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The rise of British analysis in the early 20th century: the role of G.H. Hardy and the London Mathematical Society

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Abstract

It has often been observed that the early years of the 20th century witnessed a significant and noticeable rise in both the quantity and quality of British analysis. Invariably in these accounts, the name of G.H. Hardy (1877–1947) features most prominently as the driving force behind this development. But how accurate is this interpretation? This paper attempts to reevaluate Hardy's influence on the British mathematical research community and its analysis during the early 20th century, with particular reference to his relationship with the London Mathematical Society.

Résumé

On a souvent remarqué que les premières années du 20ème siècle ont été témoins d'une augmentation significative et perceptible dans la quantité et aussi la qualité des travaux d'analyse en Grande-Bretagne. Dans ce contexte, le nom de G.H. Hardy (1877–1947) est toujours indiqué comme celui de l'instigateur principal qui était derrière ce développement. Mais, est-ce-que cette interprétation est exacte ? Cet article se propose d'analyser à nouveau l'influence d'Hardy sur la communauté britannique sur la communauté des mathématiciens et des analystes britanniques au début du 20ème siècle, en tenant compte en particulier de son rapport avec la Société Mathématique de Londres.

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1. Introduction

> In 1900 pure mathematics in this country was at a low ebb. Since the days of Newton mathematics had come to be regarded as ancillary to natural philosophy. In the nineteenth century this attitude had been confirmed by the prestige of Stokes, Clerk Maxwell, Kelvin and others. On the continent the nineteenth century was as fruitful in pure mathematics as England was barren... France, Germany and Italy had many pure mathematicians of the first rank. The leading British scholars, notably Cayley, had been solitary figures and had not led young men into research.[1] [Burkill, 1978, 323]

This extract from an obituary of J.E. Littlewood paints a gloomy picture of the state of pure mathematics in Britain at the beginning of the 20th century—and a rather inaccurate one. Although British mathematical achievements had been somewhat overshadowed by those of its European neighbours, to describe pure mathematics in 19th-century Britain as "barren" seems a little harsh. After all, British 19th-century mathematics had given the world matrices, invariant theory, Boolean algebra, quaternions, and vectors, and the country had produced several mathematicians of world class, one of whom, Arthur Cayley, arguably ranks as one of the most productive pure mathematicians of all time.

No one, however, would question the assertion that the mathematical physics of Stokes, Maxwell, and Kelvin had been the prime British mathematical area in the latter half of the 19th century, nor that Britain held a position inferior to that of its continental neighbours with regard to the number of first-rate mathematicians it produced. It is similarly true that the British university system, and the scholars produced by it, offered little or no incentive to aspiring mathematicians to undertake research [Becher, 1980]. This was in contrast to a healthy tradition of such study in Germany, for example [Biermann, 1988; Schubring, 1989], and to the beginnings of a research ethos, stimulated (ironically) by a Briton, J.J. Sylvester, in the United States[2] [Parshall and Rowe, 1994, 53–146].

Furthermore, the pure mathematics taught at Cambridge University (Britain's premier institution for mathematical instruction) not only lagged behind Europe in terms of its content, but perhaps more significantly, in terms of the importance that was placed on rigor [Becher, 1980]. Bertrand Russell, a student at Cambridge in the early 1890s, later rather harshly described the teaching of calculus there as a "tissue of fallacies" [Russell, 1967, 67], and obtained more satisfactory instruction on the continent.[3] G.H. Hardy, who graduated in 1898, five years after Russell, discovered analysis only through reading the work of the Frenchman Camille Jordan[4] [Hardy, 1940, 87].

And yet, despite these drawbacks, Britain was one of the first countries in the world to establish a professional society devoted to mathematical research [Parshall, 1996, 293]. By 1900 this body, the London Mathematical Society, was 35 years old, having an established reputation both at home and

[1] A similar view can be found in Bollobás [1986, 2].

[2] A further irony was that, when Sylvester returned to England in 1883, to take up the position of Savilian Professor of Geometry at Oxford, he tried to stimulate mathematical research there, but with little success [Fauvel, 2000b, 230–235].

[3] As he later wrote, "in England the Continental work was little known. It was only after I left Cambridge and began to live abroad that I discovered what I ought to have been taught during my three years as an undergraduate" [Russell, 1967, 67–68]. See also Griffin and Lewis [1990].

[4] Interestingly, he was encouraged to do so by the *applied* mathematician Augustus Love. A few years later, when Hardy was a lecturer at Trinity College, Cambridge, he reported that "it is very rarely indeed that I have encountered a pupil who could face the simplest problem involving the ideas of infinity, limit, or continuity, with a vestige of the confidence with which he would deal with questions of a different character and of far greater intrinsic difficulty. I have indeed in an examination asked a dozen candidates, including several future Senior Wranglers, to sum the series $1 + x + x^2 + \cdots$, and not received a single answer that was not practically worthless..." [Hardy, 1908, vi].

abroad and over 250 members. Moreover, the 914 published papers comprising the 31 volumes of the Society's *Proceedings* were predominantly on pure mathematical subjects, and although contributing members included many of the finest British mathematical physicists of the time,[5] applied mathematical papers submitted to the Society for publication before 1900 amounted to just one-fifth of its total output [Rice and Wilson, 1998, 210].

So had British pure mathematics really reached a low ebb by 1900? Or do the London Mathematical Society publication statistics indeed reveal that British pure mathematics was far stronger at that time than is suggested by Burkill? The answer seems to be a little bit of both. With the deaths of Cayley and Sylvester in the 1890s, Britain had lost its foremost pure mathematical ambassadors, two British mathematicians universally known and respected by the international community. Yet the loss was not overwhelming. Younger pure mathematicians such as Andrew Forsyth, Percy MacMahon, E.W. Hobson, and William Burnside were certainly able, and, while they hardly ranked alongside continental contemporaries such as Poincaré, Klein, or Hilbert, the overall state of British pure mathematics was certainly respectable.

Indeed, with the creation of the London Mathematical Society in 1865, a new and (then) unique venue had been provided for the dissemination of pure mathematical research, first locally, then nationally, and (gradually) internationally. The high proportion of pure mathematical papers published by the Society is evidence, not only of the significant amount of work on pure areas by British mathematicians, but also of the fact that (in contrast to applied mathematics) its *Proceedings* was one of the chief sources for publication available to the pure mathematician in Britain at that time.

