Introduction to von Neumann algebras

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The von Neumann algebra of a group

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Murray - von Neumann equivalence and the classification of factors

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A von Neumann algebra is a subalgebra of the algebra $\mathcal{B}(H)$ of all bounded linear operators on a Hilbert space H which contains the identity operator and the adjoints of its elements, and is closed under pointwise limits.

Alternatively, it can be characterised as the set of all bounded operators on H that commute with a group of unitary operators.

These algebras were born out of the need to understand unitary representations of groups as well as the mathematical foundations of Quantum Mechanics and related fields, and have evolved into an active field of current research with connections to various subjects.

In these lectures we will try to provide a self-contained introduction to the rudiments of the theory of von Neumann algebras. This will include two fundamental approximation theorems (the von Neumann Double Commutant theorem and the Kaplansky Density theorem), as well as the Borel functional calculus.

We will then apply this theory to briefly study, and give examples of

(a) the von Neumann algebra generated by a unitary representation of a (locally compact) group

(b) the classification of von Neumann algebras into types, according to the 'kinds' of projections that they contain.

The talks on *(a)* and *(b)* will be mutually independent.