Entanglement, games and quantum correlations

M. Anoussis

7th Summerschool in Operator Theory
Athens, July 2018

C^*-algebras

Definition

Let $\mathcal{A}$ be a Banach algebra. An involution on $\mathcal{A}$ is a map $a \to a^*$ on $\mathcal{A}$ s.t.

- $(a+b)^* = a^* + b^*$
- $(\lambda a)^* = \overline{\lambda} a^*, \lambda \in \mathbb{C}$
- $a^{**} = a$
- $(ab)^* = b^* a^*$

C^*-algebras

Definition

A C^*-algebra is a Banach algebra with an involution which satisfies

$$\|a^* a\| = \|a\|^2.$$

C^*-algebras

Examples

- $\mathcal{C}(X)$, for X compact.
  $\|g\| = \sup_{x\in X} |g(x)|$,
  $\overline{g}(x) = \overline{g(x)}$.
- $\mathcal{B}(H)$, for H Hilbert space
  $\|T\| = \sup_{x\in H, \|x\|\leq 1} \|Tx\|$
  $\langle Tx, y\rangle = \langle x, T^* y\rangle$.
- $\mathcal{A}$ a closed subalgebra of $\mathcal{B}(H)$ s.t. $a \in \mathcal{A} \Rightarrow a^* \in \mathcal{A}$.

C^*-algebras

Theorem

Let $\mathcal{A}$ be a C^*-algebra. Then $\mathcal{A}$ is isometrically isomorphic to a closed subalgebra of $\mathcal{B}(H)$ for some Hilbert space H.

states

Definition

Let $\mathcal{A}$ be a C^*-algebra. An element $a \in \mathcal{A}$ is selfadjoint if $a = a^*$.

Definition

Let $\mathcal{A}$ be a C^*-algebra. An element $a \in \mathcal{A}$ is positive if it is selfadjoint and $\sigma(a) \subseteq \mathbb{R}^+$.

states

Theorem

Let $\mathcal{A}$ be a C^*-algebra and $a \in \mathcal{A}$. The following are equivalent:

- *a is positive.*
- *$a = b^*b$ for some $b \in \mathcal{A}$.*
- *If $\mathcal{A} \subseteq \mathcal{B}(H)$, $\langle ax, x\rangle \geq 0, \forall x \in H$.*

states

Definition

Let $\mathcal{A}$ be a C^*-algebra. A linear form on $\mathcal{A}$ is positive if $f(a^*a) \geq 0$ $\forall a \in \mathcal{A}$.

Definition

Let $\mathcal{A}$ be a C^*-algebra. A state is a linear form on $\mathcal{A}$ which is positive and satisfies $f(e) = 1$.

states

The set of states $S(\mathcal{A})$ of a C^*-algebra $\mathcal{A}$ is a w^*-compact set of the dual of $\mathcal{A}$. It is convex, hence by the Krein-Milman theorem it has extreme points.

Definition

A state is pure if it is an extreme point of $S(\mathcal{A})$.

states

Examples

- $\mathcal{C}(X)$, for X compact. A state on $\mathcal{C}(X)$ is a probability measure. A pure state is a Dirac measure.
- $\mathcal{B}(H)$ for a Hilbert space H. If $\xi \in H$, $f(a) = \langle a\xi, \xi \rangle$ is a state. States of this form are called vector states.

states

Examples

- Let $\mathcal{D}$ be the C^*-algebra of 2×2 diagonal complex matrices. A linear form on $\mathcal{D}$ is of the form

$$f\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right) = xa + yd$$

for some $x, y \in \mathbb{C}$.
f is a state if and only if
$x + y = 1$ and $xa + yd \geq 0$ when $a \geq 0$ and $d \geq 0$.
That is $x \geq 0$ and $y \geq 0$.

states

Examples

f is pure iff $x = 0$ or $y = 0$.

