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P. L. Chebyshev (1821–1894) and His Contacts with Western European Scientists

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This paper is concerned with (a) a brief outline of Chebyshev's life; (b) certain background material in connection with Chebyshev's work on approximation and integration in finite terms; (c) the question of whether he already was in Paris in 1842, 10 years prior to his established and presumed first trip to Western Europe; d) his contacts with Catalan, Liouville, Hermite, Lucas, d'Ocagne, Laussedat, and Dwelshauvers-Déry; (e) his contacts with Dirichlet, Borchardt, Kronecker, and Weierstrass; and (f) Chebyshev's last two trips to the West. It is argued that the great Russian scientist did not work in isolation at St. Petersburg. Instead, he was in personal contact, at least until 1884, with many of the greatest European scientists of the time. © 1989 Academic Press, Inc.

Dieser Artikel enthält (a) einen kurzen Lebenslauf Chebyshevs, (b) Hintergrundinformationen zu seinem Werk über Approximationstheorie und Integration mittels Logarithmen, (c) die Frage ob er bereits 1842, d.h. 10 Jahre vor seiner nachgewiesenen und vermutlich ersten Reise nach Westeuropa, Paris besucht hat, (d) Chebyshevs Beziehungen zu Catalan, Liouville, Hermite, Lucas, d'Ocagne, Laussedat und Dwelshauvers-Déry, (e) seine Kontakte mit Dirichlet, Borchardt, Kronecker und Weierstrass, und (f) seine beiden letzten Reisen in den Westen. Ergebnis: der große russische Gelehrte arbeitete in St. Petersburg nicht in Isolation, sondern unterhielt, zumindest bis 1884, persönliche Kontakte mit vielen der größten europäischen Naturwissenschaftlern seiner Zeit. © 1989 Academic Press, Inc.

Cet article contient (a) un bref apercu de la vie de Chebyshev, (b) des informations d'arrière-plan sur ses travaux relatifs à la théorie de l'approximation et à l'intégration en termes finis, (c) le point de savoir s'il s'est déjà rendu, à Paris en 1842, 10 ans avant son voyage attesté, et censément initial, en Europe occidentale, (d) ses contacts avec Catalan, Liouville, Hermite, Lucas, d'Ocagne, Laussedat et Dwelshauvers-Déry, (e) ses contacts avec Dirichlet, Borchardt, Kronecker et Weierstrass, (f) ses deux derniers voyages en Occident. Il en ressort que le grand savant russe ne travaillait pas dans l'isolement à st. Petersbourg, mais qu'il restait en contact personnel, du moins jusqu'en 1884, avec beaucoup des principaux scientifiques européens de son temps. © 1989 Academic Press, Inc.

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INTRODUCTION

Although one of us has worked in approximation theory for many years, he long felt, as may others interested not only in approximation but also in probability and number theory, that he knew little about the life of Chebyshev, or Serge N. Bernstein (1880–1968) and Dunham Jackson (1888–1946) for that matter, all of whom rank among the great founders of approximation theory. In the case of Chebyshev, this may be a function of the limited Western appreciation of scientific progress in distant St. Petersburg, now Leningrad. Subsequently, the second author presented the other with new documents found in Liège, in connection with the *Nachlass* of E. C. Catalan (1814–1894). This led to a joint investigation concerning Chebyshev.

1. P. L. CHEBYSHEV—HIS LIFE IN SHORT

Pafnutii Lvovich Chebyshev, born of noble parentage on May 16, 1821, in Okatovo, Kaluga region, enrolled in the department of physics and mathematics of Moscow University in 1837. He studied in particular under N. D. Brashman (1796–1866) and N. E. Zernov (1804–1862), and received his candidacy (bachelor) of mathematics degree in 1841 and his master's degree in 1846 [Wassilief and Delaunay 1900; Posse 1907; Youschkevitch 1971]. Obtaining the *venia legendi* at St. Petersburg in 1847, he became lecturer there, receiving his doctorate in 1849; in 1850 he was elected extraordinary professor of mathematics, and full professor in 1860. He was also nominated a junior academician of the St. Petersburg Academy of Sciences with the chair of applied mathematics in 1853, and an ordinary academician in 1859. The chairs for pure mathematics at the Academy were then occupied by P. H. Fuss (1798–1855) (a great grandson of Euler), M. V. Ostrogradskii (1801–1862), and V. Ya. Bunyakovskii (1804–1889).

Chebyshev led a quiet life while in St. Petersburg, working steadily. He never married. In 1882 he decided to retire from active teaching at the university, but continued his research work at the Academy to the very end. He died at St. Petersburg on December 8, 1894.

Chebyshev is regarded as the creator of the largest prerevolutionary school of mathematics in Russia. Its most prominent members were A. N. Korkin (1837–1908), E. I. Zolotarev (1847–1878), A. V. Vassiliev (1853–1929), A. A. Markov (1856–1922) [Markov 1982], A. M. Lyapunov (1857–1918), C. A. Posse (1847–1928), D. A. Grave (1863–1939) [Grave 1982], V. A. Steklov (1864–1926) [Vladmirov and Markush 1981], G. F. Voronoi (1868–1908), and A. N. Krylov (1863–1945).

Chebyshev's merits were recognized early in his career. He was elected a Corresponding Member of the Société Royale des Sciences de Liège and of the Société Philomathique in 1856, of the Paris Academy of Sciences in 1860 and a Foreign Member in 1874 (the first Russian since Peter the Great), as well as a corresponding or foreign member of the Berlin Academy of Sciences (1871), the Bologna Academy (1873), the Royal Society of London (1877), the Italian Royal

Academy (1880), and the Swedish Academy of Sciences (1893). A selection of Chebyshev's research was published in two volumes by A. Markoff and N. Sonin [1899–1907], while his complete works appeared in five volumes much later [Chebyshev 1946–1951].

Chebyshev's 80 or so publications span a wide area of mathematics, namely approximation theory, probability theory, number theory, and theory of mechanisms, as well as many problems of analysis and practical mathematics.

As P. J. Davis [1983, 14, 119] argues, Chebyshev "is one of the patron saints of Russian mathematics, and it is due in no small measure to him that Russian mathematics today stands second to none in the world." Further, also citing Norman Levinson, Davis believes that the Russian success in space travel during the 1950s can be attributed to mathematicians of the Russian scientific establishment, intellectual great-grandsons of Chebyshev.

2. CHEBYSHEV'S CONTACTS ABROAD: GENERAL

As A. V. Wassilief [1900], one of his first biographers, reports, Chebyshev was in steady contact with Western European mathematicians. He was on especially friendly terms with C. Hermite (1822–1901), J. Bertrand (1822–1900), L. Kronecker (1821–1891), E. C. Catalan, and later, also, with F. E. A. Lucas (1842–1891) and C. A. Laisant (1841–1920). His collected works [Chebyshev 1946–1951] include the following further mathematicians who addressed letters to him: I. J. Bienaymé (1796–1878) (three letters), G. Mittag-Leffler (1846–1927), J. J. Sylvester (1814–1897), A. Mannheim (1831–1906) (two letters each), C. W. Borchardt (1817–1880), L. Cremona (1830–1903), M. Chasles (1793–1880), J. P. C. Duhamel (1797–1872), L. Koenigsberger (1837–1921), E. Lindelöf (1870–1946), J. J. Liouville (1809–1882) as well as Giuseppe Battaglini (1826–1894), Carl Ludwig Charlier (1862–1934), C. F. Craig (1855–1900), Charles Eugène Delaunay (1818–1872), Michel Gloesener (1794–1876), Gustav Adolf Hirn (1815–1890), J. Mention (1821–?) [1], Maurice d'Ocagne (1862–1938), Hermann Schapira (1840–1898), Francisco Gomes Teixeira (1851–1933) [2], and G. A. Zeuner (1828–1907) (one letter each). Furthermore, Wassilief [1900], as well as the biographer C. A. Posse [1907], mentions that Chebyshev spent "almost every summer" abroad, usually in France, particularly in Paris but, except for the trip he made in July to November of 1852, they give no dates. To expand his technological and mathematical knowledge, this grand tour (see his detailed report [Chebyshev 1852]) took him to the *Conservatoire des Arts et Métiers* in Paris, the railway between Paris and St. Germain, the mines and foundries of Lorraine, the paper mills of Lille, the munitions factories of Châtellerault, and the machines built by James Watt (1736–1819) [3] in Britain, as well as to the well-known London firms Maudsley Son and Field, D. Napier and Sons, and John Penn and Sons, all builders of large steam engines, including ships' engines [4]. On the mathematical side, his contacts in Paris included Liouville, Bienaymé, Hermite, J. A. Serret (1819–1885), V. A. Lebesgue (1791–1875), and J. V. Poncelet (1788–1867), and in Metz, C. A. J. de Polignac (1832–1913). In London there were Sylvester, A. Cayley (1821–1895), and the

engineer C. H. Gregory (1817–1898) [5]. In Berlin it was P. G. Lejeune Dirichlet (1805–1859).

