Reliability of a Consecutive k-out-of-r-from-n:F System

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Key Words **— Upper bound, Lower bound, Consecutive *k*-out-of-*r*-from-*n*:F system**

Reader Aids —
Purpose: Derive sharp bounds and give exact values
Special math needed for explanations: Introductory probability theory
Special math needed to use results: Same
Results useful to: Reliability analysts and probabilists.

Abstract **— The reliability of the consecutive *k*-out-of-*r*-from-*n*:F system is studied. For $k = 2$ an explicit solution is given for *n* components in line or in cycle in the i.i.d. case. For $k \geq 3$ sharp lower and upper bounds are given for the reliability of the system and demonstrated for different values of *n, k, r, p*. These bounds are exact for $r = n, n-1, n-2, n-3$ and for these values the exact analytic solution is also given.**

1. INTRODUCTION

The consecutive k-out-of-r-from-n:F system has n ordered components and fails iff at least k, out of r consecutive components, fail. This definition is extended to cover a system of n components in cycle, and of course $1 \leq k \leq r \leq n$. When $k = r$ we have the consecutive k-out-of-n:F system which was introduced by Chiang & Niu [4], Bollinger [1, 2], and a closed form solution is known, Hwang [7], Derman, Leiberman, Ross [5]. When $r = n$ we have the simple k-out-of-n:F system for which the solution is also known. The system studied in the present paper has been mentioned by Tong [19]. This problem has a number of interesting applications. Saperstein [15] & Naus [10, 11] studied this problem as the generalized birthday problem and applied it to a test for non-random clustering. Later Saperstein [14, 16] studied the same problem in connection with quality control and inspection procedures. Their solutions either are not for all values of the parameters or are quite complicated. Another application is in the radar detection "sliding window detection probabilities" Nelson [12].

Example

A telecommunication system uses n-byte messages. The last bit of every byte is a parity bit (1 when the parity of the byte is correct). An error detector indicates an error when it finds 2 or more errors in a "window" of width 4 in the parity bit sequence. This is a consecutive 2-out-of-4-from-n system.

In this paper we examine the i.i.d. case where the probability of failure is the same for each component and the components function s-independently. For $k = 2$ we give an explicit solution for the linear and the circular case. For $k \geq 3$ an explicit solution is difficult to find and we give sharp upper and lower bounds for the reliability of the system. Table 1 presents numerical results for various values of n, k, r, p. Exact solutions are given for the special cases $r = n-1, n-2, n-3$.

TABLE 1
Lower & Upper Bounds for $F_L(p;k,r,n) \equiv F$

n	r	k	p	L	LB	F	UB	U
5	3	2	.25	.958	.958	.958	.958	.958
5	3	3	.25	.633	.633	.633	.633	.633
5	3	2	.50	.719	.719	.719	.719	.719
5	3	3	.50	.250	.250	.250	.250	.250
5	3	2	.75	.288	.288	.288	.288	.288
5	3	3	.75	.039	.039	.039	.039	.039
10	7	2	.25	.999	.993	.999	1.000	.999
10	7	2	.50	.983	.982	.983	1.000	.983
10	7	2	.75	.718	.686	.718	.728	.718
10	7	5	.50	.379	.364	.379	.393	.379
10	7	5	.75	.032	.030	.032	.033	.032
15	7	3	.50	.950	.971	.976	1.000	.998
15	7	3	.75	.450	.478	.559	.736	.716
15	7	5	.75	.031	.047	.058	.068	.072
15	7	6	.75	.004	.005	.007	.007	.009
15	10	7	.50	.278	.321	.333	.421	.432
15	10	8	.50	.103	.119	.131	.156	.167
15	10	2	.75	.866	.882	.898	.982	.971
15	10	3	.75	.615	.641	.679	.775	.819
15	10	4	.75	.337	.367	.390	.471	.514
15	10	5	.75	.136	.152	.165	.196	.223
15	10	6	.75	.040	.045	.053	.058	.066
15	12	8	.25	.916	.903	.916	.926	.916
15	12	9	.25	.772	.745	.772	.781	.772
15	12	2	.75	.911	.904	.911	.924	.911
15	12	3	.75	.728	.713	.728	.747	.728
20	10	7	.50	.283	.404	.457	.622	.607
20	10	8	.50	.103	.162	.204	.279	.265
20	10	9	.50	.023	.036	.049	.058	.064
20	10	10	.50	.002	.005	.007	.007	.007
20	12	10	.50	.038	.055	.067	.086	.092
20	12	11	.50	.007	.009	.014	.015	.017
20	12	12	.50	.000	.001	.001	.001	.001

• L, U were found using conditional probability.
• LB, UB were found through the improved Bonferroni inequalities.
• For $n = r + 1$, $r + 2$ all bounds are exact, for $n = r + 3$ the L,U are exact and for $n > r + 3$ the LB, UB are sharper than L, U.

