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The 1740 Resolution of the Fermat–Descartes Controversy

LUIGI INDORATO AND PIETRO NASTASI

Department of Mathematics, University of Palermo, Via Archirafi 34, 90123 Palermo, Italy

In a booklet written in 1740 the Neapolitan mathematician Pietro Di Martino contested the opinion, shared by the most important scholars, that Fermat had deduced the same refraction law as Descartes from an opposite hypothesis on the velocity of light. He also denied that from falsehood, truth could be deduced by a direct demonstration. At the same time Di Martino stated a minimum principle for optics that was consistent with Newton's theory and identical to the principle of least action stated by Maupertuis in 1744. However, Di Martino did not extend his principle to dynamics but restricted it to optics because, unlike the French scientist, he did not look for final causes but for simple laws which provided a reasonable explanation of physical phenomena. © 1989 Academic Press, Inc.

Il matematico napoletano Pietro Di Martino, in un opuscolo del 1740, contestò l'opinione comune che Fermat avesse dedotto la stessa legge di rifrazione di Descartes pur partendo da un'ipotesi opposta sulla velocità della luce. Egli affrontò anche il problema se dal falso si possa dedurre il vero mediante una dimostrazione diretta. Di Martino stabilì, per l'Ottica, un principio di minimo che era in accordo con la teoria newtoniana e sostanzialmente identico al principio di minima azione di Maupertuis. Di Martino però, al contrario di Maupertuis, non lo estese alla Meccanica perchè piuttosto che per cause finali propendeva per leggi fisiche semplici in grado di dare spiegazioni razionali dei fenomeni. © 1989 Academic Press, Inc.

Der Neapolitanische Mathematiker Pietro Di Martino bestritt in einem kleinen Werk von 1740 die von den wichtigsten Gelehrten vertretene Meinung, Fermat habe dasselbe Brechungsgesetz wie Descartes mit Hilfe einer entgegengesetzten Hypothese über die Lichtgeschwindigkeit abgeleitet. Er bestritt ebenso, daß aus etwas Falschem die Wahrheit mit Hilfe eines direkten Beweises abgeleitet werden könne. Gleichzeitig stellte Di Martino ein Minimumprinzip für die Optik auf, das mit Newtons Theorie vereinbar war und mit dem von Maupertuis 1744 aufgestellten Prinzip der kleinsten Wirkung übereinstimmte. Di Martino übertrug jedoch sein Prinzip nicht auf die Dynamik, sondern beschränkte es auf die Optik, da er anders als die französischen Wissenschaftler nicht nach finalen Ursachen, sondern nach einfachen Gesetzen suchte, die eine vernünftige Erklärung der physikalischen Erscheinungen lieferten. © 1989 Academic Press, Inc.

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INTRODUCTION

The Fermat–Descartes controversy could be considered closed at the beginning of the 18th century, despite the apparent unawareness of the majority of scientists of the contradictory hypotheses and results of the two theories.

Kirsti Andersen, in her analysis of the mathematical technique of Fermat's law of refraction [Andersen 1983], has addressed the question of what prevented Fermat from recognizing that his law was different from that of Descartes.

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The aim of our paper is to discuss a booklet [Di Martino 1740] written by Pietro Di Martino (Naples, 1709–1746) on the difference between the hypotheses and results of Descartes and Fermat. The discussion of such a difference between the two laws leads Di Martino to address an important epistemological question: namely, whether truth can be deduced from falsehood by a direct demonstration. In Di Martino's view, attempts to derive the same physical law from opposite hypotheses have logical support only under this condition. His objective is to deny that this is possible. In order to help the reader to place Di Martino's 1740 booklet in the proper context, we summarize in Section 1 the contents of the booklet, together with those of a physics handbook written by the same author [Di Martino 1738], as well as Di Martino's strategy. In Section 2 we describe the most relevant arguments concerning optics discussed by Di Martino in his 1738 handbook, while in Section 3 we discuss Di Martino's resolution of the Fermat–Descartes controversy. Finally, in Section 4 we state our conclusions.

1. DI MARTINO'S 1738 HANDBOOK AND 1740 BOOKLET

The *Philosophiae naturalis institutionum libri tres* by Pietro Di Martino was one of the best handbooks of Newtonian physics printed in Italy, at essentially the same time as the publication of the well-known commentary of Jacquier and Le Seur (Geneva, 1739–1742). The aim of its author was to present Newton's dynamics, astronomy, and optics, and to underline the deep link between the experimental aspects of the *Opticks* and the theoretical ones of the *Principia*. The handbook, in fact, starts underlining that mathematics and physics, as reason and experience, are connected by "very close bonds" ["arctissimis vinculis coniunctis"].

