

*Seventh International Workshop
on Retrial Queues*

7th WRQ

July 17-19, 2008

*University of Athens, 'Kostis Palamas' building
Athens, Greece*

Workshop Schedule

Sponsored by the University of Athens, Greece

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Prologue

This meeting is the 7th in a series of workshops scoping to promote research and encourage interaction in the community of retrial queues. The series of workshops in retrial queues initiated in 1998 in Madrid as a forum for the communication and cooperation among the people that work in this subarea of queueing. The workshops are open to Applied Probabilists, Operation Researchers, Engineers, Computer Scientists and Statisticians with a main or side interest in retrial queues. The scope of the conference includes both theoretical papers with advances in mathematical techniques that can be useful for analyzing retrial queueing models, as well as applications of retrial queueing systems. More specifically, the topics of the 7th WRQ include analytical techniques, computational methods, optimization, control, statistical inference and applications of retrial queues.

The 7th WRQ has gathered together 32 participants from 12 countries all over the world. We would like to express our deep and sincere gratitude to all the participants for their contributions. We also thank the University of Athens for supporting the organization of this workshop.

Jesús R. Artalejo & Antonis Economou

Program Committee

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Antonis Economou (University of Athens, Greece)

Co-chairman

Jesús R. Artalejo (Complutense University of Madrid, Spain)

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Dimitris Fakinos (University of Athens), member

Spyridoula Kanta (University of Athens), secretary - booklet

Stella Kapodistria (University of Athens), secretary - webpage

Vaso Kleftaki, secretary - public relations

Workshops on Retrial Queues: A Brief History

The series of international workshops on Retrial Queues started in Madrid, September 22-24, 1998.

The 1st International Workshop on Retrial Queues (WRQ'98) gathered 25 participants from 12 countries. The Chairman was Professor Jesus R. Artalejo. The Proceedings were published in a special issue of *Top* (Volume 7, Number 2, 1999).

Professor Alexander N. Dudin chaired the 2nd International Workshop on Retrial Queues (WRQ'99), Minsk, June 22-24, 1999. The workshop was conducted jointly with the 15th Belarusian Workshop on Queueing Theory. The Proceedings were published in the monograph 'Queues, Flows, Systems and Networks', Belarus State University Publications.

The 3rd International Workshop on Retrial Queues (WRQ'00) was held in Amsterdam, March 13-15, 2000, at the Tinbergen Institute. The Program Chair was Professor Henk C. Tijms from Vrije University of Amsterdam. About 23 participants from both Western and Eastern European countries, USA, Canada, Israel, Sweden and Japan attended the meeting. The European Commission gave support to the above conferences through the INTAS project 96-0828 entitled 'Advances in Retrial Queueing Theory'.

The 4th International Workshop on Retrial Queues (WRQ'02) was held in Cochin, December 17-21, 2002, at the Cochin University of Science and Technology. The conference Chairman was Professor A. Krishnamoorthy. The Proceedings were published in the book 'Advances in Stochastic Modelling', Notable Publications, Inc., New Jersey.

Professor B.D. Choi chaired the 5th International Workshop on Retrial Queues (WRQ'04) at the Telecommunication Mathematics Research Center, Korea University. This meeting was combined with a Workshop on

Performance Evaluation of Telecommunication Systems.

The 6th International Workshop on Retrial Queues (WRQ'06) was held at 'La Cristalera', Miraflores de la Sierra, Madrid, July 8-10, 2006. The Program Chair was Prof. A Gomez-Corral. That edition gathered 23 participants from 9 countries. A selection of original, high quality contributions was published at the special issue 'Advances in Retrial Queues' of the European Journal of Operational Research.

The 7th International Workshop on Retrial Queues (WRQ'08) is organized by the University of Athens, Greece, July 17-19, 2008. The conference chairman is Prof. Antonis Economou. The workshop has gathered 32 participants from 12 countries. The participants are encouraged to submit a contribution for possible publication in the special issue 'Algorithmic and Computational Methods in Retrial Queues' of the Computers and Operations Research (COR) journal.