Indeed if $f = (1, 0)$ and $f_1 = (x_1, y_1)$, $f_2 = (x_2, y_2)$, $\alpha \in (0, 1)$ are such that

$$f = \alpha f_1 + (1 - \alpha) f_2,$$

we obtain

$$f\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right) = (\alpha x_1 + (1-\alpha)x_2)a + (\alpha y_1 + (1-\alpha)y_2)d = a.$$

It follows that

$$\alpha y_1 + (1-\alpha) y_2 = 0$$

which implies that $y_1 = y_2 = 0$ and $f_1 = f_2 = f$.

states

Examples

If $f = (x, y)$, with $x \neq 0$, $y \neq 0$, then $f = xf_1 + yf_2$ where $f_1 = (1, 0)$ and $f_2 = (0, 1)$.

states

Examples

- Let $\mathcal{A}$ be the C^*-algebra of 2×2 complex matrices. A linear form on $\mathcal{A}$ is of the form

$$f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = xa + yb + zc + wd$$

for some $x, y, z, w \in \mathbb{C}$. Hence if $A \in \mathcal{A}$, $f(A) = \mathrm{tr}(GA)$ where $G = \begin{pmatrix} x & z \\ y & w \end{pmatrix}$.

states

Notation

Let $x, y \in H$.

Define an operator $|y\rangle\langle x|$ *on* H *by:*

$$|y\rangle\langle x|(|w\rangle) = |y\rangle\langle x|w\rangle = \langle x|w\rangle|y\rangle.$$

states

Proposition

1. $(|x\rangle\langle y|)^* = |y\rangle\langle x|$.
2. $(|x\rangle\langle y|) \circ (|z\rangle\langle w|) = \langle y|z\rangle |x\rangle\langle w|$.
3. $\mathrm{tr}(|x\rangle\langle y|) = \langle y|x\rangle$.

states

Examples

$f : A \to \mathrm{tr}(GA)$ is a state if and only if
$\mathrm{tr}\, G = 1$ and G is positive. (G positive $\Leftrightarrow \langle x, Gx\rangle \geq 0, \forall x \in H$.)

Indeed, we have:

$$f \text{ is positive} \Leftrightarrow \mathrm{tr}(G|x\rangle\langle x|) \geq 0 \;\; \forall x \in H$$

$$\Leftrightarrow \langle x|G|x\rangle \geq 0 \;\; \forall x \in H \Leftrightarrow G \text{ is positive.}$$

states

Examples

$$G = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$G = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

states

Proposition

Let H be a Hilbert space with $\dim H < +\infty$ and f a state on $\mathcal{B}(H)$ determined by the matrix G. Then f is pure iff G is a rank-one operator.

proof Consider

$$G = \sum_{i=1}^{\dim H} \lambda_i |x_i\rangle\langle x_i|$$

with $\lambda_i \geq 0$ and $\sum \lambda_i = 1$. □

tensor products of linear spaces

Let E_1, E_2 be linear spaces over a field $\mathbb{K}$. Consider $E_i \hookrightarrow \mathbb{K}^{X_i}$ where X_i is some set (e.g. a basis of E_i). Set

$$\xi \otimes \eta : X_1 \times X_2 \to \mathbb{K} : (s,t) \to \xi(s)\eta(t).$$

Definition (algebraic tensor product)

$$E_1 \odot E_2 := \mathrm{span}\{\xi \otimes \eta : \xi \in E_1, \eta \in E_2\} \subseteq \mathbb{K}^{X_1 \times X_2}.$$

Remark

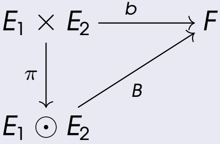
$(x_1 + x_2) \otimes y = x_1 \otimes y + x_2 \otimes y,\ x \otimes (y_1 + y_2) = x \otimes y_1 + x \otimes y_2,$
$(\lambda x) \otimes y = \lambda(x \otimes y) = x \otimes (\lambda y)\,.$

tensor products of linear spaces

Let $\pi : E_1 \times E_2 \to E_1 \odot E_2$, be the map $\pi(x, y) = x \otimes y$.