The *Sectio tertia* of the first volume (Chaps. 3–8, pp. 261–310) is devoted to optics. Di Martino follows Newton's text, apart from some significant differences, as discussed in Section 2 below. In particular, Di Martino accepts Newton's opinion that light travels faster in denser media and, as a consequence, he ends the chapter on refraction (pp. 283–285) by censoring Fermat's principle. After the book was sent to the printer, however, further reflections on this argument apparently led him to state a new minimum principle for refraction that was consistent with Newton's hypothesis. This principle is essentially the same as the one enunciated a few years later by Maupertuis and known as the principle of least action. He therefore hastened to write a preface to the book, "Monita quaedam ad lectorem," where he proves Fermat's "error" in a different way from that in the text, and announces:

> In fact I found that a minimum principle for refraction can be derived either from space & velocity, or from space & time or finally from time & velocity.

Since Newton's hypothesis on the speed of light in matter was the same as Descartes' (and therefore opposite to Fermat's), and since it was the common (although mistaken) view that Fermat had found the same refraction law as Descartes, it is easy to understand Di Martino's strategy in presenting his finding in his 1740 booklet. Rather than emphasizing the discovery of his minimum principle, the Neapolitan mathematician preferred to discuss the decidability of the Fermat–Descartes controversy. Without this, it would have been in fact impossible to explain rationally how the same empirical truth (i.e., Snell's law) could be accounted for by two theories deriving from opposite hypotheses.

This strategy is evident both in the structure of the booklet and in the accompanying letter to Francesco Maria Zanotti, Secretary of the Bologna Accademia delle Scienze [1]:

> Even though this letter is addressed to V. S. Ill.ma, I wish it to be shared by you and all Academicians of this very respected Institute, to whose careful scrutiny I submit the attached small philosophical dissertation on the refraction and motion of light rays, in which I have endeavoured to examine, and also decide, the famous controversy debated during the past century between two rare French geniuses, Descartes and Fermat. You will see that I have written with great liberty, and that with great liberty I have pointed out the errors of many very renowned mathematicians concerning this question. I wish, therefore, that this Institute gives to me the same liberty that I have given to others, amending and correcting all that will appear not to be true. Moreover, I beg you in particular, and your appreciated nephew Sig. Dr. Eustachio to read it carefully, to see if the reflections I have made on the said questions are tenable.

Anyone familiar with Di Martino's intellectual biography will not be surprised by the content and tone of this letter. To write "with great liberty," to confront himself with the great "mathematicians," and to enter the relevant scientific debates are certainly parts of his mental attitude, as we have shown in the case of his intervention in the living forces controversy [Indorato & Nastasi 1987].

The tone of the letter shows clearly that Di Martino is convinced that he has not only discussed and decided a problem of physics (formulating his minimum principle), but more than that he has touched on the related epistemological aspects that were at the center of the Italian debate on the introduction of the Newtonian synthesis. Thus, his 1740 booklet is to be regarded, using his own words, as "a small philosophical dissertation." Di Martino's stated aim was, in fact, to discuss the problem of refraction also from an epistemological point of view. Accordingly, the book ends with a revealing final paragraph: "How did Fermat succeed in deriving truth from false hypotheses?"

Here the problem of physics is linked to the retrograde motion of the planets, which is only apparent (false) in the hypothesis of the moving earth, and becomes real (true) in the hypothesis of the earth at rest.

To make this argument clearer to the reader, we wish to quote the beginning of this paragraph [Di Martino 1740, 18–19]:

> I have to explain something really singular; i.e. how could Fermat obtain the same result as Descartes, by following an opposite route. This might appear less strange to superficial people, since they think that this is usual, especially in Astronomy where celestial phenomena may be explained equally well both supposing, with Copernicus, that the Earth revolves around the Sun at the centre of the Universe, or supposing, with Tycho, that the Sun revolves around the motionless Earth. There might also be some people who, to remove the miracle, will maintain that it is sometimes possible in Geometry to deduce truth from falsehood by a