This therefore raises the question: what did Burkill mean when he wrote that British pure mathematics was at a low ebb in 1900? Some clue to this lies perhaps in what he meant by *pure* mathematics. If one surveys the papers presented to the London Mathematical Society before 1900, among the multitude of results on invariants, projective geometry, number theory, differential equations, etc., one finds practically every discipline represented, but very little in the way of what we would call *analysis*.[6] The simple reason for this is that very few British mathematicians worked in that area before the beginning of the 20th century.[7] Thus, while British pure mathematics was strong, or at least respectable, in many fields, in classical analysis it had still to make its mark. But, says Burkill, that situation was about to change:

> After 1900, the principal architect of an English school of mathematical analysis was G.H. Hardy (1877–1947). In strengthening the foundations and building on them he found a partner in ... J.E. Littlewood (1885–1977). The inspiration of their personalities, their research and their teaching established by 1930 a school of analysis second to none in the world. [Burkill, 1978, 323]

[5] A noticeable exception was Sir George Gabriel Stokes.

[6] In this paper, the term *analysis* refers to the detailed study of sequences, series and functions, as initiated by mathematicians such as Cauchy, Fourier, and Abel, and developed by Riemann, Weierstrass, etc. This includes such topics, for example, as the theory of real and complex functions, integration theory, Fourier series, and analytic number theory. Before 1900, analysis constituted less than one-seventh of all papers published by the London Mathematical Society.

[7] This is not to suggest that no analysis was undertaken by the British in the 19th century. For example, the London Mathematical Society's first President, Augustus De Morgan, had published research on divergent series in the 1840s, while shortly afterwards George Stokes was the first to publish a definition of uniform convergence [Hardy, 1949, 18–20]. Work on Fourier analysis was also undertaken by William Thomson (later Lord Kelvin), but both he and Stokes were concerned with analysis as a means towards applications, rather than for its use in foundational problems. Until the early 20th century, British mathematicians who studied analysis purely for its own sake were rare [Grattan-Guinness, 1980, 98].

Fig. 1. G.H. Hardy around 1900.

Whether or not Burkill is correct in his implication that the dearth of analysis in British pure mathematics around 1900 was symptomatic of its lowly status, the first half of the 20th century did indeed witness a major shift in the style and emphasis of pure mathematical research in Britain. But this was not the only change it experienced. It also saw the continuation of the trend towards the internationalization[8] of the British mathematical community, begun in the latter half of the previous century [Parshall, 1995]. As will be seen, these two developments were intimately related, since it was primarily through increased knowledge of, and interaction with, mathematics created outside Britain that this shift in research attitude was able to occur.

Throughout much of this post-1900 period, one figure in particular dominated the British mathematical arena. The work of Godfrey Harold Hardy (Fig. 1) established analysis as a new and fruitful research speciality for British mathematicians, a phenomenon reflected in the publications of the London Mathematical Society—not unnaturally, since by 1900 it could be regarded as representative of a significant portion of mathematical research in Britain at the time. But the question remains: given the transformation that occurred in British pure mathematics throughout the first half of the 20th century, what precise role did Hardy play, and to what extent was the London Mathematical Society involved?

[8] For a discussion on the meaning of this and other related expressions, see Parshall and Rice [2002b, 2–4].

2. Before the First World War, 1901–1914

Hardy was elected to the London Mathematical Society on 10 January 1901, the Society's first meeting of the 20th century, and his first paper was read two months later. His career was just beginning. Fourth wrangler in 1898, the young Fellow of Trinity College, Cambridge, began to publish a stream of papers on many areas of real analysis, particularly on integration theory and divergent series. Books also followed. A well-received Cambridge Tract on *The Integration of Functions of a Single Variable* in 1905 was succeeded by his most influential textbook, *A Course in Pure Mathematics* (1908). The latter was the first English textbook to present elementary analysis to students in a rigorous yet accessible way, and it revolutionized university mathematics syllabuses, not just in Cambridge, but across Britain.[9]

Hardy's involvement in the upper echelons of the British research community began in 1905 when he was elected to his first three-year term on the Council of the London Mathematical Society. Soon after came the first of many papers to be published overseas: "Some theorems concerning infinite series" appeared in the *Mathematische Annalen* for 1907. In 1910, the quality of his research was recognized by his election as a Fellow of the Royal Society of London, at the early age of 33. In the following year, he began perhaps the most prolific mathematical partnership in history when he and his Cambridge colleague J.E. Littlewood started their 35-year collaboration, which produced nearly 100 joint research papers [Wilson, 2002]. Two years later, in 1913, came the now legendary letter from the Indian genius Srinivasa Ramanujan, with whom Hardy would undertake fruitful joint research over the next four years. But what was the situation elsewhere in British mathematics at this time?

If we look at the prominent figures of the London Mathematical Society (and therefore of the British mathematical research community) at the turn of the 20th century, we see that the general situation was far from unhealthy. Applied mathematical disciplines, especially mathematical physics, were well represented by the likes of Lord Kelvin (the Society's President in 1900), Lord Rayleigh, J.J. Thompson, Alfred Greenhill, Joseph Larmor, Horace Lamb, and Augustus Love. In the newly emerging field of mathematical statistics, Karl Pearson had just helped to launch the journal *Biometrika*, and was actively pursuing groundbreaking research. However, in pure mathematics the contributions were definitely more modest. In fact, perhaps only the work of William Burnside in group theory could at this point be described as world class.

Nevertheless the London Mathematical Society, and in particular its *Proceedings*, was clearly viewed as a mathematical forum of growing international stature. Its foundation as a student society at University College London in 1865 had been followed by a dramatic growth in its standing, as it rapidly acquired the status of a national society (in all but name) in a little under two years [Rice et al., 1995]. Over the next three decades, it had consolidated its position at home, as well as providing a forum for furthering international mathematical relations by means of publications and honorary membership for foreign mathematicians [Rice and Wilson, 1998].

In the words of Edward Collingwood, "By the turn of the century the isolation [of the 19th century] had been broken" [Collingwood, 1966, 590]. For a 19th-century British mathematician to be well known and published outside Britain was relatively unusual—as well as Cayley and Sylvester, exceptions included

[9] The influence of Hardy's *Course* on British mathematics and its teaching is reflected in remarks by Burkill and E.C. Titchmarsh, both prominent British 20th-century analysts: "This work was the first rigorous English exposition of number, function, limit, and so on, adapted to the undergraduate, and thus it transformed university teaching" [Burkill, 1972, 113]. "It is to Hardy and this book that the outlook of present-day English analysts is very largely due" [Titchmarsh, 1949, 449].

Thomas Hirst, George Boole, and Sir William Rowan Hamilton. Moreover, for much of that century it had been rare for mathematicians from overseas to publish in British journals [Despeaux, 2002]. By 1900, according to Collingwood, that situation had changed—but is that really true?