Theorem (Universal property of $(E_1 \odot E_2, \otimes)$)

If F is a linear space and $b : E_1 \times E_2 \to F$ a bilinear map, then there exists a unique linear map $B : E_1 \odot E_2 \to F$ such that $B(x \otimes y) = b(x, y)$ $\forall x \in E_1, y \in E_2$. i.e. the following diagram commutes:



tensor products of Hilbert spaces

Definition

Let H_1, H_2 be Hilbert spaces. On $H_1 \odot H_2$ set

$$\langle x_1 \otimes x_2, y_1 \otimes y_2 \rangle_{hs} = \langle x_1, y_1 \rangle_1 \cdot \langle x_2, y_2 \rangle_2 .$$

Define

$$H_1 \otimes H_2 := \overline{(H_1 \odot H_2, \|\cdot\|_{hs})}.$$

If $\{e_i\}_{i\in I}$ is an orthonormal basis of H_1 and $\{f_j\}_{j\in J}$ is an orthonormal basis of H_2, then $H_1 \otimes H_2$ has $\{e_i \otimes f_j\}_{(i,j)\in I\times J}$ as orthonormal basis.

Remark

If $\dim H_1 < +\infty$ *and* $\dim H_2 < +\infty$, *then* $H_1 \odot H_2 = H_1 \otimes H_2$.
Moreover if $\{e_i\}_{i\in I}$ *is a basis of* H_1 *and* $\{f_j\}_{j\in J}$ *is a basis of* H_2, *then* $\{e_i \otimes f_j\}_{(i,j)\in I\times J}$ *is a basis of* $H_1 \otimes H_2$.

Example

$\mathbb{C}^k \otimes \mathbb{C}^n = \mathbb{C}^n \otimes \mathbb{C}^k = \mathbb{C}^{nk}$.

operators on tensor products

If $A \in \mathcal{B}(H_1)$ and $B \in \mathcal{B}(H_2)$ we define $A \otimes B : H_1 \otimes H_2 \to H_1 \otimes H_2$.
First we define $A \otimes B$ on $H_1 \odot H_2$ by:

$$(A \otimes B)(\sum_i x_i \otimes y_i) = \sum_i Ax_i \otimes By_i.$$

The operator $A \otimes B$ is well defined and we have
$\|\sum_i Ax_i \otimes By_i\| \leq \|A\| \, \|B\| \, \|\sum_i x_i \otimes y_i\|$.
Hence $A \otimes B$ defines a bounded operator
$A \otimes B : H_1 \otimes H_2 \to H_1 \otimes H_2$ with $\|A \otimes B\| = \|A\| \, \|B\|$.

separabilty and entanglement

Consider two Hilbert spaces H_1 and H_2.
Set $H = H_1 \otimes H_2$.

Definition

A vector $\chi \in H$ is a product vector if there exist $\chi_1 \in H_1$, $\chi_2 \in H_2$ s.t.

$$\chi = \chi_1 \otimes \chi_2.$$

Definition

A pure state

$$|\chi\rangle\langle\chi|$$

on $\mathcal{B}(H)$ is called pure separable if χ is a product vector.

separabilty and entanglement

Definition

A state ρ on $\mathcal{B}(H)$ is called separable if it is a convex combination of pure separable states.

Definition

A state ρ on $\mathcal{B}(H)$ is called entangled if it is not separable.

Remark

There exist vectors which are not product vectors. Hence there exist entangled states: Take a unit vector $\psi \in H$ which is not a product vector. Then the state $\rho = |\psi\rangle\langle\psi|$ is entangled.

separabilty and entanglement

Example

Take $H_1 = H_2 = \mathbb{C}^d$.

Take an orthonormal basis $\{e_i\}_{i=1}^d$ of $\mathbb{C}^d$. Then if

$$\chi = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} e_i \otimes e_i,$$

the state

$$\rho = |\chi\rangle\langle\chi|$$

is entangled. A state of this form is called maximally entangled.

games

We consider a two-person game in which there are two players Alice and Bob and a referee R.
Let I_A, I_B be finite input sets and O_A, O_B finite output sets.
The game has a rule:
$\lambda : I_A \times I_B \times O_A \times O_B \to \{0, 1\}$.
Alice, Bob and the referee are aware of the rule.

games

The game begins when the referee gives Alice an element of the set I_A and Bob an element of the set I_B. Alice and Bob do not know what the other has been given.

They produce outputs $x \in O_A$, $y \in O_B$ independently. They win if $\lambda(a, b, x, y) = 1$ and they lose if $\lambda(a, b, x, y) = 0$.

