other conic corresponds to the circle by a correspondence determined by perspective. *Brouillon*

Project's theory of conics is then unified, and the possibility condition of this unification is founded on the geometrical contributions of perspective: the existence of a determined correspondence between space and plane, the invariance of the relation of involution through it, the primacy of convergence over measure (despite the use of the relations of proportion or of Menelaus's theorem), and the innovative use of infinity for line as well as for plane – which paves the way towards the infinity of space itself, since space and plane are related to one another.[31]

3.3.2. Desargues' Brouillon Project (1639)

Now to be more precise about the possible relation between Leibniz's and Desargues' conceptions, I am going to present some elements of the *Brouillon Project*, however not technically and completely, because it is highly probable that Leibniz did not directly know this text yet had access to the methodological and general content of its theory. Then, by following the text established by René Taton (1951), in the very first lines of the text, we can find the assertion of the necessity of a correct use of terms, the usefulness of shapes, and the explicit use of two infinities: one of largeness ("*grandeur*") and one of smallness ("*petitesse*"), although they are difficult to conceive ("*entendre*") – as was actually the case in 1639. Such a use of infinity is required by reason itself, despite its unimaginable ("*inimaginable*") character which makes it awkward to "understand," that is to say to embrace in a certain way, seemingly a perceptual way as revealed by the characterization of an infinite straight line as an extendable row of aligned points (Desargues, 1639, 99). Desargues underlines this difficulty twice to understand what an infinite straight line means[32]: at the beginning of the text (Desargues, 1639, 99) and at its end (Desargues, 1639, 179), as if the Arguesian geometry of conics was contained in the larger problem of the mathematical knowledge of geometric infinity and, consequently, of space. Indeed, firstly devoted to the lengthening of a line, infinity can be applied in the same way to the spreading of a surface or of a solid. Then geometrical space appears as essentially infinite (Desargues, 1639, 179). Of course, the conception of space as infinite is certainly not new in itself, but its integration into the body, and *a fortiori* its position as the heart of a geometrical treatise is quite innovative. This integration seems epistemologically enabled by exploiting the full-potential of the identification of points at infinity and at a finite distance or, by analogy, the identification of lines at infinity and at a finite distance. Indeed, this identification implies three, quite surprising and modern, ideas.

First, it implies the identification of the cone with the cylinder as "*deux sousgenres d'un surgenre icy nommé rouleau, dont il est icy traitté principalement en général*" (Desargues, 1639, 134).[33] The theory of conics is thus unified, but in such a way that the case of the cone ("*cône*" or "*cornet*") is no more general than the case of the cylinder, so that points at infinity do not appear as an exception or a particular case. On the contrary, they could be considered as representing the more general case, since the name of the genus is "*rouleau*," which means "roll" and is not, in ordinary language, discernible from the "*cylindre*" or "*colonne*" which refers to a "roller." The integration of infinity into geometry is then implicitly but totally achieved as it seems to found the generality of the theory.

Secondly, Desargues presents the idea of a complete correspondence between a straight line and a circle, since a straight line can be considered as a circle closed in on itself at an infinite distance (Desargues, 1639, 102). The assertion of a kind of geometric identity between a circle and a straight line is absolutely remarkable and stresses Desargues' familiarity with the use of infinity. More precisely, it could appear that the identity is built by means of a correspondence between the points of a circle and the points of a straight line. But it is not entirely the case. Indeed, a circle and a straight line belong to the same kind of geometrical objects, not according to a total punctual relation, but according to a relation allowing the correspondence between a point at a finite distance and a point at infinity. As a matter of

[31] For all those analyses, see Andersen (2007, 447), and Le Goff (1994, 202–203).

[32] More precisely, he also mentions twice the difficulty of understanding the two infinities of smallness and largeness.

[33] "[...] two sub-genera of a super-genus, in that case named 'roll,' with which it here principally deals in general" (my translation).

fact, a circle and a straight line are generated by the same process: a line having a fixed point (at a finite or infinite distance) and moving in a plane represents the set of all the convergent lines (at a finite or infinite distance), that is to say the set of all secant lines or of all parallel lines. Desargues identifies the motion of this line in a succession of *loci* ("*places*") in the plane with the set of an infinity of lines placed in those *loci*. After that, he considers one point on this line and its motion during the motion of the line: in the case of secant lines, the point's motion is identical to a circular line, whereas in the case of parallel lines, it is identical to a straight line (Desargues, 1639, 101–102). This suggests several conclusions. Such a concordance of circles and straight lines, not founded in a total punctual relation, but in a relation determined by a dynamic process of *loci* change, reveals two central aspects of Desargues' geometric work: on the one hand, the heuristic power of points at infinity is the core of his method, more than the possibility of a total punctual transformation of circle into line or line into circle. On the other hand, the conception of conics as already constituted linear figures (curved or straight) determined by a certain section of the given cone is preferred to the definition of conics as the sets of projected points of the circle.