The fact is, and this is certainly due in part to the Society, by the end of the 19th century an increasing number of papers by foreign mathematicians were appearing in British journals. Papers by such mathematical luminaries as Klein, Poincaré, and Mittag-Leffler were all published by the Society, and by the opening years of the 20th century we see the pages of its *Proceedings* graced by papers from the likes of David Hilbert, L.E. Dickson, and Vito Volterra.

But British mathematics was still far from international: papers by overseas mathematicians were vastly outnumbered by those from home.[10] Furthermore, with the exception of those mentioned above, very few British mathematicians ever bothered to publish in foreign journals. The first International Congress of Mathematicians (held in Zürich in 1897) provides the perfect illustration of how far Britain still lagged behind—only three British mathematicians attended, and none gave papers [Barrow-Green, 1994, 38–39]. The British mathematical community may not have been as insular as it once was, but it could still not be described as fully international.

Nevertheless, that situation was to change markedly in a very short time. The 1904 International Congress in Heidelberg marked the first such occasion when a native English-speaker was invited to give a plenary address. Not surprisingly, given British research strengths at this time, it was one from the realm of applied mathematics, delivered by Alfred Greenhill on "The mathematical theory of the top considered historically" [Albers et al., 1987, 55]. This trend continued with the Rome Congress, four years later, when Andrew Forsyth (Fig. 2) addressed the participants on second-order partial differential equations [Albers et al., 1987, 55], on the same program as Gaston Darboux, H.A. Lorentz, Gösta Mittag-Leffler, Simon Newcomb, Emile Picard, Vito Volterra, and Henri Poincaré.

Forsyth also seems to have played a part in stimulating British interest in, and awareness of, continental work in analysis at the turn of the century. It has been previously noted that a distinguishing feature of 19th-century British mathematics was the apparent absence of interest in analysis. But it was Forsyth's *Theory of Functions of a Complex Variable* (1893), which, according to Edmund Whittaker, "had a greater influence on British mathematics than any work since Newton's *Principia*" [Whittaker, 1942, 218].[11] However, the book was not highly regarded outside Britain—not surprisingly, since its whole purpose was to bring the British up to date with recent continental work—and was soon surpassed in its standard of rigor by Whittaker's *Course of Modern Analysis* (1902) and Hobson's *Theory of Functions of a Real Variable* (1907). But it was Hardy's *Course in Pure Mathematics* (1908), together with the reform of the Cambridge Tripos system the following year,[12] which really marked the turning point in British university-level mathematical education.[13] From then on, analysis would be a fundamental component.

This new interest in analysis was motivated by several factors. First, many of the analytical methods, especially those involving complex functions, were required for the very kind of applied mathematics

[10] Compare this situation with that of the *Rendiconti del Circolo Matematico di Palermo*, which was deliberately international in nature [Brigaglia, 1993].

[11] According to Whittaker, "the majority of pure mathematicians who took their degrees in the next twenty years became function-theorists" [Whittaker, 1942, 218].

[12] This gruelling sequence of exams was radically reformed in 1909, the principal changes being the abolition of the order of merit, and, significantly, the introduction of analytic topics into the syllabus.

[13] See footnote 9.

Fig. 2. Andrew Russell Forsyth.

at which the British excelled. Yet, paradoxically, until the publication of Forsyth's book, the subject of complex variables had never been taught at Cambridge [Neville, 1942, 239].

Another factor was a growing awareness of recent work in foundational problems, inspired by the newly fashionable area of set theory. The first 10 years of the 20th century saw a rash of papers by Society members on various subjects, such as "On the infinite and the infinitesimal in mathematical analysis," by Hobson, "A theorem concerning the infinite cardinal numbers," by Hardy,[14] and a paper by Bertrand Russell, "On some difficulties in the theory of transfinite numbers and order types."[15] These papers provide firm evidence that, in this area at least, British mathematicians were up to date with mathematical developments outside their own country.

This was less true in other fields, however. For example, William Burnside lamented in 1908 that, although the London Mathematical Society had been the venue for many outstanding contributions to the theory of finite groups from himself and the American L.E. Dickson, the area had "failed, so far, to arouse the interest of any but a very small number of English mathematicians" [Burnside, 1909, 1].

Perhaps the surest sign of the growing maturity of British mathematicians in the international arena

[14] For a discussion of Hardy's interest in foundational problems, see Grattan-Guinness [2001, 412–415].

[15] Russell's paper (his only contribution to the Society's *Proceedings*) marked the first appearance of several fundamental ideas that were later incorporated in his and A.N. Whitehead's monumental treatise *Principia Mathematica* (1910–1913).

was the staging of the Fifth International Congress of Mathematicians at Cambridge in 1912.[16] "There was a time not long ago," said the algebraist and former London Mathematical Society President Edwin Elliott, "when British Mathematicians may have been thought too self-centred. If the judgment were ever correct, it is no longer. We are alive to what is being done elsewhere, and now aim at cooperation"[17] [Hobson and Love, 1913, 46].

But all this progress was dramatically interrupted by the outbreak of the First World War.

3. The First World War, 1914–1918

Up to this point, Hardy had occupied the place of a prominent, but not yet influential, research mathematician. As he later recalled, perhaps somewhat self-critically: "I wrote a great deal ... but very little of any importance; there are not more than four or five papers which I can still remember with some satisfaction" [Hardy, 1940, 87]. The years of the First World War would change that. Denied the opportunity to enter the British army, Hardy remained at Cambridge, producing over 50 papers on his own, or in collaboration with Littlewood or Ramanujan, and obtained some spectacular results, particularly in his new interests of analytic number theory and the theory of partitions.

It was during this period that his internationalist[18] tendencies and beliefs began to become apparent. Whether this attitude arose from a wish to raise the international profile of his own (and therefore British) mathematical research, because he believed that mathematical research standards abroad were higher than those observed in Britain, or purely from the desire to make a political point, it manifested itself in an attempt to maintain the pre-1914 internationalization of mathematical research regardless of contemporary world events. For example, during the war years, a time when international communication was neither easy nor routine, no fewer than 16 of Hardy's papers appeared in foreign journals, including the *Comptes Rendus*, *Acta Mathematica*, the *Rendiconti del Circolo Matematico di Palermo*, the *Journal of the Indian Mathematical Society*, the *Transactions of the American Mathematical Society*, the *Proceedings of the National Academy of Sciences*, and the *Tohoku Mathematical Journal*.