Many of Chebyshev's publications were published in major journals abroad, 17 of them in Liouville's journal, and at least 10 in other French journals. His 2nd, 4th, and 16th papers appeared in Germany in *Crelle's Journal,* and five of his contributions (after 1885) appeared in *Acta Mathematica.* Most of the remaining publications are to be found in the two journals of the St. Petersburg Academy, renowned since Euler's days at the Academy.

3. SOME BACKGROUND TO CHEBYSHEV'S WORK ON APPROXIMATION

Chebyshev's first work on approximation, namely his paper "Théorie des mécanismes connus sous le nom de parallélogrammes" [Markoff and Sonin 1899–1907 I, 111–143], was read to the St. Petersburg Academy on January 28, 1853, and published in 1854. This work, followed by "Sur les questions de minima qui se rattachent à la représentation approximative des fonctions" [Markoff and Sonin 1899–1907 I, 270–378], read October 9, 1857, but published in 1859, marked the beginning of his 40-year research on approximation and the study of mechanisms. Thematically, it included the theory of orthogonal polynomials, interpolation, theory of moments, integration, approximate quadratures, and continued fractions.

As A. P. Youschkevitch [1971] notes, Chebyshev's first paper on approximation was mainly prepared during his 1852 trip to France and England. The only work Chebyshev explicitly cites is that of Poncelet on practical mechanics and Watt's parallelogram. In particular, Watt's steam engine first led him to construct a linkage which converts circular to straight line motion with less discrepancy than that of Watt, and finally led him to new problems in approximation, as Chebyshev [1852] himself asserts. The complete mathematical solution of the link-motion problem was given by C. N. Peaucellier (1832–1913) in 1873, and independently by the Chebyshev student Lippman Lipkin (1851–1875) in 1871. (In fact, Lipkin discovered the solution in 1868 and presented it to the Petersburg Academy in 1870.) Lipkin had studied in Königsberg (1866), at the Gewerbe-Akademie in Berlin, in Jena (where he received a degree), and in St. Petersburg [Davis 1983, 31 ff] [6].

The fact that Chebyshev returned to St. Petersburg on November 7, his paper [Markoff and Sonin 1899–1907 I, 111–143] being presented to the Academy less than 3 months later, suggests that it was mainly written during the 1852 trip to France and England. Further, Watt and Poncelet take up about 2½ pages of his 12-page report of his 1852 trip [Chebyshev 1852].

Chebyshev's interests in applied mechanics go back to his earlier years. Without minimizing his natural gift for the subject, one could well imagine that Chebyshev's interests in mechanisms were enhanced not only by those of his teacher Brashman [7] but also by the outstanding contributions of the two French scientists Gabriel Lamé (1795–1870) and Benoit-Pierre-Emile Clapeyron (1799–1864) to

the development of mathematics, mechanics, applied physics, and the art of constructions at the Institute of Ways of Communication, where both taught during their "exile" in St. Petersburg (1820–1831) [Smirnov 1970; Bradley 1981] [8]. It was also at this Institute, Russia's first technical school noted for construction mechanics and civil engineering, that Chebyshev's colleague Ostrogradskii taught from 1830 on. Chebyshev's interest in applied mechanics is also documented by the series of lectures he presented on the subject between 1849 and 1851, a year before his trip [Youschkevitch 1971, 224].

4. CHEBYSHEV'S WORK ON INTEGRATION IN FINITE TERMS: HIS CONTACTS WITH LIOUVILLE

A topic of importance in connection with Chebyshev's work on integration, as well as with the beginning of his mathematical career, is his thesis *pro venia legendi,* giving him the right to teach at St. Petersburg. This thesis, entitled "On Integration by Means of Logarithms," defended in the spring of 1847, was written "at least in the first draft as early as the end of 1843," according to Youschkevitch [1971, 223]. The history around this thesis raises some open problems. Chebyshev's thesis was published only after his death (namely in 1930, reprinted in [Chebyshev 1946–1951 V, 88–140]); it was Liouville who commissioned Chebyshev on August 28, 1852, to write a revised French version for publication in the *Journal de Liouville* (where it appeared in 1853; see [Markoff and Sonin 1899–1907 I, 147–168]). Chebyshev confirms this in his report [Chebyshev 1852, xv], adding that "Liouville and Hermite suggested the idea to develop the principles on which my thesis had been based." Further, "in this paper [thesis] I considered the case where the differential under the integral contains the square root of a rational function. But it was interesting in several respects to extend those principles to a root of any degree," as suggested by the two.

In this thesis Chebyshev established a conjecture of Abel of 1826 to the effect that if the integral $\int (\rho(x)/\sqrt{R(x)})dx$, ρ and R being polynomials, is expressible by logarithms, then it can be written in the form

$$\int \frac{\rho(x)}{\sqrt{R(x)}}\,dx = c \log \frac{p + q\sqrt{R}}{p - q\sqrt{R}},$$

where p and q are entire functions and c is a constant. Upon the suggestion of Liouville and Hermite, Chebyshev in 1852 considered more generally the integral $\int (f(x)/\sqrt[m]{R(x)})\,dx$, where f is only supposed to be rational but R is still a polynomial. Now according to a result of Liouville and Abel, if this integral is expressible in finite form, it has to be of the form

$$U + c_0 \log V_0 + c_1 \log V_1 + \cdots + c_n \log V_n,$$

where U, V_0, V_1, . . . , V_n are rational functions of $x + \sqrt[m]{R(x)}$, the c_i being constants. Chebyshev himself first showed how to determine the algebraic part U, thereby generalizing Ostrogradskii's method, and then determined how many terms of the form $c_i \log V_i$ are needed. In particular, $\int (\rho(x)/\sqrt[m]{R(x)})dx$

is not expressible in finite form if $R(x)$ has no roots of multiplicity greater than m and ρ is a polynomial of degree less than the degree of $\sqrt[m]{R(x)}/x$. He also showed how to reduce the general problem to that of deciding the integrability of $\int ((x + c)/\sqrt{R(x)})dx$ in logarithms. Weierstrass who, together with Chebyshev, became the second co-founder of approximation theory (with the theorem of 1885 named after him), already criticized Chebyshev's methods of 1857 in that same year; he preferred to solve the problem using Jacobi's theory of elliptic functions, which gave a "clearer and deeper insight into the essence of the matter" [Lützen 1984b, 268].

Chebyshev wrote five further papers on the subject (appearing in 1857, 1860, 1861, 1865, and 1867) and it was followed up by Zolotarev in 1874. Whereas Dirichlet had shown vivid interest in Liouville's work (see his letter of May 6, 1840 to Liouville in [Lützen 1984b, 254]), Chebyshev was the first to become actively engaged in it. J. F. Ritt [1948] published a comprehensive book on integration in finite terms.

