Introduction to Dilation Theory and Applications

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Abstract

The purpose of this lecture series is to introduce the audience to the core elements of classical dilation theory, and to illustrate its magical powers with an application to Pick interpolation (here, "classical" means "older than 50 years"). Thus, this lecture series will also contain a crash course to Hilbert function spaces and operator theory on the Hardy space. These classical topics motivate and are part of the basis for several different modern lines of research, such as (non-selfadjoint) operator algebras as well as commutative multivariable operator theory that deals with spaces of analytic functions in several variables. A sketch of the plan for lectures is as follows:

1. **Lecture 1 - Basic dilation theory:** Overview (maximum principle), Wold decomposition, isometric and unitary dilations, von Neumann's inequality.
2. **Lecture 2 - Dilation theory in several commuting variables:** Ando's Theorem, Parrot's counter-example, commutant lifting theorem.
3. **Lecture 3 - An application to complex analysis:** The Hardy space, Pick's interpolation theorem via commutant lifting.

Time permitting, we may also cover: dilation theory in finite dimensions (finite dimensional Ando's theorem), commuting normal dilation of noncommuting operators, applications to completely positive maps.