The third important idea implied by the identification of points at infinity and points at a finite distance concerns the preservation of the involution relation in any conic section. Thanks to such a preservation, the *Brouillon Project* consists both in a "general" theory of conics – making possible the knowledge of any particular cases by general properties (Desargues, 1639, 141), and in a "simplified" method of knowing the more complicated objects through the simplest ones (Desargues, 1639, 156). Such a method presents the beginnings of a projective conception in geometry: the conception of knowledge of geometric properties as the knowledge of the invariant properties by central projection. Indeed, even if the involution relation is quantitative, as it deals with magnitudes, Desargues focuses on the importance of invariance and the possibility of "generating" ("*voir la semblable generation*") certain corresponding properties in figures from corresponding lines (or points) in corresponding figures through the section of a cone (Desargues, 1639, 147). Furthermore, the end of the *Brouillon Project* looks like a promising program for a new geometry of conics in which the qualitative nature of the approach shines through in the use of terms like "*position*," "*espece*," and "*generation*" (Desargues, 1639, 176): "*position*" accounts for the place in infinity or at a finite distance of a point or a line, "*espece*" appertains to the nature of rolls or conics depending on the position of the vertex and the projection plane, and "*generation*" refers to the geometric and projective construction of some geometrically characteristic objects which are equivalent to geometrical properties as tangents, diameters, focuses, etc. For my purpose, the point could be that this passage is just before the last paragraphs of the *Brouillon Project* that Leibniz copied out on the verso of his copy of Pascal's *Generatio Conisectionum* (Pascal, 1654), and which dealt with perspective, gnomonics, and stereotomy (Taton, 1964a, 45). If that does not prove that Leibniz read this passage in the *Brouillon Project* itself, we can nonetheless suppose that he had some understanding or intelligence of what he copied through his knowledge of Pascal's essay devoted to conics and derived from Desargues' works. In any case, there is a resemblance between Desargues' idea and the nature of *Characteristica Geometrica* as a geometry of "position" and also of "generated" objects by congruence.

Thus, the issue progressively becomes one of determining whether the lacunary data that Leibniz had about Desargues' method were sufficient to plant the seeds of the idea of a general geometry of space and situation into his mind, although at first glance geometric characteristic is not like the unified theory of conics of Desargues. Therefore, I am now going to discuss the possibility of such an influence, not only for the philological reasons that I have already partially given, but also based on theoretical concerns.

4. The irreducible originality of *Characteristica Geometrica*

4.1. Signs and difficulties of Desargues' influence on Leibniz

4.1.1. Philological signs and their limits

Some textual elements confirm the possibility of an influence of Desargues' perspective method on Leibniz when he writes his first essays on geometric characteristic. Indeed, between 1673 and 1677, Leibniz sometimes made reference either to Desargues, or to Pascal in laudatory terms (Leibniz, 1675a, 359; 1676a, 591; 1677, 62). In itself, that does not prove that Leibniz extracted the idea of a situational geometry from his knowledge of perspective. But, associated with the fact that Leibniz began to elaborate such a geometry in 1677, just after his stay in Paris where he learnt a lot about Arguesian methods, notably from La Hire (and also Pascal), that suggests the existence of a causal relation, at least partial, between his knowledge of perspective works and his creation of geometric characteristic.[34] Furthermore, the study of the geometric texts written between 1672–1676 reveals that, from the spring to the summer 1676, several texts[35] carry titles presenting some common elements with either perspective objects, or future elements of geometric characteristic. Even if those texts do not really have much to do with perspective or geometric characteristic, their common glossaries suggest that some connections happened in Leibniz's mind, maybe the idea of the family relation between those kinds of geometry. Such a chronological parallelism can also be drawn between Leibniz's later attempts for his geometric characteristic and his few manuscripts about perspective. Indeed, according to the dating by Javier Echeverría (1983), two periods can be distinguished for Leibnizian manuscripts on perspective. The first one spans from 1678 to 1685,[36] that is to say when Leibniz started to design his geometric characteristic. The second one begins around 1695, when Leibniz went over his *Geometria Situs* again and initiated with Bodenhausen a very rich correspondence about the topic of *Analysis Situs*. This seems to show that a connection exists between Leibnizian perspective and geometry of situations (or geometric characteristic).