According to Youschkevitch [1971, 223], Chebyshev's thesis solved a problem "posed shortly before by Ostrogradskii," but Youschkevitch gives neither dates nor references. However, Chebyshev, both in the French version of 1853 of his thesis and in the original Russian version, refers only to work of Abel and Liouville on the matter; also in his paper of 1857 [Markoff and Sonin 1899–1907 I, 169–200], he does not cite Ostrogradskii, but only Abel, Liouville, and Hermite. In fact, one of Liouville's chief mathematical interests between the years 1833 and 1841 was the field of integration in finite terms in which he continued work begun by Abel in 1823, as can be deduced from the outstanding work by Jesper Lützen [1984b] on Liouville. Since Ostrogradskii's first paper on integration of rational functions, presented to the St. Petersburg Academy on November 22, 1844, appeared in 1845 (see [Lützen 1984b, 262]), but Chebyshev's first draft was completed by the end of 1843, one may question the claim that Chebyshev's thesis was inspired by Ostrogradskii. In any case the latter was in far-off St. Petersburg, Chebyshev still in Moscow at the time. However, it is possible that Chebyshev's work here was also influenced by that of Brashman and O. I. Somov (1815–1876) [9] who in turn were perhaps stimulated by the publications of Abel and Liouville.

Another field of interest of Liouville to which Chebyshev was attracted is the stability of equilibrium figures of a rotating fluid mass of particles, mutually attracted according to Newton's law; that interest dates to the period 1834–1843 and has also been examined by Lützen [1984a]. Lyapunov, one of Chebyshev's most prominent disciples, wrote a superb paper on the subject in 1884 (his thesis), in which he referred to all of the few notes Liouville published on the subject. This is especially surprising since three standing mathematicians, namely Riemann (in 1860), W. Thomson (1882), and Poincaré (1885), worked on the subject without knowing of Liouville's work; Thomson was even in contact with Liouville for some 30 years. The explanation lies in the fact that in a later publication Lyapunov (1904) called the matter a problem of Chebyshev. It is known that Chebyshev gave much thought to the subject even though he published nothing on it (see [Gne-

denko 1978; Youschkevitch 1971]). V. I. Smirnov and A. P. Youchkevitch [1987], who published the correspondence between Lyapunov and Poincaré of 1885/1886 [10], as well as Dr. Lützen [11], indeed believe that Chebyshev received the idea for the problem he posed to his doctoral student from Liouville. In this respect, Liouville's chief work on the theory of stability was carried out in the winter of 1842–1843 and presented to the Paris Academy on November 14, 1842, but strangely enough it was only sent for publication in 1852 (see [Lützen 1984b, 12, 80]). (Did perhaps Chebyshev convince Liouville to send the manuscript off for publication while in Paris in 1852?)

The connections between the mathematical activities of Liouville and Chebyshev described naturally led to personal contacts between the two, chief occasions for these being Chebyshev's numerous summer trips to Paris. In fact Vassiliev states that Chebyshev spent "almost every summer abroad" and Posse [1907 II, iv] adds that when he indeed remained in Russia for his vacation, he stayed in Catherinenthal (near Reval).

From Liouville's notebooks [12] we know that from 1852 on such meetings took place in 1856, 1864, 1873, and 1878. At least in his earlier years Chebyshev's abode in Paris was the modest "Hôtel Corneille," opposite the Odéon, thus very near the rue de Condé, Liouville's lodging. On the other hand, late each summer, from August to November, Liouville left Paris for Toul, where he had a house and vineyards. Chebyshev paid him a visit in Toul at least once, in 1873. Several times a Russian geographer interested in mathematics and who settled in Paris, namely N. W. Khanykov (1819–1878), served as an intermediary between Chebyshev and Liouville, as he also did between him and other Frenchmen, Chasles for example.

One could presume that Chebyshev and Liouville exchanged many letters, but this does not seem to be so. Chebyshev was not much of a letter writer, as will be seen at length in the next section. Although Chebyshev published 17 articles in *Liouville's Journal,* Liouville's *Nachlass* in Paris, recently investigated in detail by E. Neuenschwander [1984], includes just one letter, written by Chebyshev at a time when he was about to leave Paris (October 1873) and Liouville was at Toul, so that he had no other choice. Further, in Liouville's other *Nachlass* located at Bordeaux [13] no letters by Chebyshev have been found. Conversely, only one letter by Liouville to Chebyshev, written on March 19, 1864, is recorded.

5. CHEBYSHEV'S FIRST CONTACTS WITH THE WEST

The question arises as to when and how the contacts between Chebyshev and Liouville did begin. In his report of 1852, two pages of which are devoted to his contacts with Liouville during the course of his trip, Chebyshev mentions that he "collaborated with [Liouville's] Journal since 1842," an assertion confirmed by the fact that his first published paper, "Note sur une classe d'intégrales définies multiples" [Markoff and Sonin 1899–1907 I, 3–6], appeared in this journal in 1843. The manuscript could have reached Liouville either by mail, with an explanatory letter, or by a messenger; a third possibility is that Chebyshev gave it to Liouville in person.

The first way was obviously the simplest, but an accompanying letter (possibly superfluous for later papers) is not recorded. And for Chebyshev the simplest way was not always the best. It must here be emphasized how notoriously bad a correspondent he was. Sylvester [Chebyshev 1946–1951 V, 449 (Dec. 23, 1872)] expressed this as "I know that not to answer letters is your rule"; similar sentences by Catalan ("vous n'écrivez jamais") or Hermite confirm this judgment. Whenever he could, Chebyshev preferred the use of an intermediary, such as Khanykov. To measure his dislike for letter writing, let us take the example of Catalan, who received news from Chebyshev via (the older) Bunyakovskii, as will be seen below. When Bunyakovskii ceased writing, Catalan feared that the elderly gentleman had died (which was the case), and he begged Chebyshev twice to inform him. Even then, Chebyshev did not reply. Perhaps Catalan did not fully realize that Chebyshev was unable to report on Bunyakovskii's situation precisely because their intermediary (this same Bunyakovskii) was dead.

There is one exception to this strong reluctance for letter writing: Sophie Kovalevskaya received at least six letters from Chebyshev. Perhaps this is due to some sense of gallantry, combined with the lack of intermediaries for Stockholm.

Let us now examine the possibility that Chebyshev sent the first manuscript he wrote to Liouville via some Russian traveling to Paris. In this respect there is in Catalan's *Nachlass* [Hoyoux 1974] a letter by L. B. Francoeur (1773–1849) addressed to him, dated December 13, 1842, presenting the latter a candidate for private lessons in trigonometry and logarithmic calculus. The name of this candidate reads like Tchébiatchef. Initially we thought he was our Tchebichef, the transliteration that Chebyshev himself used in France. But a closer examination led us to the conclusion [Butzer and Jongmans 1989] that Francoeur most probably was speaking of Piotr Aleksandrovich Tchihatchef (1808–1890), a Russian geographer and naturalist, who, returning from an extensive expedition to the Altai Mountains during the summer of 1842, traveled to Paris for the purpose of publishing a book devoted to his expedition. It is quite possible that this Russian scholar brought Chebyshev's manuscript along with him, showed it to Catalan, who in turn presented it to Liouville for publication in his journal. The way via Catalan would also explain the publication of Catalan's two-page complement right behind Chebyshev's article in the very same issue of June 1843 of volume 8 of *Liouville's Journal*.

Although the preceding procedure seems very plausible, one cannot exclude a further one, namely that Chebyshev himself brought his manuscript to Paris. One could well imagine that the 20-year-old Chebyshev traveled to Paris together with Tchihatchef, and so came in contact with Catalan, who introduced him to Liouville. According to Posse [1907], Chebyshev's family was far from wealthy at the time, but Tchihatchef, traveling under the commission of the Tsar with high financial means at his disposal, could have taken him as a temporary secretary. Tchihatchef, also a member of the gentry, was his senior by 13 years.