Alice and Bob are allowed to collaborate to decide any strategy before the game begins. When the game begins they are not allowed to communicate.

games

Definition

A deterministic strategy is a pair of functions (f_A, g_B)

$f_A : I_A \to O_A$

$g_B : I_B \to O_B$

such that

$\lambda(a, b, f_A(a), g_B(b)) = 1.$

If Alice and Bob have a deterministic strategy they can always win.

games

Let $\pi : I_A \times I_B \to [0, 1]$ be a probability density.
i.e.
$\pi(a, b) \geq 0$
$\sum_{a,b} \pi(a, b) = 1.$

Definition

If $f : I_A \to O_A$, $g : I_B \to O_B$ and π is a probability density, the value of (f, g) is

$$\sum_{a,b} \pi(a, b)\lambda(a, b, f(a), g(b))$$

games

Since $\sum \pi(a,b) = 1$ and $\lambda(a,b,x,y) \in \{0,1\}$,

$$\sum_{a,b} \pi(a,b)\lambda(a,b,f(a),g(b)) \leq 1$$

Remark

If $\pi(a,b) > 0 \; \forall a,b$, then:

$$\sum_{a,b} \pi(a,b)\lambda(a,b,f(a),g(b)) = 1 \Leftrightarrow \lambda(a,b,f(a),g(b)) = \forall a,b$$

$\Leftrightarrow (f,g)$ *is a deterministic strategy.*

the graph colouring game

Definition

A graph G is a pair (V, E) where V is a set and E is a subset of the set of 2-element subsets of V.

Definition

The chromatic number of G is

$$\chi(G) = \inf\{k : G \text{ has a } k \text{ colouring}\}.$$

the graph colouring game

Example

We describe the graph colouring game. We consider a graph $G = (V, E)$.
We set $I_A = I_B = V$ and
$O_A = O_B =$ a set of colours.
If $u, w \in V$, we write $u \sim w$ if u and w are adjacent.

the graph colouring game

Example

The rule is as follows:

- If $u \sim w$,
  $\lambda(u, w, a, b) = 1$ if $a \neq b$
  $\lambda(u, w, a, b) = 0$ if $a = b$.
- If $u \nsim w$ and $u \neq w$
  $\lambda(u, w, a, b) = 1$
- If $u = w$
  $\lambda(u, u, a, b) = 1$ if $a = b$
  $\lambda(u, u, a, b) = 0$ if $a \neq b$.

the graph colouring game

Alice and Bob try to convince the referee that they have a colouring of the graph G.
If $|O_A| \geq \chi(G)$ there are more colours than the chromatic number. Hence Alice and Bob can find a colouring. This gives a function
$f = g : V \to O_A = O_B$,
such that for each $u, w \in V$ we have:
$\lambda(u, w, f(u), f(w)) = 1$.
The pair (f, f) is then a deterministic strategy.

games

Definition

A probabilistic strategy is a conditional probability density $p(x, y|a, b)$, the probability that Alice and Bob produce x and y when they receive a and b.

We have $p(x, y|a, b) \geq 0$ and $\forall a, b$

$$\sum_{(x,y)\in O_A \times O_B} (x, y|a, b) = 1$$

games

Definition

$p(x, y|a, b)$ is a perfect strategy if

$$\lambda(a, b, x, y) = 0 \Rightarrow p(x, y|a, b) = 0.$$

Definition

Given a strategy p and a density $\pi(a, b)$, $\pi : I_A \times I_B \to [0, 1]$ the value of p is

$$\sum_{x,y,a,b} \pi(a, b)\lambda(a, b, x, y)p(x, y|a, b).$$

games

Remark

Since

$$\sum_{x,y,a,b} \pi(a,b)p(x,y|a,b) = \sum_{a,b} \pi(a,b)\left(\sum_{x,y} p(x,y|a,b)\right) =$$

$$\sum_{a,b} \pi(a,b) = 1.$$

the value of p is ≤ 1.