> direct demonstration; see Cardano, Clavius and Tacquet on this point. It is, however, necessary to dispel this way of reasoning as futile and totally unworthy of mathematicians. First, the fact that in Astronomy the celestial phenomena may be derived from opposite hypotheses, does not seem a stringent argument. Copernicans, for example, believe that the retrograde motions of the planets are apparent and Tychonians, on the contrary, believe them to be real. Therefore, even though both explain from opposite hypothesis why the planets, reversing every year their path, proceed toward the zodiacal signs already passed, nevertheless they do not explain the same phenomenon, but very different things, Sure enough, if one day it will be possible to know if the retrograde motions are real or apparent, a celebrated question of Natural Philosophy will come to an end. Secondly, concerning the possibility of deducing truth from falsehood by a direct demonstration, it is a shame that such an opinion is largely accepted and that it is not rejected by some scholars. This is not the right place to examine the examples that they give, taken from Euclid, Theodosius and others, in which they maintain that this way of demonstrating is used. I will possibly give elsewhere a special demonstration. For the moment, however, I cannot refrain from deriding those who are convinced that the spherical shape of the sea can be deduced directly from the very hypothesis that it is not spherical. According to this argument, the parts of the sea farther from the centre will flow until the Earth assumes a spherical and regular shape. But, either I am mistaken, or what can be legitimately deduced from this hypothesis is that the sea will oscillate as a pendulum indefinitely and will never come to rest. Their deduction is therefore false, as the hypothesis from which it is derived. From this example, it is obvious that from falsehood it is possible only to derive falsehood, using sound reasoning.

We have quoted this long passage in order to underline clearly Di Martino's aim: to show that it is impossible to deduce the same experimental physical law (for example, Snell's law) starting from two opposite hypotheses, such as the ones of Fermat and Descartes on the velocity of light in a medium.

As shown by the example of the retrograde motions, Di Martino also wanted to assume a new stand in the debate concerning the possibility of verifying experimentally a theoretical hypothesis. He was clearly against those [Generali 1986, 21] who tended to reject any astronomical hypothesis, not only the Ptolemaic, in favor of an empirical methodology and of a strictly observational astronomy.

This was, for instance, the position held by Di Martino's teacher, the Bolognese astronomer Eustachio Manfredi who, in the introduction to his astronomy handbook [Manfredi 1749, x] wrote:

> It takes a long time to ascertain the facts, and a longer one to place them into a theory, and even if at the end all the facts were found to be well described by the theory, this would not confirm the theory, because it is possible that the same facts could derive from different causes unknown to us, even if equivalent. Therefore, it is not convenient to submit Astronomy to hypotheses that, once taken as the foundation of our reasoning, could lead us into error.

Di Martino was aware that the debate on this subject had been going on for a very long time. Even if he does not explicitly quote Kepler, the reader cannot help recall the well-known passage of the *Mysterium cosmographicum* in which Kepler took a stand against the interpretation that limited astronomical theories to mathematical models apt to "save the phenomena." As is Kepler [Westman 1975, 714–715], Di Martino is critical of the claim that a true conclusion could be obtained from false premises by the rules of syllogism. Anyone accepting this could be forced to the conclusion that an experimental physical law is compatible with contradictory hypotheses. Di Martino, quoting Clavius and Tacquet, scoffs at this way of reasoning, which is today known as "Clavius' law" [Gardies 1984, 98] or also as "consequentia mirabilis" (under this name "it appears to have acquired a certain popularity among Jesuit scholars" [Lukasiewicz 1970, 168]).

Clavius' Law was well known to Di Martino: even if he does not quote its original and powerful usage [Emch 1935] made by Saccheri in his *Logica demonstrativa* [1697] and in the *Euclides ab omni naevo vindicatus* [1733], Di Martino was aware of its usage as a method of demonstration "equally scientific and that gives the same certitude as the others" (aeque scientificus nec minorem pariat certitudinem, ac reliqui) in Jacob Bernoulli's *Theses logicae* [Bernoulli 1686, 279] and in the *Elementa geometriae planae* [Napoli 1729] of his brother Nicola.

Di Martino, however, also seems to be aware that the *consequentia mirabilis* had always met with strong opposition; for example, we recall the objections of the German cartesian Daniel Lipstorp [2] to the Jesuit Tacquet [Tacquet 1754, 308–312], and the debate in the Port Royal School (Arnauld Nicole and Pascal) on reasoning by abduction [Gardies 1984, 98–106].