A possible trip by Chebyshev to Paris at the end of 1842 does not stand in contradiction to his vita. His studies at Moscow University ended in June 1841,

and it was only on April 19, 1843, that Chebyshev requested admission to the magister exams in mathematics, the first exam in literature taking place in Moscow that day (on April 19), in pure and applied mathematics on September 23, 1843. Theoretically this would leave the period July 1841 to March 1843 free for Paris. We must confess, however, that the tempting hypothesis of such a trip to Paris in 1842 (possible within the given time limits even when traveling by stagecoach and train for the completed sections of track), which speaks against the general belief that Chebyshev's first trip there took place only in 1852, cannot be corroborated by precise facts up to now. Recall that neither Posse [1907] nor Wassilief [1900] gives explicit dates when referring to Chebyshev's trips abroad, except for his 1852 tour. Nor does Chebyshev in his report to the Tsar speak directly of a previous trip of his to the West. Also Prudnikov [1964] fails to raise the question of Chebyshev's first possible trip. If Chebyshev planned a scientific career in 1841, going to Paris and the West would be in keeping with a tradition going back to Peter the Great. Thus Ostrogradskii and Bunyakovskii, who were to become his senior colleagues in 1847, studied in Paris from 1822 to 1827 and 1823 to 1825, arriving there at the ages of 21 and 19, respectively. S. N. Bernstein studied there in 1898–1902 (and in Goettingen in 1903–1904). Both Bunyakovskii and Bernstein even received their doctorates in Paris. But Chebyshev would not have been enrolled in Paris as a student. Our latter hypothesis would also help us to understand Chebyshev's mathematical development more fully, in particular that his thesis *pro venia legendi* lay in the same direction as work by both Abel (1826–1829) and Liouville (1833–1841), and that at least the first draft of his thesis was already completed by the end of 1843.

6. CHEBYSHEV'S CONTACTS WITH OTHER FRENCH OR BELGIAN SCIENTISTS

The relationships established by Chebyshev with French mathematicians, particularly during his trip of 1852, were often maintained for many years. Bienaymé, known for the Bienaymé–Chebyshev inequality in probability theory (which he formulated some 15 years before Chebyshev), may be mentioned in this respect; he has been suitably treated by Heyde and Seneta [1977, 13–14], so that we only add a comment. These authors presume that the election of Chebyshev as an associate member of the Paris Academy of Sciences (May 18, 1874) was largely due to Bienaymé's efforts. However, Bienaymé, as a "free academician," had no right to vote in such circumstances, whereas Hermite, a close friend of Chebyshev, was in a far better position to promote his candidacy [14]. Further, the very bad health of Bienaymé during his last 5 years (confirmed by three letters to Catalan in 1875, 1876, and 1877), would not have allowed him to do much, and even the writing of a letter required immense efforts on his part.

The two names just mentioned, Hermite and Catalan, are those of men who became Chebyshev's real friends. We have elsewhere treated the case of Catalan [Jongmans 1986a,b] at length. Catalan was the first mathematician who reacted to the appearance of Chebyshev on the mathematical stage: the issue of June 1843 of

Liouville's Journal presents Chebyshev's first paper (pp. 235–238) together with Catalan's comment on it (pp. 239–240). For this puzzling fact we offered in Section 5 three possible explanations: either Chebyshev sent the manuscript by mail to Liouville, the manuscript was handed to Catalan by Piotr Tchihatchef arriving in Paris at the end of 1842, or Chebyshev himself came to Paris at that time and showed the paper to Catalan, who introduced him to Liouville. Whatever the solution, no explicit mention of Catalan can be found in Chebyshev's report on his trip to France and England during the summer of 1852, a silence explicable by the danger to mention, in an official report to the Tsar, contacts with a man known for his hot republicanism and revolutionary tendencies. In fact, until recently (see below) the first explicit sign of personal contacts between Chebyshev and Catalan was a letter by the latter written in 1876, after a congress which both attended at Clermont-Ferrand, a letter quoted below in connection with Edouard Lucas; the very familiar tone of this and of the following three letters from Catalan to Chebyshev lead us to infer that the two men met well before that year, in the course of earlier summer trips by Chebyshev to Paris. It is not surprising that no letters from Chebyshev to Catalan are known, but here Chebyshev used a particular intermediary, the mathematician Bunyakovskii, whose letters to Catalan always contain short messages from Chebyshev; quotations from these letters are given below, namely about the arithmetic machine and the presence of Chebyshev at Liège on December 7, 1884, when he presided over the ceremony honoring Catalan on the occasion of his retirement from Liège University at the age of 70. Catalan addressed him as "Mon illustre ami." Catalan received many reprints of Chebyshev's works and he published two short papers by his friend in his journal *Nouvelle correspondance mathématique* (1876, 1878).

The first scientific societies outside Russia to elect Chebyshev as a corresponding member were the Société royale des sciences de Liège and the Société philomathique in Paris. According to Prudnikov [1964, 272], the respective diplomas bear the dates December 8, 1856, and December 20, 1856. At that time, Catalan was not yet professor at Liège, but he was an active member of the Société philomathique and even its president during the first trimester of 1856. In the beginning of 1856, Michel Gloesener, a professor of physics at Liège [15], proposed Catalan as corresponding member of the Société royale, and immediately afterward he himself received a similar membership in the Société philomathique; several letters from Gloesener to Catalan give details about this exchange [16]. A further agreement between the two may have led to a simultaneous proposal of Chebyshev as a member in both societies. In fact, it was Catalan who presented Chebyshev, also in the name of Bertrand and Puiseux, as a corresponding member of the Société philomathique in its session of December 6, 1856 [17]. Then on January 3, 1857, Catalan presented to the Société two precious volumes offered by Chebyshev, containing the collection of Euler's arithmetical memoirs. The apparent triangular game between Catalan, Gloesener, and Chebyshev was complemented in 1882 (after Gloesener's death) by the election of Catalan as a corresponding member of the St. Petersburg Academy of Science.

Chebyshev's closest friend in the West seems to have been Charles Hermite. Probably one of the most prolific letter writers among the mathematicians of the last century, Hermite wrote Chebyshev at least 11 times, by far the best score of the latter's correspondents, and remarkable considering that no written answer would have been expected. The content of these letters, published only in a Russian translation [Chebyshev 1946–1951 V, 424–436], is mathematical, but they allude to several personal meetings in Paris. In particular, the last letter (November 23, 1893) reports that neither time nor distance can delete from Hermite's memory the remembrance of conversations at the Hôtel du Louvre, notably about a nice discovery in which astronomy was involved. Moreover, Hermite explicitly refers to two letters received from Chebyshev, not as an answer to his own letters, but about the election of Hermite as a member of St. Petersburg Academy in 1858 and the publication in *Liouville's Journal* of a translation by Bienaymé of a paper by Chebyshev on continued fractions. The deep understanding between them could derive partly from the fact that not only Liouville but also Hermite gave Chebyshev the idea to extend the results of his thesis, and partly from the fact that they were practically of the same age and of the same mathematical level.

Another French friend of Chebyshev, Edouard Lucas, was considerably younger, being born in 1842. Chebyshev's first encounter with Lucas could have taken place in 1876, during the Clermont-Ferrand session of the Association française pour l'avancement des sciences, which both attended. A letter written near the end of that year (November 20) by Catalan to Chebyshev [18] relates an evening spent together by Chebyshev, Lucas, Catalan, and his wife during the session in the following terms.

> J'espère que tous vos voyages ont été heureux, comme celui de Clermont-Ferrand. Ma femme se réjouit, encore, quand elle se rappelle la bonne soirée que nous avons passée à l'Hôtel de la Poste. Edouard Lucas, qui nous a tant fait rire avec lame-en-table, est Professeur au Lycée Charlemagne à Paris; là où j'étais en 1847. Sans ce brigand de Bonaparte, je serais encore Parisien; du moins la chose est probable!