A consequence of this dogged internationalist approach was that, by the end of the war, Hardy was one of his country's leading mathematical ambassadors. His position within the British mathematical community had already been reinforced by his election in 1917 to the post of Secretary of the London Mathematical Society, which put him in charge of editing its *Proceedings*. In 1919, his established academic reputation, plus his growing disenchantment with Cambridge[19] and the prospect of professional advancement, prompted him to apply for the vacant Savilian Professorship of Geometry in Oxford. His appointment to this post heralded the beginning of a new period, both for Hardy and for British pure mathematics.

[16] Perhaps surprisingly, the London Mathematical Society had no official involvement with the organization of the Cambridge ICM, the official hosts being the University of Cambridge and the Cambridge Philosophical Society.

[17] Curiously, Elliott was known to be sceptical of "foreign modern" symbolic methods [Rayner, 2000, 242].

[18] Here, we distinguish between "internationalism"—the doctrine that the global community should act together for the common good—and "internationalization"—the process whereby this global society is formed. Very often, as in Hardy's case, the desire for internationalization was motivated by an adherence to internationalism. See Parshall and Rice [2002b, 2].

[19] Much of his disenchantment arose from the attitudes of his colleagues at Trinity College towards the War, which offended his pacifist beliefs; for further details, see Grattan-Guinness [1991–1992, 175, 178].

4. Hardy's Oxford years, 1919–1931

It was during the period of his Savilian Professorship that Hardy truly became the leading figure in the British mathematical community. He acknowledged as much in 1940 when he wrote: "I was at my best at a little past forty, when I was a professor at Oxford" [Hardy, 1940, 88]. For much of this time, one priority was the reestablishment of the good international mathematical relations that had been so tantalizingly within reach before the War.

As early as January 1919, Hardy was writing to Mittag-Leffler of his wish that "all scientific relationships should go back precisely to where they were before" [Dauben, 1980, 264]. As a first step, he proposed that he and Littlewood (see Fig. 3) would "offer a short account of some part of our work in the *Göttinger Nachrichten*, as a small contribution to the task of the reestablishment of friendly relations" [Dauben, 1980, 264]. This appeared as "A new solution of Waring's problem" [Hardy and Littlewood,

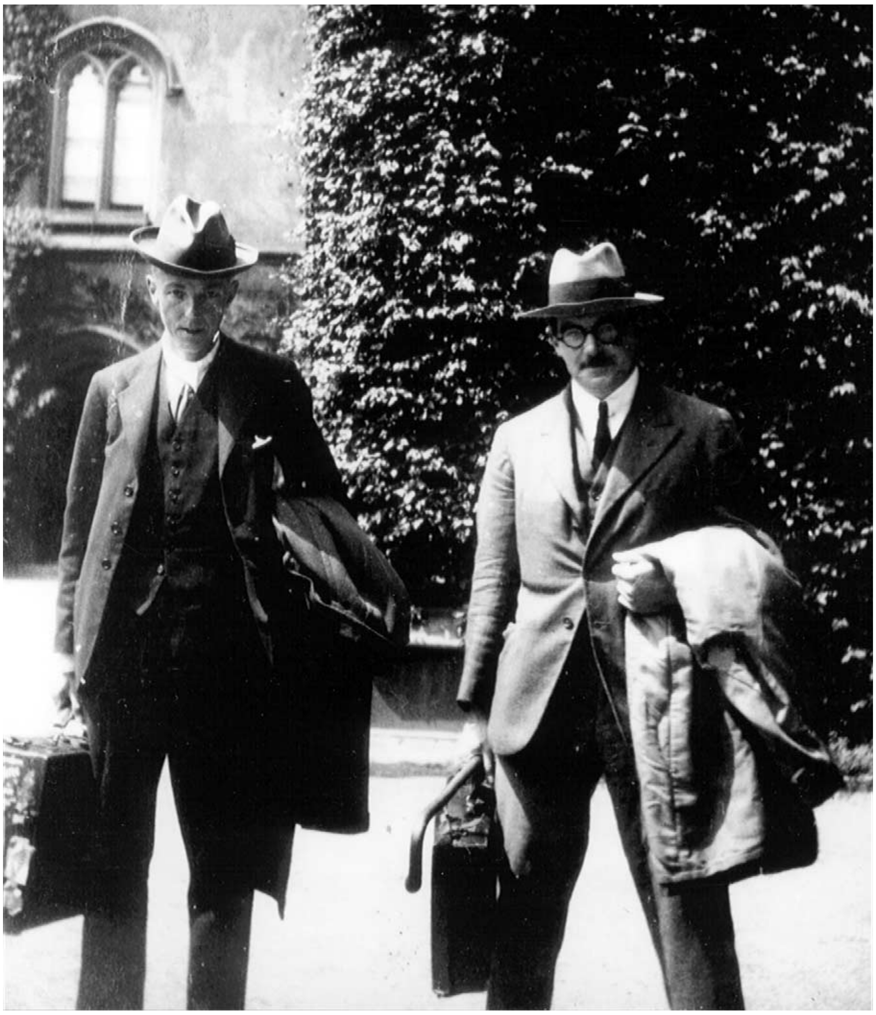

Fig. 3. Hardy and Littlewood in 1924.

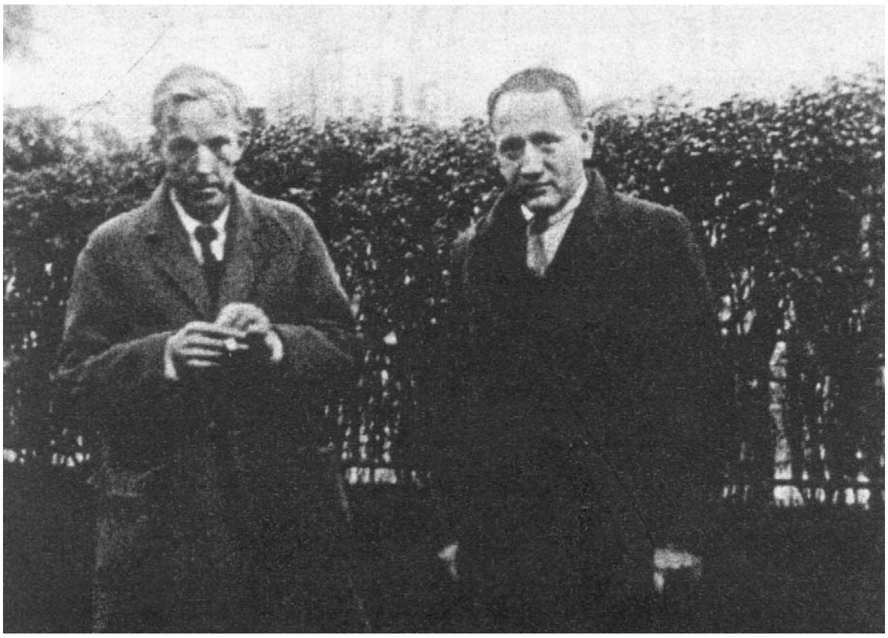

Fig. 4. Hardy and Pólya in 1924.