We think that Di Martino is wrong in deriding Tacquet's arguments and in denying any validity of Clavius' law, but we have quoted his passage above because we think that it contains, in germ, a serious objection to Tacquet's extrapolation of Clavius' law. In fact, the implicit and explicit steps in the demonstration of the sphericity of the sea surface are the following: (1) suppose that at time t the surface of the sea is not spherical; (2) suppose that, as shown by experience (Tacquet), liquids flow downward; (3) suppose that the liquid is viscous (this hypothesis is not stated by Tacquet, and lightly rejected by Di Martino); it follows that, after a long enough time δt, the surface of the liquid must necessarily assume a spherical shape (the motion is in fact oscillatory, but damped). In any case, however, this argument proves that the surface of the sea is spherical at time $t + \delta t$ if it was not spherical at time t, and not Tacquet's contention that it is spherical at time t. Di Martino's denial of Clavius' law was rather disrupting: Fathers Jacquier and Le Seur kindly asked for more arguments on the subject: "The proposition that one cannot derive truth from a false hypothesis seems to us to deserve from you a dissertation, because the matter is important, and many skillful philosophers and mathematicians, among them Mr. Wolff, have an opposite opinion" (quotation from [Di Martino 1741, 64]).

Boscovich's reaction, on the contrary, was very rash [Boscovich 1741, 4–6; Boscovich 1745, passim]: he accused Di Martino of having gratuitously dispelled what was, in his opinion, one of the most significant results of the Jesuit tradition of logic (Clavius, Tacquet, and Saccheri). Boscovich, however, was badly mistaken when he accused him of not having given the physical explanation of the constancy of the tangent component of the velocity of light in refraction. Still in 1748, 2 years after Di Martino's death, Boscovich, in his *De lumine*, again pointed out this "omission," even sharing Di Martino's criticism of Fermat and Leibniz [Boscovich 1748, 37].

On the other hand, Boscovich's self-censorship in his early papers is well known [Casini 1983, 143–58]. As an example of this necessary caution, we can consider the debate on the shape of the earth. Being aware that the Newtonian thesis of the flattening of the poles was based on the hypothesis of the spinning of the earth around its own axis, Boscovich held that Newton's thesis could also be explained from a Ptolemaic point of view.

2. PHYSICAL OPTICS IN PIETRO DI MARTINO'S 1738 HANDBOOK

As we have tried to point out, Pietro Di Martino assumes that phenomena and experiments are powerful and necessary tools to explain the basic principles of physics. As a consequence he derives the common properties of light from three main distinctive features, namely (a) light consists of a motion of matter; this property can be seen "with the evidence of the eyes" each time light is emitted by a "lucid" body, for instance, a burning one, whose "particles have an endless and violent motion"; (b) light follows rectilinear patterns by "very thin rays," so thin that they can be considered as geometrical lines; (c) light has a finite velocity of propagation.

After having stated these properties, Di Martino takes into consideration the possibility that transport of matter could be involved in the propagation of light. On this subject Di Martino shares Newton's ideas opposing Descartes' opinion that the propagation of light consisted of mere pressure without "actual motion."

Di Martino, however, does not share Newton's corpuscular theory. On the contrary, we feel that he prefers the wave theory. In fact, he says that motion of the particles emitted by a lucid body can reach our eyes in two different ways (p. 266, italic in the text):

> . . . by a *motion of translation* of some matter emitted by lucid bodies, as the *Epicureans*, and recently as the great *Isaac Newton* thought, or by *communication*, in that the particles of a lucid body communicate their motion to the nearby matter, which makes the matter in contact with it vibrate and so on, until the vibration reaches our eyes. . . . It seems more likely to me that the motion of the particles of a lucid body reaches our eyes by communication rather than by translation.

In spite of great respect for Newton, we can see a refusal of the corpuscular theory: according to Di Martino, light is propagated through the ether, an elastic medium, much more elastic than air (to explain the larger velocity of light with respect to sound), by a mechanism according to which the particles of a lucid body transmit their motion to the elastic medium. Therefore it is this perturbation of the elastic medium that is propagated, not material corpuscles.

In support of this theory, Di Martino draws an analogy with sound (see Chap. VIII, Sect. III, pp. 324–327), comparing colors to pitches. He felt authorized to draw this analogy by Newton's statement in Queries 13 and 14 of the *Opticks* that he quotes faithfully. Di Martino also quotes Query 28, but he does not seem to have understood Newton's objections to Huygens' wave theory of light. According to Newton a wave theory was inconsistent with the rectilinear propagation of light. Huygens replied to Newton's objections by asserting that, when light waves pass through an opening, the secondary waves are too weak to have any sensible effect. Newton counters that it was not clear why the same argument could not apply to sound as well.