A similar letter from Catalan to Cremona confirms the great ability of Lucas for jokes. But he was above all a good specialist of number theory, which induced Chebyshev, according to Prudnikov [1964, 240], to mention Lucas's contributions repeatedly during his lectures at St. Petersburg. Later, when he became the director of the Conservatoire national des arts et métiers in Paris, Lucas took interest in the various mechanisms devised by Chebyshev. In 1890, some of Chebyshev's models were gathered in a special showcase at the Conservatoire while others were exhibited as photographs. The five known letters from Lucas to Chebyshev treat both topics, number theory and mechanisms; unfortunately, they are not published, except for the last, written shortly before Lucas' death (June 10, 1891). We reproduce a translation from the Russian translation by Prudnikov [1964, 240–241].

> Dear and illustrious professor,
>
> I thank you for the sending of photographs. I shall transmit them to the Musée, where they will be framed and exposed. You will soon receive an extract from the review *Nature* with the

last photographs of the ship propellor, as well as drawings of your parallelograms. I shall soon send you the first volume of my number theory, which I pray you to accept in memory of me.

The calculating machine, or arithmometer, conceived by Chebyshev, deserves a special investigation in this context. The underlying principles were already explained in 1876 at Clermont-Ferrand, but it was not until 1882 that the machine was constructed in Paris by the firm Gautier, and shortly afterward presented at the La Rochelle session of the Association (August 1882); a brief notice about the machine then appeared in the *Revue scientifique* (September 22, 1882).

In the third volume of his *Récréations mathématiques*, published after his death by Gauthier-Villars (1893), Lucas adds an interesting footnote (p. 74):

M. Tchébitchef vient de nous confier l'unique exemplaire de sa machine pour quelques mois et nous autoriser à en faire prendre des dessins qui resteront exposés dans les galeries du Conservatoire. La partie principale de la machine est l'*additionneur*, qui donne la seconde solution rigoureuse du problème par le côté cinématique.

This temporary gift of the machine lasted for years, perhaps because Chebyshev did not return to Paris before Lucas' death. Whatever the reason, when Colonel Aimé Laussedat (1819–1907), the "father of photogrammetry" [19] and another correspondent of Chebyshev, became the head of the Conservatoire des arts et métiers, he gave Maurice d'Ocagne the task of writing up that part of the catalogue that concerns the arithmetic machines of the collections. The latter began a series of talks about them in 1893, and he wrote to Chebyshev, on February 25 of that year, asking for more detailed explanations about the arithmometer. Among the papers published by d'Ocagne in connection with his talks, the one appearing under the title "Le calcul simplifié par les procédés mécaniques et graphiques" [20] deserves special mention; first, it contains, in a complementary note approved by Chebyshev (pp. 269–281), a very detailed description of the arithmometer, and second, it reports that Chebyshev came to Paris in 1893 to spend the month of May, personally giving all explanations required by d'Ocagne. Its author adds the information about the construction by Gautier in 1882, in the second edition of his paper (p. 59), together with Chebyshev's approval of the transliteration of his name as Tchebichef.

At the end of this apparently last trip to France, Chebyshev donated his arithmometer to the Conservatoire. On June 2, 1893, during a visit to this institution, he was thanked for all his generous gifts by the director Laussedat [21]; on June 4, the entry of the epicycloîdal train, the essential part of the machine, was registered. In fact, Chebyshev had developed prior contacts with Laussedat; two of the four (unpublished) letters he received from Laussedat were written in June 1893, but the others bear much earlier dates, namely February 9, 1881, and May 18, 1889; a very short summary of them is given in the *Complete Works* [Chebyshev 1946–1951 V, 445–446].

One question about the arithmometer remains open, namely when—between 1882 and 1891—Chebyshev initially loaned it to Lucas "for a few months." Indirect information can be deduced from two letters by Bunyakovskii to Catalan in

1884. The first, dated January 15, 1884, contains the following sentence: "L'exécution de la machine arithmétique de M. Tchébychef avance rapidement, et il est presque certain qu'elle sera prête assez à temps pour qu'il puisse entreprendre sa tournée à l'étranger, et de Vous visiter par conséquent." The second letter (May 21, 1884) simply adds: "Mr. Tchébychef vous salue amicalement: il se propose d'entreprendre sa tournée a l'étranger, mais pas avant cet automne." This last sentence may have been an attempt to conceal Chebyshev's intention to be present at the homage ceremony to Catalan, in December 1884. But this does not alter the essential purpose of Chebyshev's trip, to demonstrate his machine in several Western cities, such as Berlin, Liège, and Paris. Since Lucas and d'Ocagne insist on the existence of a single specimen of the arithmometer, the letters by Bunyakovskii can be interpreted to infer that after its construction in 1882, Chebyshev conceived of some improvements, realized in Russia during 1883 and the beginning of 1884. The projected trip materialized in late 1884, when Chebyshev presided over the ceremony in honor of Catalan on that December 7. It may be conjectured that Chebyshev's tour ended in Paris at the very close of 1884 or during the first days of 1885, and that Lucas received the "temporary" gift at that time.

In Russia itself the arithmometer roused the interest especially of W. von Boole (1836–1899), who wrote about it in a letter to Chebyshev during the last weeks of the latter's life, i.e., on October 14, 1894. In 1896 von Boole devoted an article to Chebyshev's machine, as well as a second one to calculating machines conceived by Kummer and Bunyakovskii. Although this goes out of the frame of our topic, the first paper is of interest to us since it confirms [see review in *Fortschritte der Mathematik* **27** (1896), 40] that the single specimen of the arithmometer was in Paris. However, Youschkevitch [1971] asserts that another example is preserved in the Moscow Museum of History. Perhaps this is the simpler model constructed about 1876, intended for addition and subtraction only. To the list of Russian authors mentioned by Youschkevitch as having published books about calculating machines, let us add Apokin and Maistrov [1974] [22].

The last person to be mentioned among Chebyshev's friends in Western Europe is the engineer Dwelshauvers-Déry, a professor (and later rector) at the University of Liège. He was a specialist on steam engines and probably came in contact with Chebyshev in connection with related mechanisms. He was involved in the translation into French of two papers already published in Russian by Chebyshev, translations that appeared in the *Revue universelle des Mines*, edited in Liège. The four letters received by Chebyshev from Dwelshauvers range from 1872 to 1893. Although it was not published in the *Complete Works*, it would nevertheless be interesting to know whether the second, written on December 16, 1884, just after Chebyshev's visit to Liège, contains information about that event or about the arithmometer. Prudnikov [1964] published a Russian translation of the fourth letter, dated September 27, 1893; an English translation follows, the copy kindly forwarded to us by Dr. Yavelov:

My dear master!

You honoured Chicago with an exhibition of your mechanisms and in particular with a model which was interesting to many people and in particular to me; it was a bicycle for women. I am interested in this bicycle since, I suppose, it will help the troubles of my mother-in-law, who is almost immobilized by sciatica.

Also I should like to ask you whether you agree to exhibit the same things you presented in Chicago at Antwerp in 1894. I shall esteem it a duty to help you in it as far as it will be possible to me.

My wife and my son pass kind regards to you and, with me, they deeply regret that we are deprived of the pleasure of seeing you; however they would like that you at least do not deprive them of your letters.

Besides the usual complaints about Chebyshev's reluctance to write letters, this document shows that both Dwelshauvers and his family were quite familiar with Chebyshev, who probably visited them before December 1884; to visit Catalan and Dwelshauvers, Chebyshev merely had to step off the train at Liège when traveling to Paris, as he did in Berlin for Borchardt. Other connections must have existed, for it was from a member of the Russian Academy that Mrs. Dwelshauvers heard of Chebyshev's illness in 1886 (cf. the second letter from Catalan to Chebyshev).