1920], one of a series of joint papers on the subject in which analytic techniques were employed for the first time.

Hardy was particularly keen to determine future prospects for international collaboration. In the summer of 1919, he toured Scandinavia, giving various invited lectures and meeting (among others) the Danish mathematician Harald Bohr, who famously reported: “Nowadays, there are only three really great English mathematicians: Hardy, Littlewood and Hardy–Littlewood” [Bollobás, 1986, 8–9]. Later, in 1921, he was in Germany to attend a meeting of the Deutsche Mathematiker Vereinigung, where he noted: “The intense activity and enthusiasm in research was most impressive: nothing like it could be imagined in England” [Dauben, 1980, 276].

Hardy also used his position, both as Secretary of the London Mathematical Society and as Savilian Professor, to invite distinguished scholars from overseas to visit Britain and, in some cases, to collaborate. In 1920, L.E. Dickson stayed with Hardy in Oxford [Dauben, 1980, 276], en route from the 1920 ICM (which Hardy had boycotted[20]) in Strasbourg. Harald Bohr was another visitor, as were the Hungarian Georg Pólya in 1924 (see Fig. 4) and the Russian émigré Abram Besicovitch in 1925. Hardy spent the year 1928–1929 in the United States (mainly at Princeton and Caltech) in an exchange with the American geometer and topologist Oswald Veblen, who took up Hardy’s place on the Council of the London Mathematical Society.

From these visitors (and others) Hardy, in his capacity as editor, solicited contributions for the Society’s *Proceedings*. But he was still not satisfied with the publication opportunities for pure

[20] The policy of barring mathematicians from the countries defeated in World War I from attending the postwar Congresses aroused much opposition [Lehto, 1998, 30–37; Segal, 2002, 362–367] and Hardy was a particularly outspoken critic [Lehto, 1998, 30, 34, 37, 52–53]. The policy was reversed in 1928.

mathematicians in Britain, and to this end was instrumental in launching two journals especially for them: the *Journal of the London Mathematical Society* and the *Quarterly Journal of Mathematics*.

In 1926, apparently at his urging, the *Journal of the London Mathematical Society* was founded to provide a new venue for briefer, more succinct articles, leaving the *Proceedings* as a forum for lengthier research papers. After just two years, Hardy could report that "there has been no difficulty in getting papers, English and foreign, of the right type and quality, and the *Journal* has already a recognised position in the mathematical world as a periodical with a definite individuality" [Hardy, 1929, 62].

By the First World War, a particular feature—some might say weakness—of the *Proceedings* had been the increasing domination of pure (and particularly analytic) topics at the expense of applied mathematics. In his valedictory presidential address to the Society in 1916, the eminent mathematical physicist Sir Joseph Larmor had alluded to complaints of the "aloofness, and even aridity, of much recent work" [Larmor, 1918, 4]. Indeed, as he continued, "Of recent years the question must have presented itself to not a few of our authors whether the *Proceedings*, developing in so abstract a direction, are now quite as suitable a place for the publication of mathematical physics as they were in the days when Maxwell and Kelvin, and Rayleigh and Routh, were frequent contributors" [Larmor, 1918, 5]. "There was a time," he said, "when . . . many of us made a point of taking an interest in all the papers that [the Society] published. It would be a great thing if we could get back again towards that state of affairs" [Larmor, 1918, 4].

But if Larmor and other like-minded mathematicians had hoped that the *Journal* would provide a less one-sided view of British mathematical research, they were to be disappointed. The new publication had a strongly pure flavor from the very beginning. As Hardy reported in 1928:

> I doubt whether physics lends itself very well to treatment in terse and entertaining notes. In any case we have to face extremely formidable competition in this field: we can have our pick of English pure mathematics, but it would be silly to deny that the *Philosophical Magazine* or the *Proceedings of the Royal Society* offer better publicity to any paper of a really physical type. I think therefore that the *Journal* will probably remain a journal of pure mathematics, and I will not pretend that the prospect causes me any particular distress. [Hardy, 1929, 62]

What was of more concern to Hardy was the lack of representation of geometrical work in the Society's publications. But this, he said, was "the fault of the geometers and not of the editorial committee; it is no use trying to encourage one subject by futile attempts to stifle another" [Hardy, 1929, 63]. In any case, a significant number of papers on other subjects, such as the algebraic theory of partitions, was preventing the journal from becoming solely a resource for analysts. Moreover, he argued,

> it is not at all a bad thing for a new periodical to gain the reputation of being particularly interesting on some special subject. In this case it is quite obvious, from the foreign contributions which we receive, that the *Journal* is already regarded as a particularly appropriate medium for the publication of notes on inequalities . . . If then the *Journal* is one-sided, it is one-sided in a way which I like. [Hardy, 1929, 63]

The second new journal for which Hardy could claim some credit arose from the demise of an existing, albeit rather moribund, publication. In 1928, J.W.L. Glaisher, a veteran member and former President of the London Mathematical Society, died. For the majority of his career, he had been the editor of the *Messenger of Mathematics*, as well as the *Quarterly Journal of Pure and Applied Mathematics*, from his headquarters in Trinity College, Cambridge. Over the years, these journals had become increasingly marginalized, for a number of reasons, and with Glaisher's death it was assumed that they would die with him. Hardy, however, had other ideas.

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Fig. 5. Contents page of Volume I of the *Quarterly Journal of Mathematics*.

In 1930, again with his encouragement, the *Quarterly Journal* was relaunched—but this time it was based in Oxford, a first for any mathematical journal. As with the *Journal of the London Mathematical Society*, Hardy used his position on the editorial board to promote the best research in pure mathematics from both home and abroad. As can be seen from the contents pages of early volumes, the journal could boast papers from the best home-grown mathematicians such as Hardy and Littlewood, as well as those from abroad, such as Pólya, Zygmund, and Veblen. (See Fig. 5.)

During his time as professor at Oxford, Hardy participated in another innovation for British mathematicians. As a further concession to the growing institutionalization of research, the D.Phil. degree had been introduced into Oxford in the 1920s. Hardy acted as advisor to a number of graduate students, including Edward Titchmarsh (who went on to teach at Liverpool, before returning to Oxford as Hardy's successor), Dame Mary Cartwright (Cambridge), L.S. Bosanquet (University College London), and Sir Edward Wright (Aberdeen). These graduates quickly established themselves as well-respected mathematicians, with both Titchmarsh and Cartwright later becoming Presidents of the London Mathematical Society.