As already stated, Di Martino did not understand the core of the problem. He seems convinced that light is propagated in the ether in the same way as the sound in the air, i.e., by waves [Di Martino 1738, 324–325]. It appears that he tries to overcome Newton's objections to the wave theory by simply avoiding to relate the concept of light rays with the wave theory. He, in his handbook, never gives a definition of the concept of rays of light. In this way he is able to share Newton's dynamical proof of the law of refraction.

Di Martino's explanation of refraction can be summarized as follows (Chap. IV, p. 271 ff.): the phenomenon of diffraction (Grimaldi, but above all Newton) suggests the idea that around each body one can suppose the existence of a very thin region of an "atmosphere." When it comes to refraction then, each one of the two media, because of its own "atmosphere," will exert on the light ray a force perpendicular to the separating surface. The denser medium, however, will exert a greater force, that is to say the resultant will accelerate the normal component of the motion of light in the denser medium. In fact, Di Martino says quoting 'sGravesande [3], it is well known from experience that there is no refraction between media of equal density, and the greater the difference in density the greater the refraction.

Di Martino points out that this model can also provide an explanation of the difference between the refraction of light and the refraction of material bodies. In passing from a rarer to a denser medium, material bodies depart from the normal, while the contrary is true for light. According to Di Martino, this difference in behavior is due to the fact that the "refraction" of material bodies is mainly caused by the resistance of the media, while the refraction of light is mainly due to the attraction exerted by the media in the thin regions of their "atmospheres." Di Martino cautiously presents the explanation of this phenomenon as a model; for "if it is not true, at least it suffices to explain the phenomena of refraction" (paragraph No. 453, p. 278). This model may be summarized as follows: when a material particle or a light ray strikes the surface separating two media, two forces of different intensity, that is, an attractive force due to the "atmospheres" and a repulsive one due to the resistance of the media, arise. Both of these forces are proportional to the densities of the media. For a particle, the repulsive force owing to the resistance of the media is always greater than the attractive one owing to the "atmospheres." On the contrary, for a light ray the attractive force is greater than the repulsive. "And this is not absurd," Di Martino concludes, "in the system of attractions, where the attraction of the 'lumen' must be immense, the 'lumen' being by far the thinnest of all material particles." It is evidence that this argument is very similar to that of Newton's Query 21, where a refracting force inverse to the mass [4] is assumed.

Boscovich's criticism regarding Di Martino's failure to give an explanation of the conservation of the tangential component of velocity in the phenomenon of refraction is unjustified. In his 1740 booklet, in fact, Di Martino does not give his dynamical explanation of refraction, but only refers to it when quoting Proposition XCIV of Book I of the *Principia:* he simply takes it for granted, supposing that his interlocutor has read his 1738 handbook and, in any case, is familiar with Newton's *Principia* and *Opticks*.

3. THE RESOLUTION OF THE FERMAT–DESCARTES CONTROVERSY

As we have seen above, in his 1740 booklet, Di Martino stated a minimum principle consistent with the hypothesis that the velocity of light is greater in the denser medium. At the same time he contested the common opinion, shared by scientists such as Huygens, Leibniz, and Fermat himself, that Fermat had deduced the same refraction law as Descartes, i.e., $\sin i : \sin r = v_r : v_i$ (v_i = velocity of the incident ray, v_r = velocity of the refracted ray), from an hypothesis on the velocity of light opposite to that of Descartes. From this point of view the booklet is essentially historical; that is, the opinions of the protagonists of the dispute, starting from Fermat's famous letter to Descartes [Fermat 1894 II, 485–489], are reviewed and discussed. The question is well known and can be summarized as follows. Fermat did not accept Descartes' proof of the sine law because, he said, it was based only on an analogy and not a *true* principle. Fermat believed he had found a true principle, because "nothing is so probable and so evident" [Fermat 1894 II, 460] as that "nature always acts by the shortest courses" [Fermat 1894 II, 354].

Then Fermat stated his minimum principle which is equivalent to saying that the path of light connecting a point *A* and a point *B* is that for which the time employed is a minimum, even if the two points are in different transparent media. In other words, if the velocity of light is supposed to be a different constant in each medium (v_i or v_r) the actual path must be such that the quantity

$$1/v_i \cdot (\delta s_i) + 1/v_r \cdot (\delta s_r)$$

is a minimum (where δs_i and δs_r are the paths in the two media, respectively). Hence the sine law can be derived

$$\sin i : \sin r = v_i : v_r = \text{constant}.$$

Descartes' form was instead

$$\sin i : \sin r = v_r : v_i = \text{constant}.$$

The two laws are identical only in the sense that both affirm the constancy of the ratio of the sines, but while the former asserts that the sines are in direct ratio of the velocities, the latter asserts that they are in the inverse ratio of the velocities. Strangely enough, Fermat disregarded the diversity, and was very surprised to find a law identical to that of Descartes. Since his law was consistent with experience only if one supposes that light is faster in the rarer medium, he became even more convinced that Descartes' proof was full of paralogisms [Sabra 1981, 149].