Notation

n number of components in the system
r a "window" of r consecutive out of n components, $r \leq n$

k minimum number of failed components out of r consecutive, which cause system failure, $k \le r$

p,q probability that component is good, failed. $p+q \equiv 1$

L,C subscripts; L implies a linear system, C implies a circular system

$R_a(p;k,r,n)$ reliability of the consecutive k-out-of-r-out-of-n:F system ($a = L$ or C)

$F_a(p;k,r,n)$ $1-R_a(p;k,r,n)$: failure probability of the system ($a = L$ or C)

$N_a(j,n,r,k)$ number of ways the system works, conditional on j failed components. This is equal to the number of ways of placing at least $r-k+1$ working components among k failed components, conditional on j failed components ($a = L$ or C)

N $n-r+1$

A_i event: there are at least k failed components from i to including $i+r-1$, for $i = 1,\ldots,N$

W $\Pr\{A_i\}$,

$g(u)$ $\Pr\{A_i A_{i+u}\}$, $1 \le i \le N-1$, $1 \le u \le r-1$

$h(u,z)$ $\Pr\{A_i A_{i+u} A_{i+u+z}\}$, $1 \le i \le N-2$, $1 \le u \le r-1$, $1 \le z \le r-1$

$\overline{}$ implies the complement of an event

S_1 $\Sigma_i \Pr\{A_i\}$, for $1 \le i \le N$

S_2 $\Sigma_{ij} \Pr\{A_i A_j\}$, for $1 \le i < j \le N$

S_3 $\Sigma_{ijv} \Pr\{A_i A_j A_v\}$, for $1 \le i < j < v \le N$

LB,UB lower, upper bound for $F_L(p;k,r,n)$ using improved Bonferroni inequalities

$A_{i,\lambda}$ event: the subsystem of units $i, i+1, \ldots i+\lambda-1$ fails; $\Pr\{A_{i,\lambda}) = F_L(p;k,r,\lambda)$

$\binom{x}{j}$ binomial coefficient

$[X]$ integer part of X: trunc(X)

U,L upper, lower bound for $F_L(p;k,r,n)$ using conditional probability

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. CASE: $k = 2$

Lemma 1. For $k = 2$,

1. $$N_L(j,n,r,2) = \binom{n-(j-1)(r-1)}{j} \quad (1)$$

2. $$N_c(j,n,r,2) = \frac{n}{n-j(r-1)} \binom{n-j(r-1)}{j}. \quad \square(2)$$

Theorem 1. For $k = 2 \le r \le n$,

$$R_L(p;2,r,n) = \sum_{j=0}^{m} \binom{n-(j-1)(r-1)}{j} q^j p^{n-j}, \quad (3)$$

$m \equiv [(n+r-1)/r]$;

$$R_c(p;2,r,n) = \sum_{j=0}^{s} \frac{n}{n-j(r-1)} \binom{n-j(r-1)}{j} q^j p^{n-j} \quad (4)$$

$s \equiv [n/r]$. □

Lemma 1 and theorem 1 are proved in Naus [11]. Similar combinatorial problems involving combinations with restrictions appear in [3].

3. CASE: $n = r+\lambda$, $\lambda \le r$

The following results are proved in the appendix.

Theorem 2. For $n = r+\lambda$, $\lambda \le r$:

$$R_L(p;k,r,n) = \sum_{x=1}^{k} R_L(p;x,\lambda,2\lambda) \binom{r-\lambda}{k-x} p^{r-\lambda-k+x} q^{k-x} \quad (5)$$

$R_L(p;x,\lambda,2\lambda) \equiv 1$, if $x > \lambda$. □

Corollary 1.