Nevertheless, despite the appearance of such a relation, it is very difficult to categorically conclude that perspective constitutes the main source of geometric characteristic, even if these works appear as contemporary in Leibniz's mind, and if works on perspective (by Desargues, La Hire, Bosse or Pascal) are prior to Leibniz's essays on geometric characteristic. Indeed, in an extract from a Leibnizian manuscript on perspective, retranscribed by Javier Echeverría and dated later than 1695, Leibniz affirms the primacy of the general science of perspective over the geometry of situations:

> Haec idea perspectivae vastissima est, et totam comprehendit Geometriam situs, quae scilicet a magnitudinis (praeterquam rectarum) et motus calculo abstinet.[37]
>
> [Quotation in: Echeverría, 1983, 199]

On the one hand, this quotation could confirm that perspective is, even though certainly not exclusively so, still a major source of geometric characteristic. On the other hand nonetheless, Leibniz affirms that

[34] Indeed, in 1673, Leibniz wrote a first essay on geometric characteristic, *Characteristica Geometrica. De lineis et angulis* (Leibniz, 1673), in which he tried to elaborate a notional alphabet, but the concepts of *situs* and space were missing. At that time, such a "geometric characteristic" was still quite different from the later projects of the characteristic geometry as a geometry without magnitudes. And it was also prior to Leibniz's discovery of Desargues' (or Pascal's) works.

[35] *De sectionibus cylindrorum* (Leibniz, 1676b), *De secandis parallelis* (Leibniz, 1676c), *Nova consideratio de locis* (Leibniz, 1676d) or *Clavius constructionum et descriptionum per parallelogramma rigida* (Leibniz, 1676e).

[36] Javier Echeverría [1983, 197) considers that Leibniz wrote his first known manuscript about perspective between 1678 and 1685: *Origo Regularum Artis Perspectiva*.

[37] "The idea of perspective is the broadest one and includes the whole Geometry of situation, which of course avoids the calculus of magnitude (except of the straight lines) and of motion."

perspective is fundamental after 1695, as if he needed several decades of reflexion and work on perspective to comprehend its foundational aspect for geometry, including for the geometry of situations. Thus, it seems that the beginnings of geometric characteristic are not founded in Leibniz's introduction to perspective works, even if he had sensed the heuristic power of its method and its qualitative or relational nature.

Finally, after 1679, as far as I know, the name 'Desargues' disappears in the texts on geometric characteristic, but the terms 'perspective' and 'projection' remain, namely in the major essay from August 10th 1679 (Leibniz, 1679a, 142–144). The fact is that Leibniz read Bosse's *Maniere Universelle de Monsieur Desargues* around 1678. This reading apparently marked the end of the direct reference to Desargues. Then, between 1677 and 1679, the possible influence of Desargues petered out, giving way to the explicit notions of space and situation. Hence, it seems necessary to examine these different references to Desargues or to perspective in their conceptual context, in order to delineate the theoretical common elements between geometric characteristic and Desargues' method, depending on the periods of Leibniz's researches.

4.1.2. Theoretical signs and their limits

The Arguesian perspective method and Leibnizian geometry share several theoretical elements, but the precise examination of them shows that such commonality does not mean sameness, not even continuity, so that Desargues' influence on Leibniz appears as necessarily limited. Four elements can be pointed to: the use of infinity in the identification of parallel and secant lines, the idea of a transformation preserving an invariant, the place of perception, and the concept of "convergence" or "incidence."

First, Desargues uses two kinds of infinity, one in the small and the other in the large, as Leibniz did when he invented his differential calculus, from the infinite series and the metamorphosis method,[38] and which he was inclined to generalize to all mathematics. Thus Desargues' method appears as an applicable and practical, as well as totally innovative, use of infinity in geometry, embodied in the easily-realized identification of parallel and secant lines, which Leibniz explicitly presents as the major element of Desargues' (and Pascal's) geometry (Leibniz, 1675a, 359). Nonetheless, in his essays on geometric characteristic, Leibniz never mentions that two lines can be secant at a point at infinity or at a finite distance. He even continues distinguishing between them (Leibniz, 1679h, 138). Furthermore, even if he notes the identification between a curved line, namely a circle, and a straight line, he misses the great innovation of such a conception and its full potential for its new geometry. However he does touch upon the possibility of a consideration of geodesic analogous to the consideration of a straight line as the smallest line on a plane: in a text probably dating back to 1679, he emphasizes the problem of the knowledge of the smallest line on a curved surface, such as a sphere, a cylinder or a conic surface (Leibniz, 1679i, 272). Nevertheless, although Leibniz would have been able to identify the nature of straight lines and circles and, correspondingly, to conceive a curved space, he did not pursue such an innovative research approach. I might suggest two explicative interpretations. On the one hand, it is indubitable that Leibniz did not want to produce such a generalization which would consist in a radical surpassing of the Euclidean geometry, which is a geometry of "plane" space. On the contrary, Leibnizian space is always postulated as Euclidean. On the other hand, in the case of a real influence of perspective, the "natural" relation between a circle and a straight line would have been more significant for Leibniz, since it would have allowed a more general geometry in which spheric space and plane space would have been the two cases leading to a most general case. Therefore, according to this line of thinking, the lack of a real use of the identification of parallel and secant lines tends in my view to prove that Leibniz was not deeply influenced by the geometric innovations of perspective.

