INTRODUCTION TO COMPACT QUANTUM GROUPS

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Abstract. We will motivate and formally introduce the notion of a compact quantum group and develop rudiments of the resulting theory, trying to illustrate various possible approaches and describe several examples.

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Plan of the lectures

Lecture 1 **Basic definition and the existence of Haar state**: motivation behind the study of compact quantum groups; quantum cancellation rules; key examples; existence of the Haar state.

Lecture 2 **Representation theory**: finite-dimensional representations; right regular representation; the algebra of coefficients;

Lecture 3 **Hopf *-algebraic approach and further topics**: Peter-Weyl-Woronowicz orthogonality relations; CQG algebras (compact Hopf algebras) and their various completions; von Neumann algebraic approach; modular theory and the scaling automorphism groups; further topics.

The lectures should be accessible to the audience having a general functional analytic background and some knowledge of operator algebras.

References

[DiK] M.S. Dijkhuizen and T.H. Koornwinder, CQG algebras: a direct algebraic approach to compact quantum groups, *Lett. Math. Phys.* **32** (1994), no. 4, 315–330.

[MuT] G. Murphy and L. Tuset, Aspects of compact quantum group theory, *Proc. Amer. Math. Soc.* **132** (2004), no. 10, 3055–3067.

[Wo$_1$] S.L. Woronowicz, Compact matrix pseudogroups, *Comm. Math. Phys.* **111** (1987) no. 4, 613–665.

[Wo$_2$] S.L. Woronowicz, Compact quantum groups, *in* "Symétries Quantiques," Proceedings, Les Houches 1995, *eds. A. Connes, K. Gawedzki & J. Zinn-Justin*, North-Holland, Amsterdam 1998, pp. 845–884.