a. For $n = r+1$,

$$R_L(p;k,r,n) = \binom{r-1}{k-1} p^{r-k+2} q^{k-1} + \sum_{x=0}^{k-2} \binom{r-1}{x} p^{r-1-x} q^x. \quad (6)$$

b. For $n = r+2$,

$$R_L(p;k,r,r+2) = \binom{r-2}{k-1} p^{r-k+3} q^{k-1} + p^2(1+2q) \binom{r-2}{k-2} p^{r-k} q^{k-2} + \sum_{x=0}^{k-3} \binom{r-2}{x} p^{r-x-2} q^x. \quad (7)$$

c. For $n = r+3$,

$$R_L(p;k,r,r+3) = \sum_{x=0}^{k-4} \binom{r-3}{x} p^{r-3-x} q^x + (1-4q^3+3q^4) \binom{r-3}{k-3} p^{r-k} q^{k-3} + (p^5 q + p^4(1+2q) + qp^2(p^3+3qp^2)) \binom{r-3}{k-2} \cdot p^{r-k-1} q^{k-2} + p^6 \binom{r-3}{k-1} p^{r-k-2} q^{k-1}. \quad \square(8)$$

4. LOWER AND UPPER BOUNDS FOR $F_L(p;k,r,n)$, $k \geq 3$

Since, for $k \geq 3$, an explicit solution is difficult, we derive two types of lower and upper bounds for the reliability of the system using: 1) the improved Bonferroni inequalities in Kounias & Sotirakoglou [8], and 2) the notion of conditional probability.

4.1 *Using Improved Bonferroni Inequalities*

$$R_L(p;k,r,n) = \Pr\left\{\bigcap_{i=1}^{N} \bar{A}_i\right\} = 1 - \Pr\left\{\bigcup_{i=1}^{N} A_i\right\}, \tag{9}$$

$$\text{LB} \leq \Pr\left\{\bigcup_{j=1}^{N} A_j\right\} = F_L(p;k,r,n) \leq \text{UB}. \tag{10}$$

LB, UB are [8]:

$$\text{LB} = a_1 S_1 - a_2 S_2 + a_3 S_3$$

$$a_1 \equiv \frac{2N+t-1}{N\cdot(t+1)},$$

$$a_2 \equiv \frac{2(N+2t-2)}{N\cdot t\cdot(t+1)},$$

$$a_3 \equiv \frac{6}{N\cdot t\cdot(t+1)},$$

$$t \equiv \left[\frac{2((N-2)S_2-3S_3)}{(N-1)S_1-2S_2}\right]$$

$$\text{UB} = \min(1, S_1 - b_2 S_2 + b_3 S_3)$$

$$b_2 \equiv \frac{2(2t-1)}{t\cdot(t+1)},$$

$$b_3 \equiv \frac{6}{t\cdot(t+1)}$$

$$t \equiv [3S_3/S_2] + 2$$

The formulas for $\Pr\{A_i\}$, $\Pr\{A_iA_j\}$, $\Pr\{A_iA_jA_v\}$ are in the appendix. S_1, S_2, S_3 are:

Theorem 3.

i. $S_1 = N\cdot W$

ii. $$S_2 = \binom{N-r+1}{2}\cdot W^2 + \sum_{u=1}^{m}\binom{N-u}{1} g(u),$$

$$\cdot\ m = \min(r-1, N-1) \tag{11}$$

iii. $$S_3 = \binom{N-2r+2}{3}\cdot W^3 + 2\cdot W\sum_{u=1}^{m}\binom{N-u-r+1}{2} g(u) + \sum_{u=1}^{t}\sum_{z=1}^{s}\binom{N-u-z}{1} h(u,z), \tag{12}$$

$$m \equiv \min(r-1, N-r-1)$$

$$s \equiv \min(r-1, N-1-u)$$

$$t \equiv \min(r-1, N-2)$$ □

The proof is in the appendix. The bounds of Sobel & Uppuluri [18] are special cases of the above bounds as proved in [8].

4.2 *Using Conditional Probability*

A very simple expression for the upper bound comes from partitioning the system (linear or circular) into $s \equiv [n/(r+3)]+1$ independent linear subsystems with $r+3$ components each, but one with $u = n \bmod (r+3)$ components. If the system works then all these independent subsystems work but not vice-versa, hence:

$$\Pr\{\bar{A}_{v,u}\}\prod_{i}\Pr\{\bar{A}_{i,r+3}\} \geq R_a(p;k,r,n), \tag{13}$$

for $i = 0, r+3, 2r+6 \dots (s-2)(r+3)$

$v = n - n \bmod (r+3) + 1.$

Since the units are i.i.d., (13) becomes:

$$1 - L = R_L(p;k,r,u)\cdot(R_L(p;k,r,r+3))^{s-1} \tag{14}$$

A lower bound may be determined as follows:

From [5]:

$\Pr\{\bar{A}_{0,r+m-1}\}\ \Pr\{\bar{A}_{m,n-m}\} \leq \Pr\{\bar{A}_{0,n}\}$ for every $m = 1,\dots,n-r.$

Applying this result we have:

$$R_a(p;k,r,n) \geq \Pr\{\bar{A}_{0,r+3}\}\Pr\{\bar{A}_{4,r+3}\}\dots\Pr\{\bar{A}_{4(s-1),r+3}\}\cdot\Pr\{\bar{A}_{4s,t}\} \tag{15}$$

$w \equiv n-1$ for $a = C$, $w \equiv n-r$ for $a = L$

$t \equiv (w+1) \bmod 4$, $s \equiv [(w+1)/4]$.

The product in (15) includes s terms, each of them a consecutive k-out-of-r-out-of-$r+3$:F system given in (8) and one term having t units, when $t > 0$.

For i.i.d. units we have:

$$1-U = R_L(p;k,r,r+t-1)\ (R_L(p;k,r,r+3))^s \qquad (16)$$

For $t = 0$, the first term on the r.h.s. of (16) is omitted. The bounds calculated above give exact values of reliability, $L = U$ for $n = r,\ r+1,\ r+2,\ r+3$.

Table 1 presents numerical values for the bounds of $F_L(p;k,r,n)$. The value of $F_L(p;k,r,n)$, also in table 1, has been calculated using computer simulation and complete enumeration. The lower and upper bounds LB, L, UB, U in table 1 were calculated, using theorem 3 and (6)-(8), (14), (16), for various values of n,r,k,p.

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APPENDIX

A.1 Proof of Theorem 2.

Given that in the range $\lambda,\ldots,r-1$ ($r-\lambda$ units) exactly $k-x$ units fail, at most $x-1$ units may fail in a 'window' of length λ in the line $0,\ldots,\lambda-1,r,\ldots n$ (2λ units) for the system to function. Applying the theorem of total probability we obtain (5). *Q.E.D.*

A.2 Proof of Corollary 1.

a. Using theorem 2 for $\lambda = 1$ we get from (5),

$$R_L(p;k,r,r+1) = \sum_{x=1}^{k} R_L(p;x,1,2) \binom{r-1}{k-x} p^{r+x-k-1} q^{k-x}$$

$R_L(p;x,1,2)$ equals 1 for $x>1$ and p^2 for $x=1$. Thus, we have:

$$R_L(p;k,r,r+1) = \sum_{x=0}^{k-2} \binom{r-1}{x} p^{r-x-1} q^x + p^2 \binom{r-1}{k-1} p^{r-k} q^{k-1}.$$

b. Using theorem 2 for $\lambda = 2$ we get,

$$R_L(p;k,r,r+2) = \sum_{x=1}^{k} R_L(p;x,2,4) \binom{r-2}{k-x} p^{r+x-k-2} q^{k-x}$$

$$= \sum_{x=0}^{k-3} \binom{r-2}{x} p^{r-x-2} q^x + R_L(p;1,2,4) \binom{r-2}{k-1} \cdot p^{r-k-1} q^{k-1} + R_L(p;2,2,4) \binom{r-2}{k-2} p^{r-k} q^{k-2}. \qquad (17)$$

Since $R_L(p;1,2,4) = p^4$ using corollary 1a we get,

$$R_L(p;2,2,4) = pR_L(p;2,2,3) + pqR_L(p;2,2,2) = pq(1-q^2) + p(p^2q+p) = p^2(1+2q). \qquad (18)$$

From (12), (13) we obtain (7).

c. A result from [7] states that:

$$R_L(p;k,k,n) = \sum_{j\ge 0} (-1)^j p^{j-1} q^{kj} \left(\binom{n-kj+1}{j} - q \binom{n-kj}{j}\right)$$

Hence $R_L(p;3,3,6) = 1-4q^3+3q^4$.

Using theorem 1 we get

$$R_L(p;2,3,6) = pR_L(p;2,3,5) + qp^2R_L(p;2,3,3) = p\{qp^4 + p^3(1+2q)\} + qp^2\{p^3 + 3qp^2\}$$

Since $R_L(p;1,3,6) = p^6$ using theorem 2 we obtain relation (8). *Q.E.D.*

A.3 $\Pr\{A_i\}$, $\Pr\{A_iA_j\}$, $\Pr\{A_iA_jA_v\}$.