38 Leibniz clearly had Pascal and Desargues in mind when he presented the advantages of his new method in his famous *De quadratura circuli ellipseos et hyperbolae* from 1676, as using a transformation by means of triangles instead of rectangles (Leibniz, 1676f, 538).

Also see Debuiche (2009, 95–99 and 107–110).

Secondly, the main idea of the perspective method consists in the use of invariants for a given transformation in order to build a method of knowledge of the geometrical properties linked to those invariants. Founded on the invariance of the involution relation, such a method is both quantitative, because of the proportions involved in involution, and qualitative, since it deals with the "incidence" of lines and points. Thus, perspective appears as an instance of the production of knowledge by means of characteristics, that is to say through a determined relation of correspondence between what is used and what is to know.[39] But it is obvious that in 1679 Leibniz already had a precise idea what characteristics was or had to be.[40] More precisely, the characteristic aspect of "geometric characteristic" is at best partially embodied by the perspective model, but the text from August 1679 insists on two aspects of geometric characteristic: the choice of the best characteristics for geometry and the fact that the notion of "relation," as the situational relation of points, is the main object of geometrical knowledge (Leibniz, 1679a, 144). In fact, behind these two elements hides the critique of the Cartesian algebraic geometry. Hence, the perspective example as a geometrical model is not central and, because it is secondary, it also appears as a model that is not fundamental. And even so, Leibniz could have been convinced by the model of the perspective method that something like a geometry of space was possible, his own geometry whose object is space does not consist in a method of reduction of space in plane. On the contrary, space is conserved as a three-dimensional structure and its geometry has to be deeply distinguished from the geometry developed from perspective.

Thirdly, both methods of perspective and geometric characteristic depend on some kind of perception. Indeed, geometrical perspective is originally founded in vision theory, and connected to the rules of visual representation, even if Desargues' aim is to rid perspective of his merely perceptual or pictorial contributions. The conception of the vertex of a cone as the position of an eye maintains a sort of perception linked to positional considerations, as can also be found in Leibniz's mutual situation of two "co-perceived" points. Besides, in perspective as well as in geometric characteristic, space is considered as the place where objects can be situated in relation one to another. The Leibnizian situation is defined by means of coperception, that is to say by means of the simultaneous perception of two things distinguished by their mutual positions. The purpose of perspective is to establish rules for the plane representation of objects in space, by the reticulation of space with a system of ordinates (by means of geometral and perspective scales), so that rules of perspective representation are also rules for a positional representation according to the reticulation of space. All those common elements are nonetheless limited, because the relation between perspective, knowledge, and perception is already present in Leibniz's earlier texts. Indeed, in his letters to Thomasius in 1668–1669 (Leibniz, 1668, 1669), Leibniz used the example of perspective to explicit what the knowledge of bodies, of their essence or their qualities is. Thus it seems that his previous and ordinary knowledge of perspective was sufficient for Leibniz to make use of the perspective exemplar as he did in 1679. In other words, it is highly probable that Leibniz, by discovering Arguesian perspective, did not find any deeply innovative element for his conception of perspective as a model of knowledge through perception.

Fourthly and lastly, the perspective method is founded on the notion of 'incidence,' that is to say of the relation between "convergent" lines in points. The perspective geometric method presents an implicit passage from the conception of incidence as "convergence," to its conception as the motion of a point or a line relative to a certain law of transformation. This point could have influenced Leibniz for his own conception of "congruence." Indeed, perspective projection can be regarded as a transformation by which

[39] In 1678, in *Quid sit idea*, it is very clear that perspective is an exemplar of a more general modality of knowledge by means of a relation of correspondence between two objects, or between characters expressing these two things (Leibniz, 1678).

[40] In 1679, he had already written several essays of characteristic, even if they were not an application to geometry, namely several essays about numerical characteristic, such as *Elementa Calculi* (Leibniz, 1679j) or *Regulae ex quibus de bonitate consequentiarum formisque et modis syllogismorum categoricorum judicari potest, per numeros* (Leibniz, 1679k). Several years before that, in a letter to Mariotte from July 1676, Leibniz presented the idea of the general application of "*algebre universelle*" to any realm of knowledge: ethics, physics, mechanics or geometry (Leibniz, 1676g, 271).

points and lines can move whilst preserving some incidences: as a transformation, a perspective projection has something to do with motion, even if, as "events of convergence," it is static. Nevertheless, such a conception is more suggested than explicit: despite the idea of a unified knowledge of conics thanks to perspective, Desargues does not consider the various conics as generated by a continuous motion of the projection plane – as Leibniz would.[41] The unification is obtained by means of the identification of the points in infinity and at a finite distance – which Leibniz does not do. It seems then that the conception of "incidence" or "convergence" as the dynamical way to occupy different places one after the other in space, through a continuous variation of positions, is purely Leibnizian, and reveals several fundamental differences between Desargues and Leibniz, all connected to the question of 'incidence:' the nature of infinity and its relation to continuity, the crucial notion of congruence in Leibniz's geometry, the object of geometry as space and situation, and not as point and line, and the new structure of space. And all these differences constitute the germ of Leibniz's originality.