games

Remark

If $\pi(a,b) > 0 \ \forall a, b$
then the value of p is 1 iff p is perfect.
Since $\sum_{x,y,a,b} \pi(a,b)p(x,y|a,b) = 1$, *we have:*

$$\sum_{x,y,a,b} \pi(a,b)\lambda(a,b,x,y)p(x,y|a,b) = 1 \Leftrightarrow$$

$$\{p(x,y|a,b) \neq 0 \Rightarrow \lambda(a,b,x,y) \neq 0\} \Leftrightarrow$$

$$\{\lambda(a,b,x,y) = 0 \Rightarrow p(x,y|a,b) = 0\} \Leftrightarrow p(x,y|a,b) \text{ is perfect.}$$

games

Questions

1. *Decide whether there exists a perfect strategy.*
2. *If not, find the supremum of the values, over all allowed probabilities.*
3. *Consider different models of ``quantum probability densities''.*

correlations

Alice and Bob have a common probability space (Ω, μ) and for each $a \in I_A$ Alice has a function

$$f_a : \Omega \to O_A$$

such that

$$\mu(\{\omega \in \Omega : f_a(\omega) = x\})$$

is the probability that Alice produces x, given that she received a.

correlations

Similarly, for each $b \in O_B$ Bob has a function

$$g_b : \Omega \to O_B$$

such that

$$\mu(\{\omega \in \Omega : g_b(\omega) = y\})$$

is the probability that Bob produces y, given that he received b.

correlations

We set

$$p(x, y|a, b) = \mu(\{\omega \in \Omega : f_a(\omega) = x, g_b(\omega) = y\})$$

The set of all such p is the set of local densities.
When $I_A = I_B$ and $O_A = O_B$ with $|I_A| = n$ and $|O_A| = k$ it is contained in $\mathbb{R}^{n^2k^2}$ and it is denoted by

$$C_{loc}(n, k).$$

correlations

We have a Hilbert space H_A with $\dim H_A < +\infty$.
For each $a \in I_A$ we consider a family

$$\{E_{a,x}\}_{x \in O_A}$$

such that

- $E_{a,x} \in \mathcal{B}(H_A)\ \forall x \in O_A$
- $E_{a,x} \geq 0\ \forall x \in O_A$
- $\sum_{x \in O_A} E_{a,x} = I.$

correlations

We have a Hilbert space H_B with dim $H_B < +\infty$.
For each $b \in I_B$ we consider a family

$$\{F_{b,y}\}_{y \in O_B}$$

such that

- $F_{b,y} \in \mathcal{B}(H_B)\ \forall x \in O_B$
- $F_{b,y} \geq 0\ \forall y \in O_B$
- $\sum_{y \in O_B} F_{b,y} = I.$

correlations

The strategy is as follows:

Consider a unit vector $\psi \in H_A \otimes H_B$ and the state $|\psi\rangle\langle\psi|$.

Set

$$p(x, y, |a, b) = \langle\psi|(E_{a,x} \otimes F_{b,y})\psi\rangle.$$

When $I_A = I_B$ and $O_A = O_B$ with $|I_A| = n$ and $|O_A| = k$ these are n^2k^2-tuples.

The set of all such tuples is denoted by

$$C_q(n, k).$$

It is contained in $\mathbb{R}^{n^2k^2}$ and is called the set of quantum densities.

correlations

Remark

$$C_{loc}(n,k) \subseteq C_q(n,k)$$

Remark

There are games that have perfect q strategies but not local perfect strategies.

correlations

There is a universal state space H, two families of operators $\{E_{a,x}\}_{x \in O_A}$, $\{F_{b,y}\}_{y \in O_B}$ such that

- $E_{a,x} \in \mathcal{B}(H)\ \forall x \in O_A$
- $E_{a,x} \geq 0\ \forall x \in O_A$
- $\sum_{x \in O_A} E_{a,x} = I$
- $F_{b,y} \in \mathcal{B}(H)\ \forall y \in O_B$
- $F_{b,y} \geq 0\ \forall y \in O_B$
- $\sum_{y \in O_B} F_{b,y} = I$
- $E_{a,x}F_{b,y} = F_{b,y}E_{a,x}, \forall a, x, b, y.$

correlations

Take a unit vector $\psi \in H$ and consider:

$$p(x, y|a, b) = \langle\psi|E_{a,x}F_{b,y}\psi\rangle.$$

When $I_A = I_B$ and $O_A = O_B$ with $|I_A| = n$ and $|O_A| = k$ these are n^2k^2-tuples.
The set of all such tuples is denoted by

$$C_{qc}(n, k).$$

It is contained in $\mathbb{R}^{n^2k^2}$ and is called the set of quantum commuting densities.

correlations

We have

$$C_{loc}(n,k) \subseteq C_q(n,k) \subseteq C_{qs}(n,k) \subseteq C_{qc}(n,k).$$

Here, C_{qs} is defined as C_q, but we allow $\dim H_A$ and $\dim H_B$ to be infinite.
We have also that:

$$C_{loc}(n,k) \subsetneq C_q(n,k).$$

This follows from Bell's inequalities.