General Information

Dates

July 17 (Thursday) – July 19 (Saturday), 2008

Conference Venue

The meeting will take place at “*Kostis Palamas*” building, a historical hall of the University of Athens. The building is situated very near to the central University building, at Akadimias str. number 48 and Sina str. It is only 5 minutes walk from the metro station Panepistimio.

Website

http://users.uoa.gr/~aeconom/7thWRQ_Initial.html

Social Program

Conference Diner: The conference diner will take place on the first day of the 7th WRQ (17 July 2008) at 20:30 at a beautiful restaurant in Plaka. A meeting point for the diner will be given to all participants upon their arrival at the workshop.

Excursion - Conference Meal: The main event of the social program of the 7th WRQ will take place the last day of the workshop (19 July 2008). More concretely, an excursion to the archaeological site of ‘Delphi’ is scheduled. We will take the bus at 7.30 in the morning and we will be back at about 19.30 in the afternoon. The Conference meal of the 7th WRQ will take place in a nearby picturesque village during the excursion.

Proceedings

There will not be a proceedings volume for this workshop. However, the participants are encouraged to submit their work for possible publication in the special issue ‘Algorithmic and Computational Methods in Retrieval Queues’ of the Computers and Operations Research (COR) journal. The papers should be submitted via the electronic submission system of this journal. For the deadline, the submission procedure and further details please see:

http://www.elsevier.com/framework_products/promis_misc/cfp_caor_retqueues.pdf

Program Schedule

July 17, 2008

Session 1: Control and pricing in retrial queues

Session 2: Modeling/methodological advances in retrial queues I

Session 3: Matrix analytic methods in retrial queues I

Session 4: Matrix analytic methods in retrial queues II

09:00 – 09:30 Registration

09:30 – 09:45 Opening session

Session 1

09:45 – 10:10 Title: Structural properties of the optimal policy for the multi-server controlled retrial queueing systems

Authors: *D. Efrosinin* (Institute for Stochastics, Johannes Kepler University of Linz, Austria) and *V. Rykov* (Institute for Information Transmission Problems RAS, Moscow, Russia)

10:10 – 10:35 Title: Equilibrium customer strategies and social-profit maximization in the single server constant retrial queue

Authors: *S. Kanta* and *A. Economou* (University of Athens, Greece)

10:35 – 11:00 Title: Dynamic control of a retrial queueing system with abandonments

Author: *R.D. Nobel* (Vrije University of Amsterdam, The Netherlands)

11:00 – 11:30 Coffee break

Session 2

11:30 – 11:55 Title: Performance analysis of an (s, S) inventory with repeated attempts

Author: *M.J. López-Herrero* (Complutense University of Madrid, Spain)

11:55 – 12:20 Title: A queueing system with n phases of service, vacations and retrial customers

Authors: I. Dimitriou and Ch. Langaris (University of Ioannina, Greece)

12:20 – 12:45 Title: Level crossing ordering of semi-Markov jump processes with applications to Markovian queueing networks and retrial queues

Authors: F. Ferreira (UTAD and CEMAT, Portugal) and A. Pacheco (IST - TU Lisbon and CEMAT, Portugal)

12:45 – 13:10 Title: Mean value analysis of single server retrial queues

Authors: J. Resing (Eindhoven University of Technology, The Netherlands) and J.R. Artalejo (Complutense University of Madrid, Spain)

13:10 – 14:30 Lunch break - Sandwiches

Session 3

14:30 – 14:55 Title: The $GI/G/1$ retrial queue

Author: Q-L. Li (Tsinghua University, China)

14:55 – 15:20 Title: A tandem retrial queueing system with two Markovian flows of customers and reservation of channels

Authors: C.S. Kim (Sangji University, Korea), V.I. Klimenok and O.S. Taramin (Belarusian State University, Belarus)

15:20 – 15:45 Title: Multi-server retrial queue with phase type service process: large number of servers

Authors: C.S. Kim (Sangji University, Korea), V.V. Mushko and A.N. Dudin (Belarusian State University, Belarus)

15:45 – 16:15 Coffee break

Session 4

16:15 – 16:40 Title: A single-server retrial queueing model with MAP arrivals and catastrophic failures with repairs and customer impatience

Authors: S.R. Chakravarthy (Kettering University, USA), A.N. Dudin and V.I. Klimenok (Belarusian State University, Belarus)