4.2. Originality of Leibniz's invention

4.2.1. Characteristica Geometrica's specificity

If the differences between geometric characteristic and perspective are central for determining the possibility of an influence of Desargues on Leibniz, they are above all significant in grasping the original import of Leibniz's work about non-quantitative geometry. Leibniz's geometry is not a geometry of point, line, and plane, whereas perspective geometry is. However it has something to do with 'incidence,' just as perspective has. Nonetheless, it does not only deal with convergence conceived as meeting or section, but also with incidence as the fact that something enters a place: "*in-cidere*." The word is not exactly present in Leibniz's text, but "*coïncider*" or "*coincidentia*" are, and designate two things which are interchangeable: "*A et B coincidunt, sive sunt simul in eodem loco*" (Leibniz, 1679l, 116). Then, and naturally, 'incidence' is always a 'co-incidence,' since being incident is always having something in common: a point, a line, a surface, a solid which are consequently in the same place. Most significant is the definition that Leibniz sometimes puts forward: "*congruentia actuali sive coincidentia*" (Leibniz, 1679a, 184). Coincidence, which can also be called 'identity,' appears as the more general case of congruence, and congruence can be defined as the possibility of coincidence. This supposes two related elements. The first is the existence of a prior structure of "possible" coincidences, or congruences. The second is a dynamical approach to congruence. Indeed, congruence is the possibility of coincidence (or identity) as well as the possibility of moving whilst preserving a certain relation: the different mutual situations of points, and their order. In other words, that consists in moving to another place without losing its own form: "*Unumquodque servata forma sua moveri potest in infinitis modis*" (Leibniz, 1679a, 204). The consequence of such a definition of congruence concerns the fundamental role it plays, since congruence is simultaneously: the constitutive element of the different objects of geometry,[42] the element determining an invariant for motion – which makes the motion possible, and the relation between objects considered spatially (and not dynamically), relative to their mutual positions. Hence, congruence supposes two elements: space and *situs*. In fact, congruence requires that any

[41] Leibniz's notes on Pascal's manuscripts about conics seem to prove that Leibniz introduces some dynamical considerations – like the continuous inclination of plane projection – which are not explicitly present in Pascalian conceptions (Leibniz 1676h, 1124–1125).

[42] In geometric characteristic, congruence is also the operator of the combination of one mutual situation between *two* points to produce a mutual situation between n points, and the operation by which equations expressing geometrical *loci* are composed, noted γ: $Y\gamma(Y)$ for space defined as the *locus* of points congruent to one another; $A.Y\gamma A.(Y)$ for a sphere as the *locus* of all points in the same mutual situation with a fixed A as a certain point Y; $A.B.Y\gamma A.B.(Y)$ for a circle defined as the *locus* of all points having the same mutual situtations with two given points A and B; $A.Y\gamma B.Y\gamma C.Y$ for a straight line defined as the *locus* of all points having the same mutual situtations with three given points A, B, and C; and $A.Y\gamma B.Y\gamma C.Y\gamma D.Y$ for a point determined by the unique position in which it has the same mutual position with four given points A, B, C, and D.

geometric object has an infinity of congruent objects or that any motion always be possible. Considered as equal, these two properties necessarily imply that space is precisely what contains any possible mutual situation, an infinity of congruent situations to it, and the possibility of a continuous motion (Leibniz, 1679a, 202). As such, it can be considered as a structure in which any geometrical object can be thought, but is itself something else than these objects: it is the "situational" structure, an "order of *loci*," or an "order of coexisting," as Leibniz starts to define it in 1682: "*Spatium est continuum in ordine coexistendi*" (Leibniz, 1682, 302). Consequently, the notion of space enables one to consider two things just as what can only be distinguished by copresence or, equally, by mutual situation. This leads to the notion of *situs*.

