High-dimensional geometry, concentration of measure, geometric properties of the set of separable states

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The aim of these talks is to introduce a number of main tools from convex geometry, high-dimensional probability and the geometry of Banach spaces that are useful in quantum information theory.

In the first talk we discuss important parameters and operations related to finite-dimensional convex bodies, as well as tools that allow us measure the size and the typical behavior of such bodies; these include metric entropy and the concept of concentration of measure.

In the second talk we introduce Gaussian processes and discuss deep results that provide a link between the expectation of their suprema and the mean width of convex bodies. We also present applications of comparison theorems for Gaussian processes to the study of the spectral behavior of random matrices.

The third talk is devoted to some classical results from asymptotic geometric analysis. We discuss Dvoretzky's theorem which asserts that every convex body of sufficiently large dimension admits sections which are arbitrarily close to Euclidean balls, the ℓ-position of convex bodies and the MM^*-estimate.

Our example of applications of the above to quantum information theory will be estimates on the volume radius, the mean width and other quantitative parameters of the set of separable mixed quantum states when the dimension of the state space grows to infinity. The conclusion is that, apart from extremely few, almost all states are entangled.