16:40 – 17:05 Title: Title: The $MAP/PH/1$ multi-priority retrial queue

Authors: A. Krishnamoorthy (Cochin University of Science and Technology, India) and S. Babu (University College, Thiruvananthapuram, India)

17:05 – 17:30 Title: Retrial queues with service dependent on the number of retrials I

Authors: G. Mytalis and M. Zazanis (Athens University of Economics and Business, Greece)

17:30 – 17:55 Title: Retrial queues with service dependent on the number of retrials II

Authors: *G. Mytalis and M. Zazanis* (Athens University of Economics and Business, Greece)

July 18, 2008

Session 5: Generating function methods in retrial queues

Session 6: Modeling/methodological advances in retrial queues II

Session 7: Application of retrial queues in cellular mobile systems

Session 5

09:00 – 09:25 Title: Tail asymptotics for queue sizes in retrial queues

Authors: *Jerim Kim, Jeongsim Kim and Bara Kim* (Korea University, Korea)

09:25 – 09:50 Title: Synchronized reneging in a queueing system with constant retrials

Authors: *S. Kapodistria and A. Economou* (University of Athens, Greece)

09:50 – 10:15 Title: New distributions of the discrete-time single-server retrial queueing system with general retrial times

Authors: *A.G. Hernandez-Diaz and P. Moreno* (Pablo de Olavide University, Spain)

10:15 – 10:40 A discrete-time retrial queueing system with recurrent customers

Authors: *I. Atencia* (University of Malaga, Spain), *I. Fortes* (University of Malaga, Spain), *S. Nishimura* (Tokyo University of Science, Japan) and *S. Sanchez* (University of Malaga, Spain)

10:40 – 11:10 Coffee break

Session 6

11:10 – 11:35 Title: Queue length and waiting time in a cyclic-waiting queue

Author: *L. Lakatos* (Eotvos Lorand University, Hungary)

11:35 – 12:00 Title: Distribution of new descriptors of the customer's behaviour in the single server retrial queue

Author: *J. Amador* (Complutense University of Madrid, Spain)

12:00 – 12:25 Title: A single server feedback retrial queue with collisions
Authors: *B. Krishna Kumar, S. Sadiq Basha and G. Vijayalakshmi*
(Anna University, India)

12:25 – 12:50 Retrial queues with limited number of retrials
Author: *Y.W. Shin* (Changwon National University, Korea)

12:50 – 14:00 Lunch and coffee break - Sandwiches

Session 7

14:00 – 14:25 Title: How to control retrials?
Author: *F. Machihara* (Tokyo Denki University, Japan)

14:25 – 14:50 Title: On the applicability of the number of collisions in
slotted p-persistent CSMA/CD protocols
Author: *A. Gómez-Corral* (Complutense University of Madrid, Spain)

14:50 – 15:15 Title: Cellular mobile networks with repeated calls operating
in random environment
Author: *J.R. Artalejo* (Complutense University of Madrid, Spain)

15:15 – 15:45 Closing session

Abstracts

In this section, abstracts are listed in alphabetical order of the presenting author.

Distribution of new descriptors of the customer's behaviour in the single server retrial queue

J. Amador – Complutense University of Madrid, Spain

Abstract.– We deal with the single server retrial queue with exponential repeated attempts. It should be pointed out that in the majority of applications it is very difficult to observe the retrial group; hence the main difference between a retrial queue and a standard waiting line is that the retrial group performs as an invisible queue. As a result, the original flow of primary arrivals and the flow of repeated attempts become undistinguished. Our aim is to consider aspects of this problem. Thus, we investigate the distribution of the successful and blocked events made by the primary customers and the retrial customers in the single server retrial queue.

Cellular mobile networks with repeated calls operating in random environment

J.R. Artalejo – Complutense University of Madrid, Spain

Abstract.– Many of the currently used cellular networks have been constructed on the premise that the retrial phenomenon is negligible and the operating environment is static. However, a proper modelling of the mobile cellular network cannot ignore the existence of repeated calls. Moreover, real systems often operate in varying environment conditions.

