Crossed products of dual operator spaces and the approximation property

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We compare the Fubini crossed product and the spatial crossed product of a locally compact group G acting on a dual operator space X, studied by Hamana and others. The two notions coincide when X is a von Neumann algebra (Takesaki-Digernes). We show that they coincide if the dual action (of the group von Neumann algebra $L(G)$) on the Fubini crossed product satisfies a certain non-degeneracy property. This yields an alternative proof of the recent result of Crann and Neufang that the two notions coincide when G has the approximation property of Haagerup-Kraus.

On synthetic and transference properties of group homomorphisms

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We study Borel homomorphisms $\theta : G \to H$ for arbitrary locally compact second countable groups G and for which the Borel measure

$$\theta_*(\mu)(\alpha)) = \mu(\theta^{-1}(\alpha)$$

is absolutely continuous with respect to ν, where μ (resp. ν) is a Haar measure for G, (resp. H). We prove that either $\theta_*(\mu)$ is a Haar measure for the group $\overline{\theta(G)}$ or $\theta_*(\mu)(U)$ is equal to 0 or infinity for all open sets U. We define a natural mapping γ from the class of maximal abelian selfadjoint algebra bimodules (masa bimodules) in $B(L^2(H))$ into the class of masa bimodules in $B(L^2(G))$ and we use it to prove that if k is a set of operator synthesis, then $(\theta \times \theta)^{-1}(k)$ is also a set of operator synthesis and if $E \subseteq H$ is a set of local synthesis for the Fourier algebra $A(H)$, then $\theta^{-1}(E)$ is a set of local synthesis for $A(G)$. We also prove that if $Bim(I^\perp)$ is the masa bimodule generated by the annihilator of the ideal I in $VN(G)$, then there exists an ideal J such that $\gamma(Bim(I^\perp)) = Bim(J^\perp)$. In case $\theta_*(\mu)$ is a Haar measure for $\overline{\theta(G)}$ we show that J is equal to the ideal $\rho_*(I)$ generated by $\rho(I))$, where $\rho(u) = u \circ \theta$, for all $u \in I$.

Elementary operators on the algebra of adjointable operators on a Hilbert module

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Let A be a prime unital C*-algebra, X a countably generated Hillbert A-module, $B(X)$ the C*-algebra of adjointable operators on X and $K(X)$ the C*-algebra of (generalised) compact operators on X. We characterise multiplication operators and elementary operators on $B(X)$ in terms of the size of their images. To obtain these characterisations we introduce the concept of a uniformly approximable subset of a C*-algebra. We show that $M_{A,B}(\mathcal{B}(\mathcal{X})) \subseteq \mathcal{K}(\mathcal{X})$ if and only if at least one of A or B belongs to $\mathcal{K}(\mathcal{X})$. We show that the set $M_{A,B}(\mathcal{B}(\mathcal{X})_1)$ is a uniformly approximable subset of $\mathcal{K}(\mathcal{X})$, $(\mathcal{B}(\mathcal{X})_1$ is the unit ball of $\mathcal{B}(\mathcal{X})$), if and only if $A, B \in \mathcal{K}(\mathcal{X})$. If Φ is an elementary operator on $\mathcal{B}(\mathcal{X})$, we show that $\Phi(\mathcal{B}(\mathcal{X})) \subseteq \mathcal{K}(\mathcal{X})$ (resp. is a uniformly approximable subset of $\mathcal{K}(\mathcal{X})$) if and only if there exist $\{A_i\}_{i=1}^k, \{B_i\}_{i=1}^k \subseteq \mathcal{B}(\mathcal{X})$ such that at least one of A_i or B_i (resp. both) belong to $\mathcal{K}(\mathcal{X})$ for $i = 1, \dots, k$ and $\Phi = \sum_{i=1}^k M_{A_i,B_i}$. Finally, we assume that $\mathcal{A}$ is a separable unital C^*-algebra. We show that if Φ is an elementary operator on $\mathcal{B}(\mathcal{X})$, then the set $\Phi(\mathcal{B}(\mathcal{X})_1)$ is separable if and only if $\Phi(\mathcal{B}(\mathcal{X})) \subseteq \mathcal{K}(\mathcal{X})$.