Subspaces of C^*-algebras and the Shilov boundary

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Abstract

The Shilov boundary of a compact Hausdorff space X relative to a uniform algebra $\mathcal{A}$ in $C(X)$ is the smallest closed subset $K \subset X$ such that every function in $\mathcal{A}$ achieves its maximum modulus on K. This notion is encountered, in particular, in the theory of analytic functions in relation to the maximum modules principle. I will discuss its non-commutative analog introduced by W. Arveson. I will start with basic definitions and facts of the theory of operator spaces and operator systems and proceed with its fundamental theorems - Arveson's extension theorem (for completely positive and completely contractive maps), Choi-Effros' theorem and finally Hamana's result on injective envelope of an operator system from which the existence of the Shilov boundary will be derived. Another approach to Shilov boundary is via maximal dilations that will be demonstrated as well. I will conclude with some natural examples of (non-commutative) algebras and their Shilov boundaries.

References

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