Since Pietro Di Martino shared Newton's theory of light, it was obvious to him that the velocity of light was greater in the denser medium. He therefore believed that Descartes' form of the sine law was correct. It is for this reason that Di Martino proved first that Fermat's assumption concerning the velocity of light was wrong [1740, Sect. II] and second that his minimum principle was false [1740, Sect. III].

According to Di Martino, since Fermat's hypothesis on the velocity of light was false, no true consequence could be derived from it. Contemporary scientists had not realized, in Di Martino's opinion, that Fermat had made two errors: his very minimum principle, and the assumption concerning the velocity of light in the media. These two errors, which compensated each other, had misled Fermat, and the Cartesians as well, to believe that Fermat's law of refraction was the same as Descartes', and therefore correct. To prove that Fermat had actually erred twice, it suffices, according to Di Martino, to prove that light is faster in the denser medium. As a consequence, the principle of least time must be false because it would cause a light ray, when passing, for example, from air to water, to turn toward the normal, which is contrary to experience. However, Di Martino does not give the dynamical proof we have sketched in Section 2, but that given by Descartes, which, as we know, is based on a mechanical analogy, that is, on the conservation of the tangential component of the velocity of light at the surface of separation of the two media [5].

Di Martino then considers the minimum principle proposed by Leibniz against Fermat's principle of least time [Leibniz 1682]. According to Leibniz, light followed the easiest way, i.e., a path where the "difficulty" was a minimum. Moreover, in a homogenous medium, the "difficulty" had to be computed as the length of the path δs times the "resistance" R of the medium; therefore, in the case of two homogenous media the relation

$$R_i \cdot \delta s_i + R_r \cdot \delta s_r = \text{a minimum}$$

should hold, and the sine law should be

$$\sin i : \sin r = R_r : R_i.$$

Leibniz's principle is therefore consistent with experience if the "resistance" of the denser medium was supposed to be greater than that of the rarer medium, which seems quite reasonable. But Leibniz assumed further, in agreement with Descartes, that light is faster in the denser medium. According to Leibniz, the greater the resistance, the greater the velocity. Di Martino judged this assumption could not be maintained because it was against any physical common sense. Where "resistance" is greater, by no means can velocity be greater as well. In fact, he objects that the greater the resistance, the smaller must be the velocity. Di Martino is convinced that light is actually more "impeded" by water than by air, even if it is faster in water, and he rebukes Descartes [Di Martino 1740, 10–11]. For this reason, in his 1738 handbook he tried to prove that the "atmosphere" of the medium, rather than its "resistance," must be taken into account.

According to Di Martino, since light is faster in the denser medium, ''resistance'' cannot be proportional to density, but rather must be inversely proportional to density. Consequently, Leibniz's principle was equivalent to the principle of least time and hence erroneous [6].

Finally, Di Martino asserts his own minimum principle to be consistent with experience and with Newton's theory of light. Once he ascertained that a minimum principle, in which only space, or velocity, or time was concerned, yielded incorrect results, he realized that he could try to minimize a combination of two of these quantities. Di Martino's principle is identical to what will be known as Maupertuis' principle of least action, that is

$$v_i \cdot \delta s_i + v_r \cdot \delta s_r = \text{a minimum}.$$

From this principle Di Martino derived the sine law in Descartes' form, but by a different method than in the introduction to the 1738 handbook, where he gave an analytical proof (the same that may be found in many books on the history of science). In the 1740 booklet he preferred to give a ''synthetic'' proof. We shall omit both of them for the sake of brevity.

4. CONCLUSIONS

In 1744 Maupertuis stated the principle of least action in a paper entitled ''The Agreement between the Different Laws of Nature That Had, until Now, Seemed Incompatible'' [Maupertuis 1753]. Maupertuis' aim was to state a principle, consistent with Newton's theory and valid both for light and material bodies.