"Situation" is quite often defined by means of distance, so that it would be easy to think that Leibniz considers it as a quantitative notion, even if he claims that his geometry is not relative to magnitudes. Indeed, as the "rigid *extensum*" between two extremities (or a combination of such rigid *extensa*), situation can be identified with the straight line between two points (or the plane surface between two lines). It is then the shortest line, and consequently, the unique line between them. Nonetheless, situation is also conceived through qualitative considerations, since it is related to congruence. Indeed, saying that situation is a "rigid" *extensum* is equivalent to saying that it is congruent to an infinity of situations (and also that it could be moved), so that a situation is what can be distinguished from an infinity of other (congruent) situations only by coperception. Thus, two interpretations of situation are possible. One is quantitative and considers that congruence is something more particular than similarity: objects are similar when they have the same form and can be distinguished only by copresence (Leibniz, 1677, 56; 1679a, 182; 1679f, 88; 1680, 280), and similar objects are congruent when they are also equal (Leibniz, 1679a, 184; 1679l, 118). The other is more qualitative: the most general concept is coincidence (or identity) which indicates the case of two things occupying the same place, that is to say with no (spatial) difference (Leibniz, 1679a, 170 and 184). Less general is the concept of congruence, since it refers to the case of two things only spatially distinct, by their position (Leibniz, 1679a, 172 and 184). Hence, similarity is more particular than congruence, for two similar things are distinct spatially, by their position, and quantitatively, by their magnitude. What is then manifest is that the notional alphabet of geometric characteristic is not completed, including in the body of the essay from August 1679 itself (Leibniz, 1679a). That implies *a minima* that it is not obvious that Leibniz considers that the definition of congruence stems from similarity, as similarity *and* equality. In other words, it is not evident that congruence contains an irreducible quantitative element. In fact, by conceiving similarity as the difference by position and by magnitude, and congruence as the possibility of a substitution *servato situ*, similarity appears as a more quantitative concept than congruence, since it requires both a change of place and a change of size. Between identity, which indicates the absence of any possibility of transformation, and similarity, which designates the possibility of a transformation preserving forms but not magnitudes, congruence embodies the possibility of a transformation preserving any quality of the object, except its position. As such, it defines what *situs* is in terms of a "rigid" *extensum*: what can change its position without modifying anything else.

From all these previously analyzed elements we found crucial differences between perspective and geometric characteristic. First, perspective obviously is a method of transformation which preserves something, since without preservation, transformation is both irregular and useless for knowledge. But perspective does not preserve similarities, even if it preserves a "*mesme forme*," because a correspondence exists between the object and its perspective representation. Nonetheless, according to Bosse and Desargues, "geometral" scale is the most fundamental scale of perspective and perspective art depends on the correspondence between that scale and the two-perspective ones. Besides, "geometral" scale preserves forms, but not magnitudes: it is then the way to maintain similarities. In that case, as founded in congruence, early geometric characteristic appears as a more general science than perspective method. Such seems to be the case of a perspective at infinity: not a scenographic representation, but an ichnographic one. Thus, geometric characteristic seems to embrace perspective, as Leibniz claims in his notes on Pascal's manuscripts about conics (Leibniz, 1676h, 1127) and reformulates around five to seven years later:

> Videamus an non commodius sit Motum adhibere quam Sectiones, cum revera Sectiones sint moti generantis vestigia. Et ita poterimus nihilominus abstinere a consideratione similitudinis; adhibita sola consideratione congruentiae.[43]
>
> [Leibniz, 1682, 300]

That quotation shows that Leibniz distinguishes between the static conception of sections (about which he cannot ignore they are the core of Arguesian perspective, as they are a traditional object of geometry since antiquity) and the dynamical considerations of his own geometry of situations. Besides, he explicitly connects the foundation of his geometry in the notion of motion and the central place which he gives to congruence, rather than to similarity. Hence, it seems that Leibniz makes absolutely his the idea of a geometric characteristic as a geometry of space having to allow motions and incidences and which, to achieve that effect, has to be the structure of all possible situations and congruences.

Secondly, in that framework, I am inclined to interpret Leibniz's relative disinterest in the identification of parallel and secant lines as a sign of his specific conception of "incidence." Indeed, although Leibniz praises this identification as a great advance in geometry, and encourages the use of infinity in mathematics, he never uses the identification of points (or lines) at a finite distance or at infinity. More particularly, in some examples or demonstrations of geometric characteristic, he explicitly deals with incidence without mentioning such an identification. To me, that seems to prove that Leibniz considers (co)incidence as something else than geometrical intersection: (co)incidence is mainly an actual congruence which always implies the motion of a given object (point, mutual situation, or combination of mutual situations) and a *locus* in space which the object occupies. Thus, this allows one to assert two ideas: the limited influence of perspective on Leibniz's invention of geometric characteristic, since what Leibniz recognizes as the most important perspective innovation is left out of geometric characteristic, and the nature of geometry as the science of space. Indeed, Leibniz is not interested in considerations about geometrical intersections at infinity or at a finite distance, on the one hand because his geometry does not deal with 'intersections' (or 'sections') like perspective, and on the other hand because the difference between what occurs at infinity and at a finite distance is not significant anymore, for Leibniz's geometry is a science of the whole of space and of the dynamical concept of congruence, that is to say of possible coincidence. As such, infinity is all over space and also in any parts of it, since space is continuous and a yet fully-present structure of possible situations, congruences and motions defining, or drawing, shapes.