Hardy also influenced mathematical developments at the institutional level. He argued that one way of improving the status of mathematical studies in Oxford was to appoint more research fellowships in mathematics, and thus create better job prospects for the most promising graduates. In an article in *The Oxford Magazine* in 1930, he said, "Mathematicians are reasonably cheap, but they cannot be had for nothing"[21] [Rayner, 2000, 246] and went on to make a strong case for the foundation of a mathematical institute, influenced by recent developments elsewhere, such as in Princeton and Göttingen. Whether or not Hardy's views were motivated by the onset of the Great Depression in 1929, their timing was particularly apposite and his pleas did not fall on deaf ears: the establishment of a mathematics institute in 1934 [Morrell, 1997, 314] marked the beginning of institutional support for mathematics at Oxford.

By the end of the 1920s, Hardy's influence on British mathematics was at its peak. His election in 1926 as President of the London Mathematical Society was yet further corroboration of his status,[22] and one that he must have regarded as a high point of his career: "This Society has always meant much more to me than any other scientific society to which I have belonged. My record of attendances, since I became secretary in 1917, has no blemish; I have been at every meeting both of the Council and of the Society, and have sat through every word of every paper" [Hardy, 1929, 61–62]. He was awarded the Society's De Morgan Medal, its highest accolade, in 1929.

True, other British mathematicians had previously held such a position and received such recognition. They too had published many papers in prestigious foreign journals, while simultaneously exercising editorial power over major British publications. The difference is that by 1930, largely through Hardy's efforts, a distinct English mathematical "research school" had come into being. Centered on Hardy in Oxford and Littlewood in Cambridge, these mathematical researchers had established analysis as a speciality in which, for the first time, Britain led the world.

5. The influx, 1931–1940

In 1931, Hardy returned to Cambridge, where he and Littlewood continued to advise postgraduates: Frank Smithies and Robert Rankin were two of Hardy's students [Rankin, 1998], while Harold Davenport and the brilliant analyst Raymond Paley[23] studied under Littlewood. As with Hardy's Oxford postgraduates, these students quickly became active contributors to London Mathematical Society meetings and, to further their training for this and other professional venues, Hardy and Littlewood ran weekly seminars or "conversation classes" for them in Littlewood's rooms. This was apparently a

[21] The article appeared in the 5 June 1930 edition of *The Oxford Magazine*, pp. 819–821.
[22] He also served as President of the Mathematical Association from 1925–1927.
[23] Before his death at the tragically early age of 26, Paley also collaborated with Antoni Zygmund, Norbert Wiener, and Pólya.

highly productive and popular event: “Mathematicians of all nationalities and ages were encouraged to hold forth on their own work, and the whole thing was conducted with a delightful informality that gave ample scope for free discussion after each paper” [Titchmarsh, 1949, 453].

By now mathematical research environments were established at both Oxford and Cambridge, but activity was also under way elsewhere in Britain. In Manchester, the Cambridge-trained American-born number theorist Louis Mordell was fostering a vibrant and energetic mathematics department, which already included Edward Milne, and was soon to recruit Davenport to its faculty; in Edinburgh, the analyst and applied mathematician Edmund Whittaker held the chair; his student, the analyst George Watson, held the professorship in Birmingham from 1918 until 1951; meanwhile in London, Hardy’s Oxford student L.S. Bosanquet was teaching at University College. The active role these mathematicians all took at meetings of the London Mathematical Society and the prominent positions they held within it further enhanced their status as leading and influential members of the British mathematical research community.

The 1930s saw the British mathematical community become more internationally diverse than ever before, and once again Hardy played a key role in this process. Following the rise to power of the Nazis in Germany in 1933 and the worsening political situation throughout Europe in the following years, many German and eastern European scholars found themselves either dismissed as “undesirable” or forced to emigrate for fear of persecution [Rider, 1984; Segal, 2002, 368–371; Siegmund-Schultze, 1998; Siegmund-Schultze, 2002, 339–341].

Many countries set up relief organizations, one of the earliest being the Academic Assistance Council (AAC), founded in Britain in May 1933; its aims were to raise funds to enable academic refugees to find employment in Britain or elsewhere. One of its most active supporters was Hardy, who worked energetically to recruit many of the displaced mathematicians for British universities, in order to strengthen British mathematics further.[24] He wrote to the AAC from Cambridge:

> There are several men whom I should wish to recommend very strongly—for example, [Hans] Heilbronn (perhaps the best of all the mathematical refugees) and [Richard] Rado. But I should wish to see them here, or at Oxford, and not in Canada and Australia. [Fletcher, 1986, 18]

Hardy, together with Davenport,[25] was instrumental in obtaining a place for Heilbronn (Fig. 6) at the University of Bristol. By 1934, Hardy could report that the AAC’s money had been well spent, since while at Bristol Heilbronn had “finished an exceptional piece of work which will make a considerable sensation when it appears and add greatly to his status” [Fletcher, 1986, 20]. This was Heilbronn’s proof of the Gauss conjecture on class numbers of imaginary quadratic number fields [Heilbronn, 1934].

In the case of the Hungarian Richard Rado, Hardy initially took him on as a graduate student. After Rado obtained his Ph.D., Hardy used his influence to try to obtain him a position first at Calcutta, then Reading, then Imperial College London. Eventually, in 1936, Rado obtained an assistant lectureship at Sheffield, while Louis Mordell in Manchester took in the number theorists Paul Erdős and Kurt Mahler. The German group theorists Kurt Hirsch and Bernhard Neumann became Ph.D. students at Cambridge under Philip Hall, while their compatriot Walter Ledermann completed his doctoral work at the University of St. Andrews. As with the mathematicians Hardy had trained at Oxford and Cambridge,

[24] In all, Hardy created 18 posts at Cambridge for refugee mathematicians [Morrell, 1997, 308].

[25] Davenport had met Heilbronn at Göttingen in 1933 while the latter was working as assistant to Edmund Landau.

Fig. 6. Hans Heilbronn.

these refugee mathematicians followed his lead and took an active interest in the affairs and publications of the London Mathematical Society, attending its meetings, publishing papers in its journals, and becoming officers on its Council.

The inevitable outcome was that the British mathematical community now contained not only the best of home-grown talent, but also many of the finest young mathematicians from central Europe. True, Britain may have missed out on Richard Courant[26] and Emmy Noether,[27] but it had nevertheless succeeded in strengthening its mathematical resources immeasurably.