In this talk, we show how the matrix-analytic formalism gives one the ability to construct and study versatile cellular mobile networks with user retrials operating in random environment. More concretely, we investigate two three-dimensional Markovian models which allow us to represent two

different options for the use of the guard channel concept. We put emphasis on the numerical evaluation of the redial behaviour and the environmental factors on the system performance. This implies the performance analysis of a variety of descriptors including blocking probabilities (handover and fresh calls), mean average analysis, waiting time in orbit, etc.

Discrete-time retrial queueing system with recurrent customers

I. Atencia – University of Málaga, Spain

I. Fortes – University of Málaga, Spain

S. Nishimura – Tokyo University of Science, Japan

S. Sanchez – University of Málaga, Spain

Abstract.– The object of this paper is to analyze a discrete time retrial queueing system with two types of customers: transit and recurrent customers. Transit customers arrive according to a geometrical distribution of parameter p . We consider a fixed number of recurrent customers in the orbit where the successive interretrial time for this type of customers follows a geometrical law with probability r_1 while the interretrial time for the transit customers follows a geometrical distribution with probability r_2 . The service time for both types of customers is considered general. A model of the system is derived and used to calculate the main performance characteristics and numerical examples.

A single-server retrial queueing model with MAP arrivals and catastrophic failures with repairs and customer impatience

S. Chakravarthy – Kettering University, USA

A.N. Dudin – Belarusian State University, Belarus

V.I. Klimenok – Belarusian State University, Belarus

Abstract.– We consider a single server retrial queueing system in which arrivals occur according to a Markovian arrival process. An arriving customer finding an idle server will get into service immediately; otherwise the customer enters into a retrial orbit of finite (or infinite) size. The system is subject to catastrophic failures (that are assumed to be exponentially distributed) at which time all customers in the system are lost. The system undergoes a repair (which is assumed to be exponentially distributed) and after completion of a repair the server in the system will be available for service. The customers in the orbit try to reach the server by sending a signal at random times (exponentially distributed) and get service if the

server is idle at those times. While the customers are in the orbit, they may become impatient after waiting a random amount of time (exponentially distributed) and leave the system never to return. In the case of finite orbit size, any arrival finding the orbit full with the server busy is considered lost. We assume that the system can fail due to catastrophic failures only when the system is busy (i.e., the server is busy offering a service). However, this assumption can easily be modified to take into account the failure of the system when it is idle. This queueing model is studied using matrix-analytic methods and the qualitative nature of the model is brought out through some illustrative numerical examples. We also compare the results for finite and infinite (orbit) cases.

A queueing system with n phases of service, vacations and re-trial customers

I. Dimitriou – University of Ioannina, Greece

C. Langaris – University of Ioannina, Greece

Abstract.– A queueing system with a single server providing n phases of service in succession is considered. Every customer receives service in all phases. When a customer completes his service in the i^{th} phase he decides either to proceed to the next phase of service or to join the K_i retrial box ($i = 1, 2, \dots, n - 1$), from where he repeats the demand for the $(i + 1)^{th}$ phase of service after a random amount of time and independently to the other customers in the system. When there are no more customers waiting in the ordinary queue (first stage), the server departs for a single vacation of an arbitrarily distributed length. The arrival process is assumed to be Poisson and all service times are arbitrarily distributed. For such a system, the mean number of customers in the ordinary queue and in each retrial box separately are obtained, and used to investigate numerically system performance.

Multi-server retrial queue with phase type service process: large number of servers

A.N. Dudin – Belarusian State University, Belarus

C.S. Kim – Sangji University, Korea

V.V. Mushko – Belarusian State University, Belarus

Abstract.– We consider a multi-server retrial queueing system with non-persistent customers. The input flow is described by the *BMAP* (Batch Markovian Arrival Process). The service time has *PH* (phase type) distri-

bution. Known approach by V. Ramaswami, which assumes counting the number of servers currently providing the service at each phase of the PH service process instead of keeping the track of the service at each server, is applied. This allows to increase essentially the number of servers for which performance measures of the retrial queue under study can be computed within a reasonable time. Numerical illustrations are presented.

