J. Phys. D: Appl. Phys. **35** (2002) L88–L89 PII: S0022-3727(02)34541-8

RAPID COMMUNICATION

The power law dependence of the a.c. conductivity on frequency in amorphous solids

A N Papathanassiou

Solid State Physics Section, Department of Physics, University of Athens, Panepistimiopolis, Zografos, GR-15784 Athens, Greece

E-mail: apapathan@in.gr

Received 5 March 2002
Published 16 August 2002
Online at stacks.iop.org/JPhysD/35/L88

Abstract
The a.c. conductivity σ_{ac} of polyaniline prepared with a.c. plasma polymerization was studied recently at low angular frequencies ($\omega \leqslant 1$ MHz) (Mathai J *et al* 2002 *J. Phys. D: Appl. Phys.* **35** 240). The data were analysed considering a power dependence of the a.c. conductivity on frequency $\sigma_{ac} \propto \omega^n$, $n \leqslant 1$. It was found that the index n varies between 0.4 and 1. In this work, we show that these values conflict with the bounds of the n parameter predictions of the Austin and Mott model, which assumes a single mode of charge hopping in amorphous media. We suggest that multiple a.c. conductivity mechanisms of the Austin and Mott type may contribute to the measured a.c. conductivity. A numerical calculation for a couple of mechanisms showed that the resulting a.c. conductivity curve exhibits *effective* n values which can explain the wide distribution of the indexes evaluated in polyaniline.

The a.c. conductivity of polyaniline thin films synthesized by a.c. plasma polymerization technique was measured recently [1]. Complex impedance measurements were performed in the frequency range 10^2–10^6 Hz. It was observed that the a.c. conductivity σ_{ac} increased with the (angular) frequency ω. The a.c. conductivity versus frequency plots were analysed by considering a power law:

$$\sigma_{ac} \propto \omega^n, \tag{1}$$

where the value of the exponent n ($n \leqslant 1$) evidences for the type of the dominant conductivity mechanism in amorphous materials. The power law (equation (1)) is an approximation of the Austin and Mott model that describes the a.c. conductivity in amorphous media. The exponent is evidently obtained by taking the logarithm of equation (1):

$$n = \frac{\mathrm{d}\log\sigma_{ac}}{\mathrm{d}\log\omega}. \tag{2}$$

The analyses of the experimental results indicated that n lie between 0.4 and 1 for low frequencies ($<10^4$ Hz). The authors concluded that these values are typical for a.c. conduction through hopping.

The physics underlying the power law relationship predicted by the Austin and Mott model is based on phonon-assisted hopping of charge carriers through tunnelling from a localized site to another one. At low temperatures, where hopping near the Fermi level is dominant, the a.c. conductivity σ_{ac} increases upon frequency ω is described by the Mott and Austin relation [2, 3]:

$$\sigma_{ac}(\omega) = A\left(\frac{e^2}{\alpha^5}\right)\{N(E_F)\}^2 kT\omega\left\{\ln\left(\frac{\nu_{ph}}{\omega}\right)\right\}^4, \tag{3}$$

where k is Boltzmann's constant, $N(E_F)$ the density of states per unit volume, ν_{ph} a typical phonon frequency, α the localization length of the (localized) wave function and $A = \pi^4/96$ [4].

By differentiating equation (3) with respect to ω, we get

$$\frac{\mathrm{d}\ln\sigma_{ac}(\omega)}{\mathrm{d}\ln\omega} = 1 - \frac{4}{\ln(\nu_{ph}/\omega)}. \tag{4}$$

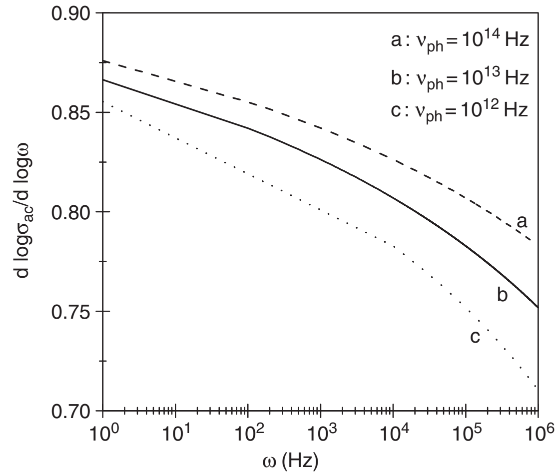

Figure 1. $\mathrm{d}\log\sigma_{\mathrm{ac}}(\omega)/\mathrm{d}\log\omega$ as a function of the angular frequency ω for three typical phonon frequencies. The numerical calculations were based on the Austin and Mott model (equation (4)).

