Προσαρ. \((X_i, B_i, \mu_i)\), \(i \in N\), \(\text{kato} \; \text{πιθανότητας}\)

\[ X = \prod_{i=0}^{\infty} X_i, \quad B_i = 2^{B_i \times B_{i-1} \times B_{i-2} \times \ldots} \]

για \(B_i \in B_i\), \(X_i = 0, 1\), \(\mu_i(B_i) = 1\)

\[ \prod_{i=0}^{\infty} \mu_i(B_i) = 1 \]

Παραδείγματα:

1) Μονοκλονιαίο διανυσματικό, Μακρογενή, Βεννουλλί

\[ X_i = \{0, 1, \ldots, k-1\}, \quad B_i = 2 \times i \]

\(p_i \geq 0, \sum_{i=0}^{\infty} p_i = 1\)

\[ \mu_i = \sum_{j=0}^{k-1} p_j \delta_{i,j} \]  \(\mu_i(A) = \sum_{j=0}^{k-1} p_j \delta_{i,j}\)

\[ X = \prod_{i=0}^{\infty} X_i = \{ (x_0, x_1, \ldots), x_i \in \{0, 1, \ldots, k-1\} \} \]
$T : \mathcal{X} \rightarrow \mathcal{X}$ shift

$T(x_0, x_1, \ldots) = (x_1, x_2, \ldots) \quad \mu \left( \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right) = \mu \left( \left\{ x_1 = c_1, x_2 = c_2, \ldots, x_{n+1} = c_{n+1} \right\} \right)

= \sum \mu \left( \left\{ x_0 = c, x_1 = c_0, \ldots, x_{n+1} = c_{n+1} \right\} \right)

= \left( \sum_{i=0}^{\infty} p_{i,0} \right) p_{0,1} \cdots p_{n,0} = \mu \left( \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right)

= \sum \mu \left( \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right)

= \sum_{i=0}^{\infty} p_{i,0} p_{i+1,0} \cdots p_{n,0} = \sum_{i=0}^{\infty} p_{i,0} p_{i+1,0} \cdots p_{n,0}

b) Markov, shift, Markov

$X_{\omega_0}$, $\omega_100s$

$p = (p_{ij})$, $\mathcal{X} \times \mathcal{X}$

$\sum_{j=0}^{\infty} p_{ij} = 1$

$\prod_{i=0}^{\infty} p_{i,j} > 0$ such that $\sum_{i=0}^{\infty} p_{i,j} = 1$

$\mu \left( \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right) = \prod_{i=0}^{n} p_{i,0}, p_{i+1,0} \cdots p_{n,0} - p_{i,0} - p_{i+1,0} \cdots - p_{n,0}$

$\mu \left( \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right) = \prod_{i=0}^{n} p_{i,0}, p_{i+1,0} \cdots p_{n,0}$

$p(x_1 = c_1 / x_0 = c_0)$

To see other cases are $\mathbb{B}$ invariant and Kolmogorov consistency form.

$\mu \left( T^{-1} \left\{ x_0 = c_0, \ldots, x_n = c_n \right\} \right)

= \sum_{i=0}^{\infty} \mu \left( \left\{ x_0 = c, x_1 = c_0, \ldots, x_{n+1} = c_{n+1} \right\} \right)

= \sum_{i=0}^{\infty} p_{i,0} p_{i+1,0} \cdots p_{n,0}$
\[ |I_{v_{11}}^*| = 3^{-n} \]

\[ \Omega \quad \{v_{11} \rightarrow \Omega_{v_{11}} = \{ \alpha \} \} \]

\[ \Pi : \{0, 1, 2\}^\infty \rightarrow x \]

\[ \Pi(c_1, c_2, \ldots) = \Omega \{v_{11} \rightarrow \Omega_{v_{11}} \} \]

\[ T \big( \Pi(c_1, c_2, \ldots) \big) = \Omega \{v_{11} \rightarrow \Omega_{v_{11}} \} \]

\[ T \circ \Omega = \Omega \circ T \]

\[ T \text{ is semi-conjugacy} \]

\[ \mu_{c_{1,1}} \text{ is a proper (non-unit)} \]

\[ \mu_{c_{1,1}}(\{x_0 = 1\}) = \mu(\{x_0 = 2\}) = \frac{1}{2} \]

\[ \mu \circ T_{c_{1,1}} = \mu \quad \text{ACX} \]

\[ \mu(A) = \mu_{c_{1,1}}(\{x_0 = 1\}) \quad \text{mass shift} = \text{mass} \]

\[ \text{addition to } \mu_{c_{1,1}} \]

\[ T \text{ again addultion to } \mu \]

\[ \mu(T^{-1}(A)) = \mu_{c_{1,1}}\left( \mu^{-1}(T^{-1}(A)) \right) = \mu_{c_{1,1}}(\sigma^{-1}(\mu^{-1}(A))) \]

\[ \mu(A) = \mu_{c_{1,1}}(\{x_0 = 1\}) = \mu_{c_{1,1}}(A) \]
Subshift of finite type

\[ X = \prod_{i=1}^{\infty} \{0, 1, \ldots, k-1\} \]

Define a relation \( A(x, x_n) = 1 \) \( \forall x \in X \)

\[ X = \{ x \in X : A(x, x_n) = 1 \} \]

\[ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

The associated matrix is Markov.

Additive measure \( \mu \) is Markovian if \( \mu \) is Markovian.

Thus, \( \mu \) is \( A \)-invariant.