Structural properties of optimal policy for the multi-server controlled retrial queueing systems

D. Efrosinin – Johannes Kepler University of Linz, Austria

V. Rykov – Institute for Information Transmission Problems RAS, Russia

Abstract.– This paper introduces convexity results and related properties for the dynamic programming value functions of retrial queueing systems with one common queue and several heterogeneous servers. The presented approach uses the concept of event based dynamic programming which consists in dividing of the dynamic programming operator into separate event operators. Each operator corresponds to some event, e.g. primary and retrial arrivals, service completions.

We have two main objectives: to prove some monotonicity properties of the value function; to calculate the optimal control policies that minimize the mean number of customers in the system. According to the event based approach, analysis of monotonicity properties of dynamic programming operator reduces to the analysis of each event operators separately.

It is shown that the optimal control policy is of threshold and monotone type, i.e. the slower servers must be switched on only if the orbit capacity exceeds some prespecified level. Using these properties we describe the system by quasi-birth-and-death (QBD) process with policy dependent block-three diagonal infinitesimal matrices, and derive the steady-state distribution together with explicit expression for the mean number of customers as a function of threshold levels.

For exact evaluation of the optimal thresholds we use the policy iteration algorithm or minimize the explicit expression for the mean number of customers. Using the latter case we construct the regions where the certain threshold levels are optimal and derive the approximate relations to the boundaries between the neighboring regions as functions of system's parameters, and hence we get the approximate explicit relations for the threshold levels.

Level crossing ordering of semi-Markov jump processes with applications to Markovian queueing networks and retrial queues

F. Ferreira – University of Trás-os-Montes e Alto Douro and CEMAT, Portugal

A. Pacheco – Instituto Superior Técnico - Technical University of Lisbon and CEMAT, Portugal

Abstract.– We address the level crossing ordering of stochastic processes with partially ordered state spaces, which compares directly processes in terms of the times they take to reach upper sets when departing from a common state. We derive sufficient conditions for the level crossing ordering of general homogeneous Markov and semi-Markov jump processes with partially ordered Polish state spaces, and put those into use to derive specific conditions for the comparison of some Markovian queueing networks and retrial queues. We relate some of the conditions derived with those obtained by A. Economou for the usual stochastic ordering of the same networks.

On the applicability of the number of collisions in slotted p -persistent CSMA/CD protocols

A. Gómez-Corral – Complutense University of Madrid, Spain

Abstract.– One of the main concerns in wireless networks such as wireless LANs (WLANs) is the limited power supply at mobile terminals. The medium access control (MAC) protocol employed by the 802.11 WLAN belongs to the class termed CSMA (carrier sense multiple access). In particular, the 802.11 WLAN MAC protocol has been shown to be closely approximated by a p -persistent CSMA, where the transmit power consumption of a WLAN can be reduced by optimizing the access probability p . Classically, WLANs have been analyzed using Poisson arrivals and deterministic transmission times. In this context, the most commonly used approach is based on throughput-delay characteristics.

In this talk, we seek to show how the value of p can be evaluated, in an alternative manner, from the use of the number of collisions. To that end, we focus on the slotted version of a p -persistent CSMA protocol with collision detection (CSMA/CD) and an infinite number of terminals. More specifically, we first note that this protocol can be thought of as a level-dependent Markov chain of $M/G/1$ -type. The Markovian framework is formulated in terms of a $D - BMAP$, discrete phase-type transmission and collision-recovery times, and a binomial process for retransmission times. The length of the slot is fixed as a function of the propagation delay. The mass function of the number of collisions during a LAN cycle is derived by means of an

iterative scheme based on a finite assumption over the number of blocked terminals. Numerical results provide interesting insight into the choice of the access probability p for LANs with general distributions for inter-arrival times, and transmission and recovery-collision times.

Equilibrium customer strategies and social-profit maximization in the single server constant retrial queue

S. Kanta – University of Athens, Greece

A. Economou – University of Athens, Greece

Abstract.– We consider the single server constant retrial queue with a Poisson arrival process and exponential service and retrial times. This system has not waiting space so the customers that find the server busy are forced to abandon the system but they can leave their contact details. Hence, after a service completion, the server seeks for a customer among those that have unsuccessfully applied for service at a constant retrial rate.