The last main difference between perspective and geometric characteristic also concerns the nature of space. In Leibniz's conception, space is a structure, since it is the space of all possible relative positions (of points or of mutual situations of points). As such, it appears as a system, but not like Desargues' system of space. Indeed, by developing a set of corresponding scales, Desargues designs a system of ordinates which constitutes a reticulation of space. This system obviously is a relational one, for it is independent of objects whereas any object can be represented in it, by its relative position to others and by the relative position of its parts. But Desargues' system is also a quantitative representation of spatial positions, since it requires the consideration of magnitudes, not in themselves, of course, but in their ratios ("geometral" scale) or with regard to the involution relation (perspective scales). Leibniz's view is quite different, even if he can integrate any consideration about distance into his geometry. As a matter of fact, a straight line is a major problem for Leibniz, but this has something to do with his aim to embrace Euclidean geometry. I argue that, in geometric characteristic, the relational and combinatorial nature of *situs* is more fundamental, a feature which allows Leibniz generally to leave out quantitative considerations or to revisit them – when it is

[43] "Let us examine whether it is not more convenient to use motion rather than sections, for sections are the traces of the motion which generates them. And then, we will no less be able to avoid considering similarity, the only consideration of congruence being used."

necessary. Therefore, if Arguesian space is essentially a 'metric' space, based on magnitudes and distances, on the contrary, Leibnizian space is primarily relational, and only secondarily quantitative.

These three analyses point to the idea of an irreducible originality of Leibniz's work, for which it would be interesting to explore the reasons.

4.2.2. Reasons for the originality of Leibniz's geometry of situations

Leibniz is quite honest and usually admits whom he was influenced by. Here, he clearly claims that his invention is totally original (Leibniz, 1679c, 853–854; 1679a, 150). This implies that perspective cannot be a major inspiring source for Leibniz. Besides, as already mentioned, the critique of the imperfect analysis of Cartesian geometry (Leibniz, 1677, 50) coincides with the assertion of the necessity for geometry to deal with space and *situs*. Indeed, according to Leibniz, the main consequence of Cartesian geometry's weakness consists in the need for figures and a diverted use of the imagination (Leibniz, 1677, 50; 1679a, 144–146; 1679c, 851). On the contrary, an completed geometric characteristic supposes a correct use of the imagination ("*sans se servir de figures ny de modelles et sans gêner l'imagination*") (Leibniz, 1679c, 852). Thereby, the imagination does not consist in representing objects but, on the contrary, is a modality of science, in that case of the science of space, of *situs*, and of mutual positional relations.

The nature of the imagination seems to me a crucial issue for understanding how the Leibnizian idea of geometric characteristic comes up, and we can conceive this as possibly conditioning the emergence of this idea in two ways. The first concerns the heuristic role of the imagination. On the one hand, in Desargues' *Brouillon Project*, the imagination appears as something playing a role in knowledge and having to be coordinated with the capacity of understanding (Desargues, 1639, 99 and 179). Such an aim leads Desargues to employ perspective art in geometrical knowledge. On the other hand, for Leibniz, the use of the imagination has also to be limited in geometry, but is nonetheless central, since geometry has to become the science of the imagination which both employs the imagination and concerns the objects of the imagination (Leibniz, 1679a, 148). Thus, there is a kind of similarity between the conceptions of Desargues and Leibniz: the imagination is connected to the geometry as a science of space and relative to positions in it. However, this does not prove that Leibniz was influenced by Desargues, but rather that Desargues and Leibniz belong to a common culture which calls for overcoming figurative imagination in order to represent objects only in relation to their mutual situations.[44]

Therefore, the question becomes to define what kind of new imagination is at play in geometry, which leads me to the second possible way I mentioned. According to Leibniz, the imagination is a tool for knowledge, and knowledge is based on characteristic – which deeply distinguishes Desargues from Leibniz, since there is nothing like a demand for characteristic in Desargues' works. The main idea is that geometry which is considered as the science of the imagination, and whose objects are space and *situs*, is also characteristic. Consequently, there is necessarily a correspondence between what geometry deals with, what it operates by, and how it works. First, the nature of a characteristic demands rethinking geometry by determining its primary elements (notional alphabet) and the modalities of their combination (derived notions or composed elements) by means of suitable operations. Secondly, elements and operations have to be appropriate for the combinatorial nature of characteristics: indeed, characteristics in general is a science of relations, not of elements, and correspondence does not occur between characters and objects, but between character relations and object relations. Thirdly, calculation consists in the combination of characters according to the preservation of relations. Thus, as geometry has to become characteristic, it has also to fulfil the demands for a notional alphabet, relational nature, and combinatorial synthesis. Hence, in my view, the main Leibnizian ideas of geometric characteristic come from his characteristic project: primary elements (point, space, and situation) are researched and finally established as such, the main object is a relation (the mutual

[44] See De Risi (2007).

situation of points), congruence is the operation preserving the relation of situation and constituting the combinations of situational relations, and, lastly, as combinatorial, geometrical calculus preserves situational relations and their order. Thus, because of the correspondence between a symbolic characteristic and the objects known by it, space cannot be anything other than an order: the order of what is preserved by congruence, that is to say of situational relations.