Actually his minimum principle, one stated for light, in which case it is wrong, was extended, rather arbitrarily, into dynamics where it was successful. It is unlikely that Maupertuis was aware of Di Martino's 1740 booklet when stating his principle, but it is a matter of fact that his principle of least action is similar to the minimum principle of the Neapolitan mathematician. Furthermore, Di Martino's and Maupertuis' memoirs are very similar in this historical treatment and they had the same aim: to look for a minimum principle compatible with Newton's theory, which they were advocating. Both rejected Fermat's principle of least time, using almost the same words, asserting that it could not be maintained because light is faster in denser media. However, unlike Maupertuis, Di Martino shares the wave theory of light. Moreover Di Martino's scientific works do not have those metaphysical aspects which permeate Maupertuis' thought. While Maupertuis rebuked those scientists who wanted to bar final causes from the laws of Nature, Di Martino was rather pragmatic and looked for simple laws that were logically consistent and supplied a reasonable explanation of physical phenomena. It is for this reason that Di Martino did not attach any peculiar significance to the quantity $[m] \cdot v \cdot s$, while Maupertuis claimed that

> it is the quantity of action which is Nature's true storehouse, and which she spares as much as possible in the motion of light.

The metaphysical nature of his principle, and therefore its universality, enabled Maupertuis to extend it into mechanics, a step that was not possible for Di Martino, for the reasons we have tried to make clear.

On the other hand, owing perhaps to the peculiar historical and cultural Italian milieu, it was the epistemological aspect of the controversy which mainly interested Di Martino. If the same "truth" could be derived from two opposite hypotheses, no certainty could be reached in natural philosophy, no general principle could be stated, and only simple experimentalism would have been possible. Supporters of this point of view could use the Fermat–Descartes controversy to strengthen their position. Di Martino thought that his "resolution" of the dispute, i.e., that Fermat's and Descartes' laws were actually different, would have blunted this weapon of his adversaries.

Di Martino's epistemology is certainly naive, but we feel that his attempt to blend together physics, mathematics, and logic, in a framework in which natural philosophy acquired the dignity of a rational science, deserves to be pointed out. This is in fact one of the few Italian attempts in this direction that we are aware of.

NOTES

1. In Archives of the Academy of Sciences of Bologna, Titolo III, p. II, "Lettere missive di F. M. Zanotti," fasc. 5. The letter is dated 1740.

2. On Lipstorp, professor at the University of Lübeck and author of the *Specimina philosophiae cartesianae, quibus accedit* (. . .) *Copernicus redivivus* (Lugduni Batavorum: Elzevier, 1653), cf. [Mouy 1934, 97–98, 184–185].

3. Cf. ['sGravesande 1748 II, 745–746]: "then there is no refraction if the two media have equal density; and the refraction is greater, the greater the difference between the two densities." With regard to this notion of atmosphere, clearly derived from the Dutch physicist, Di Martino's point of view was very similar to Clairaut's, apart from the corpuscular hypothesis. Compare, for example, Di Martino's explanation with the analogous paragraph of the important paper of [Clairaut 1741].

4. About the difficult formulation of this Query, cf. [Hall 1978, 187–191].

5. We think it is due to the fact that Di Martino took the thesis for granted, that he gave the simplest proof. His aim, as we said, was not to prove that Fermat was wrong, but to understand why he seemed to obtain a correct result.

6. As Di Martino notes, the reducibility of the Leibnizian principle to Fermat's is borrowed by Jacquier and Le Seur. However, we think, it is only a formal homage, because he seems rather to paraphrase a well-known paragraph of Mairan on this argument [Mairan 1725, 380].

REFERENCES

Andersen, K. 1983. The mathematical technique in Fermat's deduction of the law of refraction. *Historia Mathematica* **10,** 48–62.

Bernoulli, J. 1686. *Theses logicae de conversione et oppositione enunciationum. . . .* Basel: Bertschianis. The entry [Bernoulli 1686] appears in [Bernoulli 1969, 275–281].

——— 1969. *Die Werke von Jakob Bernoulli,* J. O. Fleckenstein, Ed., Band 1. Basel: Birkhäuser.

Boscovich, R. 1741. *De natura et usu infinitorum et infinite parvorum.* Roma: Komarek.

——— 1745. *De Viribus Vivis Dissertatio.* Roma: Komarek.

——— 1748. *Dissertatio de Lumine pars secunda. . . .* Roma: Komarek.

Brunet, P. 1938. *Etude historique sur le principe de la moindre action.* Paris: Hermann.

Casini, P. 1983. *Newton e la coscienza europea.* Bologna: Il Mulino.