6. Epilogue

By 1940, the mathematical environment in Britain was markedly different from that of 40 years before.[28] During the intervening period, for a variety of reasons (not all to do with mathematics), a definite research ethos had come into being in British universities. In 1900, not even the most prestigious professor at Oxford or Cambridge was required to undertake research (although the expectation was no doubt there), while the high teaching and examining load imposed on the more junior tutors and lecturers

[26] Courant spent the academic year 1933–1934 in Cambridge, before emigrating to the United States [Reid, 1976, 155–163].

[27] There was an abortive attempt to recruit her for Somerville College, Oxford, in 1935 [Rayner, 2000, 247].

[28] Writing in 1942, Hardy marvelled that "The Cambridge of those days [c.1900] would seem a strange place to a research student transplanted into it from to-day" [Hardy, 1942, 220].

severely restricted their creative mathematical activity. In the Britain of 1900, then, neither research nor training in research techniques was a high institutional or professional priority.

By 1940, research had become a definite part of a faculty member's duties. Indeed, this modification of academic priorities had been recognized by mathematicians even earlier. In a talk given to the Oxford Mathematical and Physical Society in 1925, the elderly Edwin Elliott contrasted the new research-orientated outlook of British mathematicians with that which existed when he began his career several decades earlier, in which teaching and examining were paramount: "But how about research and original work under this famous system of yours, I can fancy someone saying. You do not seem to have promoted it much. Perhaps not! It had not yet occurred to people that systematic training for it was possible" [Fauvel, 2000a, 22].

The British recognition of the role of research as a fundamental feature of mathematical academic life coincided with increasing recognition and awareness of their mathematical work overseas. This resulted in further interaction between British and foreign mathematicians, to the extent that by 1940 the following practices (now accepted as standard) were firmly in place:

- British mathematicians were regularly publishing in foreign journals.
- Overseas mathematicians were regularly publishing in British journals.
- British mathematicians were spending active periods abroad.[29]
- Overseas mathematicians were spending active periods in Britain.[30]
- There were multiple instances of British collaboration with foreign mathematicians.[31]

Equally noticeable during this period was the change in British mathematical research interests, as illustrated by published papers. In particular, we see substantial growth in the number of papers on analysis published between 1900 and 1940, especially when compared with other subjects. If we look at the publications of the London Mathematical Society, we see that from 1865 to 1900, analytical topics counted for just under 14% of all papers published in the Society's *Proceedings*, but by 1940, analysis amounted to nearly 40% of the Society's output, followed by 22.5% on algebraic subjects, 13.6% on geometry, and 16.4% on applied mathematics. Furthermore, whereas the pre-1900 papers on analysis had consisted mainly of studies on elliptic functions, series, and harmonic analysis, the range of subjects covered between 1900 and 1940 expanded dramatically to include complex functions, theory of integration, Fourier integrals, Dirichlet series, and special functions. Thus, not only had the variety of analytical topics increased, but the subject of analysis itself now constituted the largest mathematical discipline represented in the *Journal* and *Proceedings* of the London Mathematical Society.

To be sure, many of the above developments can be attributed, at least in part, to Hardy; he can certainly be seen as actively promoting many of them. But it should be stressed that, influential as Hardy may have been among the British at this time, he was not alone in his efforts to promote pure mathematical research.

For example, for much of this period Henry Baker was equally hard at work founding a British research school in geometry. Lowndean Professor at Cambridge from 1914–1936, Baker supervised a multitude

[29] For example, Hardy, Paley, and Davenport.

[30] Such as Veblen, Wiener, Landau, Pólya, and Besicovitch, plus the mathematical refugees of the 1930s.

[31] Examples include the collaborations of Hardy and Littlewood with Georg Pólya, Harold Davenport with Hans Heilbronn, and Henry Whitehead with Oswald Veblen.

of research students, including Donald Coxeter, William Hodge, Jack Semple, Leonard Roth, William Edge, Patrick du Val, and John Todd. His Saturday afternoon "tea parties" for his students were well under way before Hardy and Littlewood's "conversation classes" got going.

Slightly later (from 1927), Philip Hall belatedly answered Burnside's plea for an increased British awareness in group theory, igniting British interest in that area and supervising students for over 30 years. In Oxford a young graduate student Henry Whitehead was encouraged by Veblen (during the latter's 1928–1929 sabbatical year replacing Hardy) to take up differential geometry [Morrell, 1997, 309]. Together, they authored the influential text *The Foundations of Differential Geometry* (1932), and Whitehead went on to lead a British topological school at Oxford, with students such as Ioan James and Peter Hilton.

Although Hardy was the best-known British analyst of the early 20th century, he was neither the first nor the only British mathematician to undertake groundbreaking analytic research during this period. Another example was William Henry Young (Fig. 7).[32] One of the most original British mathematicians of the time, Young, like Hardy, was a brilliant analyst. In particular, his work on the theory of integration produced a definition of the Lebesgue integral independently of Lebesgue (although Lebesgue anticipated Young's work by two years). Like Hardy, Young also received numerous academic honors and distinctions in recognition of his work, including the Presidency of the London Mathematical Society from 1922–1924, and his tenure as President of the International Mathematical Union from 1928 to 1932 indicates that he too was highly regarded internationally. Why then does Hardy receive so much attention?

The first reason is one of personality. Hardy was a tremendously good self-publicist. He went out of his way to establish and cultivate mathematical networks and relationships both nationally and internationally, regularly attending meetings at home and abroad, as well as inviting visitors from overseas to Britain. By contrast, Littlewood (to whom one would think that equal credit is due) was seldom seen outside Cambridge. In fact, there were jokes at the time among European mathematicians "that Hardy had invented him so as to take the blame in case there turned out anything wrong with one of their theorems" [Snow, 1967, 29].

Similarly, the reason for Baker's relatively low profile is that, compared to Hardy, he was far less gregarious. While certainly a keen proponent of British mathematical research, and a leading figure in the British mathematical community,[33] he never occupied the influential position of the Society's Secretary, neither was he as active internationally as Hardy. Likewise, William Young lived and worked abroad throughout this period, and never held a permanent academic post in Britain for any significant duration;[34] consequently, even if he had been so inclined, he did not have the opportunity to animate a school of research students. In the case of Hall and Whitehead, by the time they were attracting students in the 1930s, the research momentum was already fully under way.

[32] For a study of the mathematical partnership of Young and his wife Grace Chisholm, see Grattan-Guinness [1972].

[33] He was, for example, President of the London Mathematical Society from 1910 to 1912.