We assume that the arriving customers that find the server busy decide whether to leave their contact details or to balk based on a natural reward-cost structure, which incorporates their desire for service as well as their unwillingness to wait. We examine the customers' behavior and we identify the Nash equilibrium joining strategies. We also study the corresponding social and profit maximization problems.

Synchronized reneging in a queueing system with constant retrials

S. Kapodistria – University of Athens, Greece

A. Economou – University of Athens, Greece

Abstract.– The phenomenon of synchronized reneging, i.e. impatient customers that perform synchronized abandonments, has been recently studied for several Markovian models. In these models we have developed an overall methodology approach based on probability generating functions. We will show how this methodology can be extended in retrial queues with synchronized abandonments. More specifically, we study the standard $M/M/1$ retrial queue, in which customers arrive according to a Poisson process at rate λ and the service times are exponentially distributed with rate μ . The retrial queue is characterized by the requirement that customers finding the server busy must leave the service area and join a group of unsatisfied customers called orbit. Customers in orbit retry after a constant time α . We suppose that the abandonments are possible only at the event epochs of a Poisson process at rate ζ which is set on every time that there is at least

one customer in orbit. At the decision epochs, every present customer remains in the orbit with probability q or abandons the orbit with probability $p = 1 - q$, independently of the others. For this model we obtain the p.g.f. of the number of customers and an algorithm for the calculation of the factorial moments.

Tail asymptotics for queue sizes in retrial queues

Bara Kim – Korea University, Korea

Jeongsim Kim – Korea University, Korea

Jerim Kim – Korea University, Korea

Abstract.– First, we consider an $M/G/1$ retrial queue, where the service time distribution has a finite exponential moment. We show that the tail of the queue size distribution is asymptotically given by a geometric function multiplied by a power function. The result is obtained by investigating analytic properties of probability generating functions for the queue size and the server state.

Second, we consider a $Geo/G/1$ retrial queue with light-tailed service time distribution. Under appropriate assumptions, we show that the tail of the queue size distribution is asymptotically geometric. It is remarkable that the results for an $M/G/1$ retrial queue and a $Geo/G/1$ retrial queue are inconsistent.

A tandem retrial queuing system with two Markovian flows of customers and reservation of channels

V.I. Klimenok – Belarusian State University, Belarus

O.S. Taramin – Belarusian State University, Belarus

C.S. Kim – Sangji University, Korea

Abstract.– We consider a tandem queueing system with a single-server first stage and multi-server second stage. The input flow at the first stage is described by the $BMAP$. The customers from this flow are considered as non-priority customers. The customers of an arriving group which meet the server being busy go to the orbit of infinite size. From the orbit, they try their luck in exponentially distributed random time. The service time at the first stage has general distribution. After service at the first stage a non-priority customer proceeds to the second stage.

The second stage consists of N identical independent servers. The service time by a server is exponentially distributed. Besides the customers proceeding from the first stage, an additionally MAP flow of priority cus-

tomers arrives at the servers of the second stage directly, not entering into the first stage. If a priority customer meets a free server upon arrival, it starts a service immediately. In the contrary case it leaves the system forever.

We assume that R of N servers at the second stage are reserved to serve the priority customer only. If a non-priority customer completes its service at the first stage and meets more than R free servers at the second stage it occupies an idle server immediately. In the contrary case the customer leaves the system forever with some probability or goes to the orbit with the complementary probability.

We calculate the stationary distribution and the main performance measures of the system. The problem of optimal design is numerically investigated.

A Single server feedback retrial queue with collisions

B. Krishna Kumar – Anna University, India

S. Sadiq Basha – Anna University, India

G. Vijayalakshmi – Anna University, India

Abstract.– A Markovian retrial queue with collision of customers is studied. The primary customers arrive in a Poisson manner and service times follow exponential distribution at a single server facility. When a customer arrives, it either encounters a free server and immediately begins service or interrupts (collides with) a customer in service; the result is that both customers are affected and enter into an orbit. The orbit customers are allowed to make retrials to get their service at the service facility under a linear retrial rate. The joint distribution of the server state and orbit length in steady state, for this system, is investigated. Some interesting performance measures are obtained. Numerical illustrations are provided.