For $1\le i<j<v\le N$ we have:

$$\Pr\{A_i\} = \sum_{x=k}^{r} \binom{r}{x} q^x \cdot p^{r-x} = W. \qquad (19)$$

$$\Pr\{A_iA_j\} = \Pr\{A_i\}\Pr\{A_j\} = W^2 \text{ if } i+r-1<j. \qquad (20)$$

$$\Pr\{A_iA_j\} = \sum_{x_1=t_1}^{m_1} \sum_{x_2=t_2}^{m_2} \sum_{x_3=t_3}^{m_3} \binom{r+i-j}{x_1} \binom{j-i}{x_2} \binom{j-i}{x_3} q^x \cdot p^{r+j-i-x}. \qquad (21)$$

$x \equiv x_1+x_2+x_3,\ i<j\le r-1,\ k\le x_1+x_2\le r,\ k\le x_1+x_3\le r$

$t_1 \equiv \max(0,k-j+i),\ m_1 = r+i-j$

$t_2 \equiv \max(0,k-x_1),\ m_2 = j-i$

$t_3 \equiv \max(0,k-x_1),\ m_3 = j-i$

$\Pr\{A_i A_j A_v\} =$

i. $\Pr\{A_i\}\cdot\Pr\{A_j\}\cdot\Pr\{A_v\} = W^3,$

$$\text{if } i+r-1<j,\ j+r-1<v \tag{22}$$

ii. $\Pr\{A_iA_j\}\cdot\Pr\{A_v\} = \Pr\{A_iA_j\}\cdot W,$

$$\text{if } j\le i+r-1,\ v>j+r-1 \tag{23}$$

iii. $\Pr\{A_i\}\cdot\Pr\{A_jA_v\} = W\cdot\Pr\{A_jA_v\},$

$$\text{if } i+r-1<j,\ v\le j+r-1 \tag{24}$$

iv. $$\sum_{x_1=t_1}^{m_1}\sum_{x_2=t_2}^{m_2}\sum_{x_3=t_3}^{m_3}\sum_{x_4=t_4}^{m_4}\sum_{x_5=t_5}^{m_5}\binom{i+r-v}{x_1}\binom{v-j}{x_2}\cdot\binom{j-i}{x_3}\binom{j-i}{x_4}\binom{v-j}{x_5}\cdot q^x\cdot p^{v+r-i-x}, \tag{25}$$

if $j\le i+r-1,\ v\le j+r-1$

$x_1+x_2+x_3\ge k,\ x_1+x_2+x_4\ge k,\ x_1+x_3+x_5\ge k,$

$x \equiv x_1+x_2+x_3+x_4+x_5,$

$t_1 \equiv \max(0,k-v+i),\ m_1 = i+r-v$

$t_2 \equiv \max(0,k-x_1-j+i),\ m_2 = v-j$

$t_3 \equiv \max(0,k-x_1-x_2),\ m_3 = j-i$

$t_4 \equiv \max(0,k-x_1-x_2),\ m_4 = j-i$

$t_5 \equiv \max(0,k-x_1-x_3),\ m_5 = v-j$

$\Pr\{A_i\} = W$ is independent of i $1\le i\le N$, $\Pr\{A_iA_j\}$ is a function of $j-i = u$ and $\Pr\{A_iA_{i+u}\}$ is given in (20) for $r-1<u\le N-1$ and in (21) for $1\le u\le r-1$. Also $\Pr\{A_iA_jA_v\}$ is given in (22), (23), (24), (25) for various values of u and z.

A.4 Proof of Theorem 3

From (19) we have $S_1 = N\cdot W$. There are

$$\binom{n-2r+2}{2} = \binom{N-r+1}{2}$$

cases with $1\le i<j\le N,\ i+r-1<j$ and $N-u$ cases with $1\le i<j = i+u\le N$.

There are

$$\binom{n-3r+3}{3} = \binom{N-2r+2}{3}$$

cases with $1\le i<j<v\le N,\ j>i+r-1,\ v>j+r-1,$

$$\binom{N-u-r+1}{2}$$

cases with $1\le i<i+u+r-1<v,\ v+r\le n$

$N-u-z$ cases with $1\le i<i+u+r\le N.$ *Q.E.D.*

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