[34] As Hardy later reported, Young "was still not properly appreciated, and I can remember that, when he was a candidate for the Sadleirian chair in 1910, no one in Cambridge seemed to take his candidature very seriously" [Hardy, 1942, 221]. Young held a part-time post of Professor of the Philosophy and History of Mathematics at the University of Liverpool from 1916 until 1919, when he was an unsuccessful candidate against Hardy for the vacant Savilian Professorship at Oxford [Grattan-Guinness, 1972, 161–162]. He did however manage to obtain the Professorship of Pure Mathematics at Aberystwyth, but various disagreements led to his resignation in 1923.

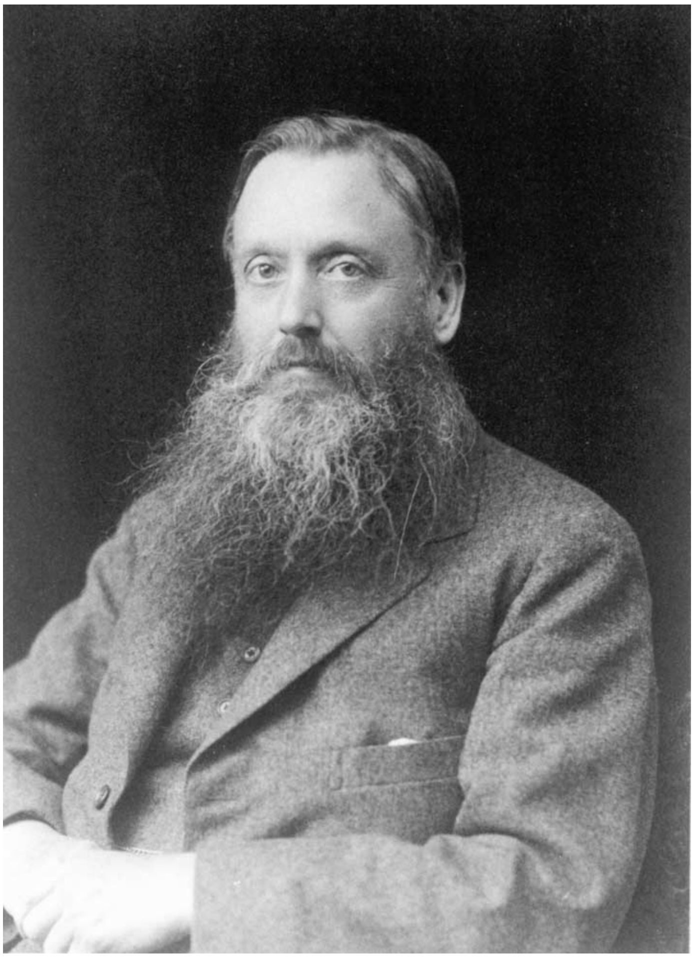

Fig. 7. William Henry Young.

Another reason for the prominence of Hardy's name in any account of British mathematics in the early 20th century is his sheer productivity during this period, especially in comparison with his contemporaries. Turning again to publications in the London Mathematical Society's two journals, we find that among the most frequent contributors between 1900 and 1940 were Burnside (31 papers), Hobson (32 papers), and Young (69 papers[35]). But it was Hardy who was by far the most prolific, with a yield of 106 papers,[36] a figure that amounts to nearly 10% of the Society's entire output over this whole period. When the sheer quantity of this work is considered together with its high quality, it is hardly surprising that Hardy demands so much attention.

So, to re-state our original question: what were the roles of Hardy and the London Mathematical Society in the changes that occurred in British pure mathematics in the first half of the 20th century? We

[35] Sixty-three of these papers were by Young alone, with six coauthored by his wife Grace.

[36] Of Hardy's 106 publications, 56 were solo efforts and 50 were joint papers (including 34 with Littlewood).

highlight four main features of Hardy's work which helped to change attitudes to the way mathematics was practised in Britain:

- He vigorously promoted research into various areas of analysis, particularly integrals and Fourier series, as well as topics in analytic number theory.
- He helped to found new publication outlets for pure mathematicians working in Britain and elsewhere.
- He consistently advocated an internationalist approach to mathematical research activity.
- He urged academic institutions to increase financial support for the emergent profession of research-level mathematics.

How was the London Mathematical Society involved in these activities? In the first place, Hardy used the Society as a vehicle to promote his own mathematical research interests, which lay chiefly in analysis. He did so by a sustained and prolific series of publications in the Society's *Proceedings* and *Journal*, encouraging like-minded mathematicians (whose interests frequently overlapped with his) to do likewise. Such mathematicians included those whose research he had personally supervised and who went on to emulate his example of graduate supervision, close involvement with the Society, and prodigious publication.

Second, to increase the number of outlets for British pure mathematical research, as well as the speed with which papers could appear, Hardy was actively concerned with the establishment of not one but two major journals, both of which quickly became foci for the publication of research on areas in which he and his followers worked. In the case of one of these journals, the London Mathematical Society was explicitly involved, with Hardy using his place on the Society's Council and the journal's editorial board to influence developments that had a direct and major impact on the landscape of mathematical research-level publication in Britain.

Third, Hardy used his influence to propagate the doctrine of internationalism among his colleagues by going out of his way to publish large amounts of his work overseas, to attend meetings abroad, to arrange foreign exchanges and visits, and to collaborate with foreign mathematicians. Interestingly, despite its well-established national position, as well as its international reputation by this point, Hardy does not seem to have used the London Mathematical Society in an explicit way in his quest towards internationalization. This is perhaps explained by the fact that the Society's activities were strictly limited to holding meetings (in London) and publishing papers. Anything else would have been beyond its rules of operation and financial capability.

A similar reason perhaps explains the non-involvement of the Society in the last of Hardy's activities during our period, the advocacy and promotion of professional and institutional support for mathematics and mathematicians, as illustrated by his calls for a mathematical institute at Oxford in the 1930s. As we have seen, the recognition of research as a legitimate profession was a defining feature of British academic life between 1900 and 1940, and one in which Hardy took a deep interest. But again, it was something he had to pursue independent of the London Mathematical Society, the subject being well beyond its remit.

British mathematics underwent tremendous changes during the early 20th century; these included a huge growth in the professionalization of research and the development of a more cosmopolitan and outward-looking international nature. As we have seen, Hardy was a key figure in these developments, and it is perhaps not unnatural for him to have received much of the credit for the transformation of the British mathematical environment during this period. But other factors were clearly also at work—social, economic, and political. Mathematically, Hardy's role was to encourage and inspire his colleagues to carry out research in his chosen fields, to emulate his style of research and publication, and to accept his standards as those to aspire to. As the premier body for research mathematicians in the country, the London Mathematical Society was the prime instrument by which many of these values were transmitted. Thus, facilitated by the Society's meetings and publications, Hardy's work served as a catalyst that resulted in the emergence of analysis as the dominant mathematical research discipline in early 20th-century Britain.

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