Perhaps the most interesting part of the foregoing letter is the reference to the World's Columbian Exposition held in Chicago during 1893. According to Prudnikov [1964, 242, 465], the Pedagogic Museum in St. Petersburg, with which Chebyshev collaborated, was invited to take part in this exposition. The Chicago exhibits of this Museum included seven mechanisms due to Chebyshev, which aroused great interest and won a prize. We cannot exclude that Chebyshev went to Chicago as a private visitor of the exposition; being in Paris at the beginning of June, part of the way was already accomplished. This may even have been a pretense for a last sojourn in Paris, a halt full of remembrances and therefore prolonged beyond necessity. At any rate, we know that Chebyshev was not in Chicago as a delegate of his government, like the Prussian commissioner Felix Klein. Moreover, Chebyshev attended neither the Chicago Congress of Mathematics and Astronomy (August 21–26) nor the following Evanston Congress (August 28–September 9) [23]. As for Chebyshev's participation in an exhibition in Antwerp that next year, we are inclined to exclude it for the simple reason that there are no additional letters concerning it between Chebyshev and Dwelshauvers in 1894.

7. CHEBYSHEV AND BERLIN MATHEMATICIANS

In [Chebyshev 1852], a half page is devoted to Lejeune Dirichlet, Germany's most renowned mathematician at the time [Butzer 1988]. He writes that "it was of great interest for me to become acquainted with the celebrated geometer Lejeune-Dirichlet," that the most important of this "savant's" investigations were the applications of infinitesimal calculus to number theory, that he himself "found an occasion each day to talk with this geometer concerning this research as well as

other questions on pure and applied analysis," and that he attended "with particular pleasure one of his lectures on theoretical mechanics." He further regretted deeply that he already had to leave Berlin on October 30, due to the unexpected setting in of ice in the Gulf of Finland.

The French translation of Chebyshev's report seems to imply that Chebyshev met Dirichlet for the first time in 1852 but, oddly enough, whether such possible meetings are noted in Dirichlet's *Nachlass* has not been checked [24].

Furthermore, Chebyshev states explicitly that he talked to Dirichlet about his investigations of 1848–1852, concerning the distribution of prime numbers [Tchebichef 1852a,b]. There, using combinatorial methods, he proved that the function $\pi(x)$—the number of primes not exceeding x—satisfies the inequality

$$c_1 < \frac{\pi(x)}{x/\log x} < c_2, \qquad (*)$$

where $0.921 < c_1 < 1 < c_2 < 1.106$. He mentions that Legendre [1785] had surmised the result. Chebyshev in his work used the zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ for real s, already employed by Euler [1737] and Dirichlet in his work [1837, 1839]. Note that (*) enables one to prove Bertrand's postulate of 1845, namely that for $n > 3$ there is always at least one prime between n and $2n - 2$. We concur with E. Landau's assessment [1909]: "Concerning the general prime number problem, Chebyshev was the first after Euclid to make the first sound steps and to prove important theorems."

A question arises whether Chebyshev ever met August Leopold Crelle (1780–1855), founder (in 1826) of the *Journal für reine und angewandte Mathematik*. Thus Chebyshev's papers on convergence of Taylor series [1844] and on a proposition in probability theory [1846] appeared there. Did Chebyshev send his manuscripts to Crelle by post, messenger, or in person in the years 1843–1844 and 1845–1846? There is a slight possibility that he presented the first paper to Crelle in person, considering that Dirichlet was absent from Berlin at the time, being in Italy from July 1843 to June 1844. In fact, it is unlikely that Chebyshev would meet with Crelle without also trying to see Dirichlet, since the former played a relatively minor role in mathematics whereas Dirichlet and Crelle had close relations, at least through Crelle's journal. See [Eccarius 1972, 1976].

On the other hand, no direct contacts between Chebyshev and Crelle have been documented, at least not in German sources. Thus in the *Nachlass Crelle*, in the archives of the Akademie der Wissenschaften der DDR, no material is listed in connection with Chebyshev. But this *Nachlass* is incomplete since Crelle's correspondence was sold in 1856 and scattered, and thus practically lost. That which remains includes manuscripts submitted for possible publication in *Crelle's Journal*, as Dr. W. Eccarius (Eisenach, GDR) has kindly informed the authors. The only original manuscript of Chebyshev's in this archive is "Note sur la convergence . . . ," as Dr. Christa Kirsten, Director of the Zentrales Akademie-Archiv, has kindly written us. However, its title page is missing, so that possible remarks by Crelle concerning the form of its transmittance (by post, messenger, or directly

from Chebyshev) cannot be ascertained. The archive lacks any letters by Chebyshev to either Crelle or Borchardt, the latter being editor of the said journal in 1856, when Chebyshev's third paper (a note of a single page) was published there. It is probable that Chebyshev sent his papers of 1844 and 1846 to Crelle by mail or perhaps via a Russian traveler, such as Khanykov.

Concerning the third paper of 1856, we can consider the connection between Chebyshev and Borchardt in greater detail. In [Chebyshev 1946–1951 V, 452] one finds a letter written by Borchardt to Chebyshev on December 14, 1879. First, we give an English translation of the French retranslation by M. Hyart [25] of the Russian translation of the original letter, written in French by Borchardt.

Berlin, December 17, 1879

Dear Mr. Tchebychev,

I am very moved by the honour conferred to me to be elected as a corresponding member of your famous Academy, and at the same time—a fact increasing the importance attached to it by me—by the fact that I learned this from you. I infer from this that you are at the origin of this great honour.

I reserve to myself the right to express my gratitude to your Academy when I shall be officially informed. In the meantime, I beg you to be the interpreter of my sincere gratitude among your colleagues.

Some years ago I had the great pleasure to receive the visit of your fellow-countryman and pupil Zolotarev. I later sent him a reprint of my memoir on the arithmetic–geometric means of four numbers, and I was deeply grieved when the post-office sent it back with the mention of his death.

I hope, dear Mr. Tchebychev, that you will soon find time for the trip that will lead you to Berlin for some time. I know few geometers with the same ability as you for giving their friends, during the conversation, an insight into the main ideas of their works.

It will be a great pleasure for me to receive you. In the meantime believe in the devotedness of a sincere friend.

C. W. Borchardt

The friendly tone of the letter, as well as the phrase that Borchardt knew few geometers with Chebyshev's ability to convey insights into the main ideas of their works during a short conversation, suggest that Borchardt must have met Chebyshev several times. In fact, in the proposal that Chebyshev be elected as a corresponding member of the Berlin Academy [see Biermann 1960], signed by Borchardt, Kronecker, Kummer, Helmholtz, and Weierstrass on June 19, 1871, one reads: "In his last stay in Berlin he [Tschébychew] demonstrated a model of a mechanism he designed, which exceeds with a relatively simple construction the precision of Watts' parallelogram [in the theory of practical mechanics] as well as all similar devices up to now." This proves that Chebyshev had stopped over more than once in Berlin well before June 1871 to meet these German mathematicians. Moreover, it was Borchardt himself (and not Kronecker or Weierstrass)

who wrote up the foregoing proposal. The most natural starting point of the relations between Borchardt and Chebyshev was the year 1856 when Borchardt became Crelle's successor as editor. Perhaps Chebyshev took this occasion to introduce himself to Borchardt as well as present him his paper of 1856, noting that his trip to Paris in 1856 allowed him to stop over at Berlin. Further, Borchardt had the opportunity to meet Chebyshev again on any of the latter's many trips to and from Paris. However, in his letter of December 1879, Borchardt writes that he hopes Chebyshev will soon find time to stay in Berlin for a longer period, which implies that Chebyshev never stayed in Berlin very long, at least in the presence of Borchardt. Borchardt's wish could hardly be realized since he died shortly afterward (June 27, 1880).

Returning to the election proposal of 1871, Borchardt began his appraisal with the words "Mr. Pafnutij Tschébychew, member of the Petersburg Academy of Sciences, belongs to the most gifted mathematicians of the present time." He continues with an excellent, two-page description of Chebyshev's achievements—Chebyshev was only 50 years old at the time. The Berlin Academy was the fourth foreign society (after the Liège Society, the Société Philomathique, and the Paris Academy) to honor him in this way. The fact that Borchardt was elected a corresponding member of the St. Petersburg Academy on December 7, 1879, suggests that Chebyshev in turn thought highly of him. A. P. Juschkewitsch [1981] reports that he could not find any written election proposal concerning Borchardt.