In conclusion, as a geometry of space and situation, based on the notion of congruence, *Geometrica Characteristica* appears as a part of the most general characteristic project and, as such, aroused by epistemological reasons. Besides, because of the conditions of the "well-done" characteristics as corresponding to the nature of its objects, and because of the nature of the Leibnizian concept of space as a relational ordered *extensum*, geometric characteristic is both a geometry of corresponding situations and a method of correspondence. In that context, the similarity between Desargues' perspective and Leibniz's geometry can be explained as the necessary family relationship between, on the one hand, the perspective method of projectively corresponding properties, and on the other hand, the more general method of correspondence of situations by congruence. Finally, even if Leibniz did not find the idea of geometric characteristic in perspective, he did nevertheless at least recognize in the projective method a way to reach what he intended to do create: a geometry relative to spatial position and founded in relational correspondences.

5. Conclusion: The invention of Leibnizian *Characteristica Geometrica*, toward other sources

The question of Desargues' possible "influence" on Leibniz's invention of geometric characteristic sheds light on the general issue of Leibniz's ingenuity. Indeed, Leibniz always read his predecessors or contemporaries in the light of his own ideas. In that case, one should question the relevance of the term 'influence,' since Leibniz's thought is both shaped by a plurality of readings and endowed with an at least partial, but absolute originality. Thus his ingenuity supposes the continuity with the world of ideas which he belongs to, and perhaps this continuity is also a condition for the emergence of Leibniz's innovative ideas. However, such a continuity does not constitute an authority: Leibniz never merely uses what others have claimed or thought, and there is no univocal action by which others' ideas would modify Leibniz's. Thereby, if something like an 'influence' exists, it can only be conceived of as the existence of an occasion for Leibniz to think of some given objects, conceptions, definitions, demonstrative processes by himself, to envision them in a properly Leibnizian way, marked by the seal of its exceptional intelligence. Then, if there are some philological obstacles to determining the impact of his knowledge of the Arguesian perspective on Leibniz, the main obstacle is Leibniz himself, who never repeats an idea without changing it and often is interested in such or such an idea precisely because it already existed in his mind, sometimes merely implicitly or unformed. Hence, it is not possible to determine exactly and absolutely a boundary separing where Leibniz would create something original and from where he would inherit from others. Besides, even if influence is conceived as an occasion to think the ideas of others by himself, it is not obvious that there is an influence of Desargues on Leibniz. Indeed, by comparing geometric characteristic with perspective, it appears that what is common also is what distinguishes these two kinds of geometrical methods: the role of quantity, the place of figures and the imagination, the relation between similarity and congruence, the use of the identification of parallel and secant lines, and the conception of space through perceptive considerations. Moreover, all these elements coincide with the qualitative nature of geometric characteristic and Leibniz's rejection of magnitudes. Such a conception reveals at least two other main sources for Leibniz's invention.

First, the nature of situation, as relative and perceptive, recalls the more general Leibnizian conception of knowledge, and his metaphysics, namely through the concepts of perception, expression, uniformity, and through the use of the principle of sufficient reason (for instance, Leibniz, 1679a, 156–158). Then it gives a philosophical basis to the invention of *Characteristica Geometrica*. Hence, some questions become crucial: in what relation does the geometry of space stand with a more general and philosophical theory of space in

Leibniz's "early" Parisian thought? And how much is the invention of geometric characteristic generated by Leibniz's philosophy?

Secondly, and finally, the relation between Leibniz and Pascal seems to have played a key role in Leibniz's initiation to the geometry of conics and, eventually, in his invention of a geometry of situation. Indeed, some aspects of the requirements of characteristic are common to Leibniz and Pascal (such as the role of the primary definitions). Besides, Pascal and Desargues were friends and influenced one another, particularly Pascal by his elder. Moreover, the reading of Pascal's texts on geometry by Leibniz in Paris is established by the various copies which he made of some of them, or of parts (Pascal 1654, 1657). Lastly, Leibniz and Pascal share the notion of situation and the idea of geometry as a science of space. And Leibniz interprets Pascal's conics as dynamically continuous (Leibniz, 1676h, 1124–1125). Then, was Leibniz "influenced" by Pascal's innovative ideas, more than by his master Desargues' conceptions? Was the reading of Pascal's essays a source of inspiration for Leibniz?

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