Tsirelson's problem

Tsirelson's problem is the following: Is

$$C_q(n,k)^- = C_{qc}(n,k)$$

for all n, k? Here $^-$ is the closure in $\mathbb{R}^{n^2k^2}$.

Theorem (Ozawa)

The following are equivalent:

1. *Connes' Embedding Conjecture has an affirmative answer.*
2. $$C_q(n,k)^- = C_{qc}(n,k)$$ *for all n, k.*

quantum chromatic numbers

Definition

Let $t \in \{\mathrm{loc}, \mathrm{q}, \mathrm{qs}, \mathrm{qc}\}$. A game $G = \{I_A, I_B, O_A, O_B, \lambda\}$ has a perfect t-strategy if there exists $p \in C_t(n, k)$ s.t. $\lambda(a, b, x, y) = 0 \Rightarrow p(x, y|a, b) = 0$.

Definition

Given a probability density $\pi : I_a \times I_B \to [0, 1]$ and t as above the t-value of the game G is

$$w_t(G, \pi) = \sup\{\sum \pi(a, b)p(x, y|a, b)\lambda(a, b, x, y) : p \in C_t(n, k)\}.$$

quantum chromatic numbers

Idea:
Distinguish $C_t(n,k)$ by finding a game with perfect strategies for one t but without perfect strategies for another t.

Theorem (Slofstra, 2017)

$C_q(n,k)$ is not closed for $n \sim 100$, $k = 8$.

He constructed a game with a perfect qa-strategy but no perfect q-strategy ($C_{qa} = C_q^-$). The construction is based on group theoretic techniques.
Dykema-Paulsen-Prakash: $C_q(5,2)$ is not closed.

quantum chromatic numbers

Consider the graph colouring game.

Definition

For $t \in \{\mathrm{loc}, \mathrm{q}, \mathrm{qs}, \mathrm{qc}\}$ we set

$$\chi_t(G) = \min\{c \in \mathbb{N} : \exists\, p \in C_t(n, c), p \ \text{ perfect}\}.$$

Since $C_{loc} \subseteq C_q$, we have

$$\chi_{loc}(G) \geq \chi_q(G).$$

quantum chromatic numbers

question

Calculate $\chi_t(G)$ for different graphs.

Example

Tsirelson's problem has a positive answer $\Rightarrow \chi_{qa} = \chi_{qc}$.

quantum chromatic numbers

The Hadamard graph:

Let $N \in \mathbb{N}$. The set of vertices V of the Hadamard graph Ω_N is the set of N-tuples with entries ± 1 and, for $u, w \in V$, $u \sim w \Leftrightarrow \langle u, w \rangle = 0$. That is, $d_H(u, w) = N/2$. The graph Ω_N has 2^N vertices.

Theorem (Frankl-Rodl, 1987)

For all large enough n, $\chi(\Omega_{2^n}) > 2^n$.

Theorem

$\chi_{loc}(G) = \chi(G)$.

Theorem (Broussard-Cleve-Tapp, 1999)

$\chi_q(\Omega_{2^n}) \leq 2^n$.

quantum chromatic numbers

Corollary

For all large enough n, $\chi(\Omega_{2^n}) \neq \chi_q(\Omega_{2^n})$.

Corollary

For all large enough n, $C_{loc}(2^N, N) \subsetneq C_q(2^N, N)$*, where* $N = 2^n$.

More general results were obtained by Avis-Hasegawa-Kikuchi-Sasaki (2006) and Paulsen-Todorov (2015).

bibliography

V. Paulsen, Entanglement and nonlocality, PMATH 990/QIC 890,
(Notes by S. J. Harris and S. K. Pandey)
http://www.math.uwaterloo.ca/ vpaulsen/