Kronecker is the only non-French-speaking mathematician who, according to Wassilief [1900], belonged to Chebyshev's circle of foreign friends. Yet there is only one letter addressed by him to Chebyshev, dated November 15, 1889 (see [Chebyshev 1946–1951 V, 453]). The contents are insignificant: Kronecker thanks Chebyshev for a translation of his "Theory of Equations" and announces the Berlin Academy will in return send him a copy of the first volume of the collected works of Dirichlet that he had just edited. However, Kronecker was elected a corresponding member of the St. Petersburg Academy on December 8, 1872, and the election proposal (of October 25, 1872) was written by Chebyshev himself, cosigned by the academicians D. M. Perevoshtshikov (1790–1880), an astronomer, Bunyakovskii, and Somov. The proposal, composed in Chebyshev's characteristically terse style, is reproduced in [Juschkewitsch 1981]; it does little more than cite the titles of four papers by Kronecker.

It is possible that Chebyshev was attracted to Kronecker in his capacity as a number theorist. Kronecker, one of the great students of Dirichlet, continued the latter's work in the field. Another of Dirichlet's students also became a corresponding member of the Academy, namely E. E. Kummer (1810–1893), on December 7, 1862. The proposal was most probably written by Bunyakovskii, cosigned by Chebyshev and Somov [Juschkewitsch 1981, 251].

No letters by Weierstrass, Germany's most influential mathematician between about 1866 and 1890, to Chebyshev are included among the correspondence [Chebyshev 1946–1951 V]. Further, the election proposal of November 30, 1864, for Weierstrass to become a corresponding member of the St. Petersburg Academy was signed by O. Somov, Bunyakovskii, and the academician A. N. Savich (1811–

1883), an astronomer, but not by Chebyshev. A. P. Juschkewitsch [1981] adds to this sentence the words "and this was, we believe, not by chance." In addition, Weierstrass became an honorary member of the Academy on December 2, 1895, one year after Chebyshev's death. Weierstrass had criticized Chebyshev's paper of 1857. Did Chebyshev know of the deep differences between Kronecker and Weierstrass which began about 1875 and reached their peak in 1885–1888? Did he perhaps side with Kronecker? On the other hand, Juschkewitsch [1981] believes that in later years Chebyshev's attitude changed. He cites a letter by S. Kovalevskaya of November 21, 1881, addressed to Mittag-Leffler to the effect that "there is here [Berlin] among the *Zuhörer* [class audience] of Weierstrass a young Russian especially sent by Chebyshev." This probably was D. F. Selivanov (1855–1932), later Professor in St. Petersburg. However, Weierstrass' students L. Fuchs and H. A. Schwarz only became corresponding members after Chebyshev's death, in 1895 and 1897, respectively.

The fundamental differences between Weierstrass and Chebyshev in regard to their outlook on mathematics as a whole have already been outlined by Wassilief [1900, 47].

> In contrast to Weierstrass, ["whose whole mathematical work is devoted exclusively to pure mathematics"] Chebyshev's papers and those of his school are characterized by the endeavour to present methods that are applicable to practical problems.

Indeed, the characteristic feature of Chebyshev's work was the unity of theory and practice, the linkage of mathematical theory with problems of engineering and of the natural sciences; his solution to a problem was often given in terms of an approximation together with an error estimate (evaluated in the form of inequalities or in terms of a computational algorithm). Thus Chebyshev had more in common with the earlier Euler, the contemporary Riemann, and the later Runge and Poincaré.

8. THE DECLINE OF CONTACTS WITH THE WEST

The fully documented trips of Chebyshev to Paris, with or without stopovers at Liège or Berlin, took place in 1852, 1856, 1864, 1873, 1875, 1878, and 1893; in 1875 he addressed the Société mathématique de France, during Bienaymé's presidency. He attended four sessions of the Association française pour l'avancement des sciences, namely in Lyon (1873), Clermont-Ferrand (1876), Paris (1878), and La Rochelle (1882). So he must also have passed through Paris during the summers of 1876 and 1882, as well as during his trip to Liège in December 1884.

During these trips he also made friends with scientists other than those mentioned so far. Among the corresponding members of the St. Petersburg Academy of Science it is not surprising to find the names of Liouville [26], Hermite, Bienaymé, Catalan, Borchardt, and Kronecker; their election was largely due to Chebyshev's efforts as referee. But he intervened also in the elections of other scientists, including mathematicians whom he had already met in 1852: Poncelet (elected in 1857), Bertrand (1859), Duhamel (1859), Chasles (1862), Cayley (1870), Delaunay (1871), Sylvester (1872), Hirn (1886) [27], and the astronomer Maurice Loewy (1833–1907; in 1889).

The contacts with Western European mathematicians seem to have gradually declined after 1884. Bienaymé, Borchardt, and Liouville, as well as the helpful Khanykov, were no longer there. Chebyshev did not pay Catalan the visit he had promised for the latter's 50th wedding anniversary; in 1887 his name disappeared from the membership list of the Association française. The contacts which Hermite, Catalan, and Lucas tried to maintain by mail were crowned with little success. Nor can we find any explicit reference for a trip by Chebyshev to the West between the years 1884 and 1893. In fact, neither Prudnikov [1964] nor other Russian sources known to us refer to any trips at all after 1884. With the death of Bunyakovskii in 1889 and Lucas in 1891, two further connecting links were broken. Then unexpectedly, in 1893, Chebyshev's sojourn in Paris took place, the real purpose of which we do not know. In any case, he spent a lot of time in the company of the newcomer Maurice d'Ocagne in regard to explanations about the arithmometer. Whether he met Hermite at that time is not clear; the latter's letter of November 1893 leaves open whether the meeting at the Hôtel du Louvre took place that year or several years earlier. When Catalan died in February 1894, only Hermite and Dwelshauvers-Déry of his old companions remained; but they received no more news from him. It seems that, after 1884, the great Russian mathematician cut himself loose from the Western European mathematicians. His age is probably not the sole explanation for this phenomenon. Maybe Chebyshev had finally found in Russia what he had previously been searching for in Paris, and perhaps Berlin, namely participation in an active mathematical life, now centered around his own students. The school that he founded was growing steadily and it has since received international fame [28].

NOTES

1. Little is known about J. Mention (not to be mistaken for Paul Mansion). He was a pupil of Catalan to whom he wrote two letters in 1850. He taught at St. Barbe and St. Geneviève schools at Paris. His lessons to a Princess Trubezkoi led him to St. Petersburg where he wrote numerous mathematical works for the Academy. He came back to Paris in 1860 and taught at the Jesuit College of Rue de Vaugirard. He collaborated with *Nouvelles Annales de Mathématiques* until 1867; there he published in particular in 1860 a French translation of Chebyshev's talk on geographical maps.

2. Concerning Gomes Teixeira, who taught at Porto and was Portugal's most influential mathematician of his time, see H. de Vilhena, *O Professor Doutor Francisco Gomes Teixeira (Elogio, notas, notas de biografia, bibliografia, documentos)* (Oficinas Fernandes Lisboa, 1936). He was in contact with Hermite, Bellavitis, Birger, Hansted, Le Paige, d'Ocagne, Le Pont, G. Loria, Sibirani, Lerch, Cesàro, de La Vallée Poussin, and Pincherle.

3. The many references to Watt in Chebyshev's report may be to engines built to James Watt's designs (or exploiting some of Watt's ideas such as pantograph linkages using parallelograms in machines; see Section 4), or to those of his son James Watt (1769–1848), or to their firm James Watt and Company (which continued until 1895).

4. Chebyshev refers to the firm Nipper. This may be a wrong transliteration from the Russian for the Napier company. Napier had trained at Maudsley's factory. Penn and Napier were both Presidents of the Institution of Mechanical Engineers.