Clairaut, A. C. 1741. Sur les explications cartésienne et newtonienne de la réfraction de la lumière. *Histoire de l'Académie des Sciences de Paris pour l'année 1739 Mémoires*, 259–275.

Di Martino, P. 1729. *Elementa geometriae planae seu elementarum Euclidus priores Sexlibri.* Napoli: Mosco.

Di Martino, P. 1738. *Philosophiae naturalis institutionum libri tres.* 3 vols. Napoli: Mosca.

——— 1740. *De luminis refractione et motu, brevis lucubratio ad ornatissimos eruditissimosque PP. Thomas Seur et Franciscum Jacquier.* Napoli: without typographical notes.

——— 1741. *De corporum quae moventur viribus. . . .* Napoli: without typographical notes.

Dugas, R. 1955. *A history of mechanics.* Neuchatel: Griffon.

Emch, A. F. 1935. The logica demonstrativa of Girolamo Saccheri. *Scripta Mathematica* **3**, No. 1, 51–60; No. 2, 143–152; No. 3, 221–233.

Fermat, P. 1891–1912. *Oeuvres de Fermat*, P. Tannery & C. Henry, Eds. 4 vols. Paris: Gauthier-Villars.

Gardies, J. L. 1984. *Pascal entre Eudoxe et Cantor.* Paris: Vrin.

Generali, D. 1986. Periodici eruditi ed epistolari nell'Italia del primo Settecento: Il caso veneto-bolognese. *Materiali della Società italiana di studi sul secolo XVIII*, Roma.

'sGravesande, W. G. 1748. *Physices elementa mathematica, experimentis confirmata: Sive introductio ad philosophiam Newtonianam.* 2 vols. Leiden, 1720–1721: P. van der Aa & B. J. van der Aa; 2nd ed., 1725: A Langerak & J. and H. Verbeek; 3rd ed., Leiden, 1742; and 4th ed., 1748 (the ed. we used): A. Langerak & J. and H. Verbeek.

Hall, A. 1978. Le problème de la vitesse de la lumière dans l'oeuvre de Newton. In *Roëmer et la vitesse de la lumière*, R. Taton, Ed., pp. 179–194. Paris: Vrin.

Indorato, L., & Nastasi, P. 1987. De re vel de nomine: La controverse des forces vives en Italie à peu près en 1740. *Dijalektika* **22**, 59–80.

Kepler J. 1981. *Mysterium cosmographicum (The secret of the universe).* Translation by A. M. Duncan. Introduction and commentary by E. J. Aiton. With a preface by I. Bernard Cohen. New York: Abaris Books.

Leibniz, G. W. 1682. Unicum opticae, catoptricae et dioptricae principium. *Acta Eruditorum*, 185–190.

Lukasiewicz, J. 1970. Philosophical remarks on many-valued systems on propositional logic. In *Selected works*, L. Borkowski, Ed., pp. 153–169. Amsterdam/London: North-Holland.

Mairan, J. J. Dortous de 1725. Suite des recherches physico-mathématiques sur la réflexion des corps. *Histoire de l'Académie Royale des Sciences, année 1723 Mémoires*, 343–386.

Manfredi, E. 1749. *Instituzioni astronomiche.* Bologna: Lelio dalla Volpe.

Maupertuis, P. 1753. Accord de différentes loix de la nature, qui avoient jusqu'ici paru incompatibles. In *Les oeuvres de M.r Maupertuis*, 2 vols., Vol. 1, pp. 87–104. Berlin: De Bourdeaux.

Mouy, P. 1934. *Le développement de la physique cartésienne.* Paris: Vrin.

Sabra, A. I. 1981. *Theories of light from Descartes to Newton.* Cambridge: Cambridge Univ. Press.

Saccheri, G. 1647. *Logica demonstrativa.* Torino: Zappata.

——— 1733. *Euclides ab omni naevo vindicatus.* Milano: Montano.

Tacquet, A. 1754. Appendix qua demonstratur, ex falso posse directe deduci verum. In *Elementa geometriae planae ac solidae quibus accedunt selecta ex Archimede theoremata . . .*, pp. 308–312. Padova: Manfrè. Note that we have consulted the 1754 edition of Tacquet because the original edition of 1654 cannot be found in Palermo.

Westman, R. S. 1975. Kepler's theory of hypothesis. In *Kepler: Four hundred years*, A. Beer and P. Beer, Eds., pp. 713–720. Oxford/New York: Pergamon.