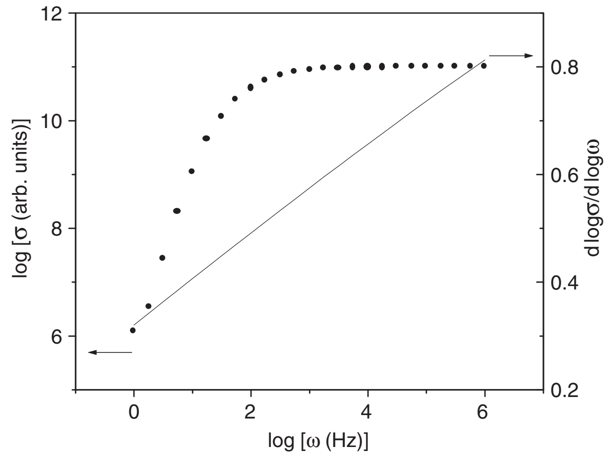

Figure 2. Logarithm of the a.c. conductivity versus the logarithm of frequency (——) for a material were the (total) conductivity results from two different conductivity modes, i.e. $o_{\mathrm{ac}} = 1 \times \omega^{0.80} + 1 \times \omega^{0.85}$. Circles represent the $\mathrm{d}\log\sigma_{\mathrm{ac}}(\omega)\mathrm{d}\log\omega$ values.

The slope of a $\log\sigma_{\mathrm{ac}}$ versus $\log\omega$ plot (which, according with equations (2) and (4), is represented by the exponent n) is determined by the phonon frequency and depends on the a.c. frequency ω. In figure 1, $\mathrm{d}\log\sigma_{\mathrm{ac}}(\omega)/\mathrm{d}\log\omega$ is plotted as a function of ω for three typical values of the parameter ν_{ph}: 10^{12}, 10^{13} and 10^{14} Hz. The angular frequency ω ranges from 1 Hz to 1 MHz. We observe that, depending on the phonon frequency, $\mathrm{d}\log\sigma_{\mathrm{ac}}(\omega)/\mathrm{d}\log\omega$ ranges between 0.85 and 0.88 at $\omega = 1$ Hz and between 0.71 and 0.78 at $\omega = 1$ MHz. The analysis of the experimental data should yield n values compatible with those depicted in figure 1, provided that a *single* hopping a.c. conductivity mechanism operates. The values of the exponent n reported for polyaniline thin films over a wide range ($0.4 \leqslant n \leqslant 1$) [1] and violate the limitations for the slope of a $\log\sigma_{\mathrm{ac}}(\log\omega)$ diagram predicted within the model of Austin and Mott.

A plausible explanation in terms of the Austin and Mott model for the observed widespread distribution of n is the following: the n parameter can take values far from the boundary values predicted by the model of Austin and Mott if multiple conductivity mechanisms operate, i.e.

$$\sigma_{\mathrm{ac}} = \sum_{i=1}^{m} \sigma_{\mathrm{ac},i} = \sum_{i=1}^{m} c_i \omega^{n_i}, \qquad i = 2, 3, 4, \ldots, m, \quad (5)$$

where the integer i labels the ith mechanism. Each elementary mechanism is governed by the power law of equation (1), i.e. $\sigma_{\mathrm{ac},i} = c_i \omega^{n_i}$. We have performed numerical calculations for the case of a couple of a.c. conductivity measurements ($m = 2$) with exponents $n_1 = 0.80$ and $n_2 = 0.85$. Note that n_i are physically acceptable with respect to the Austin and Mott model (see figure 1). The (total) a.c. conductivity is plotted as a function of the frequency in figure 2. In the same diagram, the slope of the curve is shown. $\mathrm{d}\log\sigma_{\mathrm{ac}}(\omega)/\mathrm{d}\log\omega$ (which coincides with the *effective* parameter n characterizing the measured a.c. conductivity) ranges from 0.31 (at $\omega = 1$ Hz) to 0.80 (at $\omega = 1$ MHz). Equation (5) explains the low n values observed in the low-frequency limit ($\omega \to 1$ Hz). In the high-frequency region, the slope coincides with the value n_1. n_2 cannot be traced at all. Our results indicate that the usual $\log\sigma_{\mathrm{ac}}$ versus $\log\omega$ representation is incapable of revealing the values of n associated with the modified Austin and Mott model unless a unique a.c. conductivity mechanism proceeds. It is necessary to compare the effective n values resulting from the experimental curves with those predicted through equation (4) (see figure 1).

References

[1] Mathai J *et al* 2002 *J. Phys. D: Appl. Phys.* **35** 240
[2] Austin I G and Mott N F 1969 *Adv. Phys.* **18** 41
[3] Mott N E and Davis E A 1979 *Electronic Processes in Non-Crystalline Materials* (Oxford: Clarendon)
[4] Pollak M 1977 *Phil. Mag.* **36** 1157