5. The Gregory in question was most likely Charles Hutton G., a railway engineer who made significant developments in railway signaling. He was elected to the Council of the Institution of Civil Engineers in 1849, and was president in 1867–1868. For an obituary see *The Engineer* **85** (1898), p. 39.

This information concerning Gregory as well as the many details in regard to Watt and the firms of Maudsley, Napier, and Penn was very kindly communicated to the authors by J. H. Andrew, Keeper of Technology, Museums and Art Gallery, Department of Science and Industry, City of Birmingham. The first-named author had the fortune to be able to visit the James Watt section of the Birmingham Museum of Science and Industry on the occasion of the conference "Inequalities: Fifty Years from Hardy, Littlewood and Polya," held at Birmingham, July 13–17, 1987.

6. As Davis [1983, 31 ff] mentions, the question of priority between Lipkin and Peaucellier raised a controversy between the Russian and French schools, due to the fact that Peaucellier already had announced the discovery of his inversor in 1864, but without giving details. This controversy, underlined by the *Prix Monthyon* conferred on Peaucellier by the French Academy of Science, was the more foolish since the inversor, though geometrically perfect as a straightforward application of the plane transformation called inversion, had the mechanical disadvantage of too many (seven and six) bars and joints; in the long run it provoked discrepancies by which the precise rectilinear motion became less accurate than in the simpler approximate devices, due to Watt or Chebyshev. The idleness of this quarrel seems even greater if one recalls with Cajori [1919, 301] that another "exact" solution had been found as early as 1853 by P. F. Sarrus (1798–1861) and reported on by Poncelet, but had then been forgotten until 1905. For a description of Watt's parallelogram, Chebyshev's linkage mechanism, and its connection with the theory of best approximation, see, e.g., Wassilief and Delaunay [1900, 43 ff, 63 ff], Ferguson [1962], Richenhagen [1985, 183–190], Geronimus [1954], and Talbot [1970].

7. Nikolai Dimitrievich Bras(c)hman(n) (*Neurausnitz (= Rousinov), Moravia, near Brno), in Russia since 1824, Professor at Moscow University since 1834, founder of the Moscow Mathematical Society, was a mathematician whose special interests lay in mechanics, specifically hydromechanics and the principle of least action. See *Great Soviet Encyclopedia,* vol. 4, p. 52 (New York/London: Macmillan, 1976); Poggendorff, vol. 1, p. 281; and A. T. Grigorian, "Brashman, Nikolai Dimitrievich," in *Dictionary of Scientific Biography,* C. C. Gillispie, Ed., vol. 2, pp. 424–425 (New York: Scribner's, 1970).

8. Thanks are due to Dr. I. Grattan-Guinness for pointing out the reference to Bradley [1981].

9. (I)osif Ivanovich Somov, an outstanding student of Brashman, graduated from Moscow University in 1835, taught at engineering schools, became Professor at St. Petersburg University in 1847, and became a member of the Academy in 1862. See A. Somoff, "Nécrologie de Josepf Ivanowitsch Somoff" (Traduit du Russe par M. J. Hoüel), *Bullettino Boncompagni* **11** (1878), 453–459 (486); *Great Soviet Encyclopedia,* vol. 24, p. 318. The fact that Chebyshev's work in mechanics and integration in finite terms was also influenced by his teacher Brashman (and older fellow-student Somov) was kindly pointed out by a French referee.

10. Lyapunov, who found a proof by Poincaré of 1885 obscure, exchanged five letters with Poincaré between October 9, 1885, and December 1886. Their views diverged sharply. In a paper of 1918 Lyapunov wrote that Chebyshev first gave the problem to E. I. Zolotarev and S. A. Kovalevskaya, and then to him in 1882.

11. Thanks are due to Dr. J. Lützen, Copenhagen University, Denmark, for his letter of June 13, 1984, containing this information.

12. This information can be taken from the Ms. 3620/2, 3620/5, 3629/14, 3630/14, 3639/6, and 3640/1841 at the Bibliothèque de l'Institut de France, Paris, cited by Neuenschwander [1984, 70].

13. Kindly communicated by Dr. Neuenschwander, who is just now examining this part of Liouville's *Nachlass*.

14. Bienaymé and Hermite congratulated Chebyshev, in letters respectively dated May 27 and June 6, 1874. In [Chebyshev 1946–1951 V, 431], the second date is written as "6 June 1875 (?)," but the hypothetic 1875 makes no sense in our context and must obviously be changed to 1874. This is not the sole confusion resulting from the almost indecipherable handwriting of Hermite.

15. Gloesener (* Haut Charage, Luxemburg) taught at Louvain, was Professor of Physics at Liège (1830–1861), and was a member of the Brussels Academy. He wrote many papers on mathematical topics, published a booklet on the electrical telegraph, and worked on refraction. See F. Folie, "Notice sur M. Gloesener," *Annuaire de l'Académie Royale de Belgique* **44** (1870), 277–364.

16. See the letters II 124, 126, 127, 128 in [Hoyoux 1974].

17. This information was found in the Archives of the Société Philomatique [Ms. 2081–2093 at the Bibliothèque de la Sorbonne]. Catalan gave a report on Chebyshev's works at the session of December 13; the election took place on December 20. No details about the report or the election are known; matters concerning future corresponding members were treated in secret committees.

18. A copy of this letter, located in Chebyshev's *Nachlass* in the Archive of the USSR Academy of Science (Moscow), was kindly forwarded to the authors by Dr. B. E. Yavelov, Moscow.

19. Cf. Biswass, A. and M., "Laussedat, Aimé," in *Dictionary of Scientific Biography*, C. C. Gillispie, Ed., vol. 8, pp. 64–65 (New York: Scribner's, 1973).

20. Appeared in *Annales du Conservatoire des arts et métiers, Series* 2 **V** (1893), 231–281; 2nd ed., Gauthier-Villars, 1905.

21. Private communication from the Conservatoire des arts et métiers, Paris.

22. This information was kindly supplied by a Russian referee.

23. See James B. Campbell, *Campbell's Illustrated History of the World's Columbian Exposition . . . .*, 4 vols. (Chicago, IL: N. Juul and Col, 1984). Also: H. W. Tylor, "The Mathematical Congress in Chicago," *Bulletin of the New York Mathematical Society* (October 1893); reproduced in Battaglini's *Giornale di Matematiche* **31** (1883), 370–377. By the way, Chebyshev's mechanisms are not mentioned in Campbell.

24. The Dirichlet *Nachlass*, dispersed in three locations, Berlin (GDR), Berlin (West), and Kassel (FRG), has not yet been intensively studied. See Schubring [1986]. A list of the main scientific correspondents of the second part of the *Nachlass*, located in the Staatsbibliothek Preussischer Kulturbesitz, Berlin (West), has been published by P. L. Butzer et al. [1982].

25. The authors are greatly indebted to Dr. Charles Hyart (Emeritus Professor of Russian Literature), Liège, not only for this translation but for many others of Russian texts, in particular of passages from Prudnikov's book [1964] and Chebyshev's *Collected Works*.

26. Liouville already had been elected a corresponding member in 1840. Lucas probably was not elected since he died at the age of 49. The list of members here and below is taken from Akademia Nauk SSSR—Personal'nyj sostav (= Directory of Members of the Academy of Sciences of the USSR), 2 vols. (Moscow: Nauka, 1974) (Nauka reprint, Tokyo, 1978)

27. Hirn (* near Colmar), a physicist and engineer, wrote basic publications on the theory of heat, steam engines, ventilators, and friction. He founded a metereological observatory in Colmar.

28. For an early and excellent assessment of the work of Chebyshev and especially his school, see S. N. Bernstein [1913], "On the Best Approximation of Continuous Functions by Means of Polynomials" (Kharkov Univ.), in *Collected Works*, I, pp. 109–114 (Oak Ridge, TN: U.S. Atomic Energy Commission Translation, Series 3460, 1958).

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