The MAP/PH/1 multi-priority retrial queue

A. Krishnamoorthy – Cochin University of Science and Technology, India

S. Babu – University College, India

Abstract.– In this paper a multi-priority (a total of m priorities) retrial queue, with self-generation of priorities of is considered. Customers arrival is governed by a marked *MAP* and service times are Phase type distributed. We designate priorities in the descending order as: $1, 2, \dots, m-1, m$. Waiting space of finite capacities are available for customers of priority 1 to $m-1$. However, customers of the least priority, namely m , will have to

leave the service area on arrival, if they encounter a busy server at that time. Such customers proceed to an orbit of infinite capacity and retry for service according to an exponentially distributed duration with parameter proportional to the number of customers in the orbit. In addition these customers also generate into "super-priority", designated by the numeral '0', while in orbit and immediately proceed to the service station; if at this epoch the server is idle this customer is immediately admitted. Else it waits in a waiting space of capacity exactly 1, exclusively for super-priority customers. If this is also occupied, then the present super-priority turned customer will leave the system immediately in search of service elsewhere. This process of super-priority generation by orbital customers also takes place according to an exponentially distributed time with parameter, depending on the number of orbital customers. Thus our system becomes stable in a natural fashion. This is analytically established in this paper. In addition customers of priority i generate into one of higher priority j according an exponentially distributed duration with parameter depending on the number of i^{th} priority customers available at that instant. If the waiting space in priority j is full at that epoch, the present customer leaves the system for ever. Thus we have a *LDQBD* whose infinitesimal generator has a finite matrix, namely, the priority generation rate matrix, as its generator. We compute several performance measures. We also provide optimal values for the waiting space of customers of priority 1 to $m - 1$. Finally quite a bit of numerical results are provided. Some applications to communications and other disciplines are indicated.

Queue length and waiting time in a cyclic-waiting queue

L. Lakatos – Eotvos Lorand University, Hungary

Abstract.— We consider a queueing system in which the request for service may be done at the moment of arrival or, in case of refusal, at moments differing from it by the multiples of a given cycle time T . Such problems constitute a special class among retrial systems called cyclic-waiting ones. As example one can mention the landing process of airplanes and functioning of optical buffers realized via fiber delay lines. Generally, the queueing system may be characterized from the viewpoint of its owner and the customers determining the distributions of queue length and waiting time. By using the embedded Markov chain technique in case of exponential distributions we find the generating functions for both of them, show that the two approaches lead to the same stability condition.

The *GI/G/1* retrial queue

Q.-L. Li – Tsinghua University, China

Abstract.– In this talk, we study the $GI/G/1$ retrial queue. To this end, the embedded Markov chain is developed as a Markov chain with continuous state space. Using the censoring technique, we construct the RG -factorization with continuous phase. We give the queue length, waiting time and busy period. At the same time, we also provide some numerical examples.

Performance analysis of an (s, S) inventory with repeated attempts

M.J. López-Herrero – Complutense University of Madrid, Spain

Abstract.– An important feature in probabilistic inventory models is the specification of what is happening when a demand occurs but the system is out of stock. The following two cases are usually considered in the existing literature: (i) Lost sales case, where any un-served demand is lost. (ii) Backlog case, where any demand finding upon arrival the system out of stock is backlogged and is filled as soon as an adequate replenishment arrives.

The present study provides another alternative where any blocked demand leaves temporarily the service area; joins a pool of pending demands and some time later the demand is reinitiated. We consider a single-item inventory model controlled according to a continuous (s, S) policy with repeated demands exponentially distributed. The stochastic model formulation is based on a bidimensional Markov Process. We obtain performance measures directly computable from the stationary probabilities of the number of items stored in the inventory but also study the waiting time of a customer until finding a positive stock level.

How to control retrials?

F. Machihara – Tokyo Denki University, Japan

Abstract.– In cellular mobile networks, active customers on calls may take retrials, when their call services are unexpectedly interrupted by the hand-off failures. Some customers are psychologically persistent and repeat their retrials over and over. Their retrials themselves produce new retrials. When some area becomes congested, the enormous retrials cause some serious problems on the control plane stability. Some priority should be given to mobile active customers on calls over inactive customers in order to decrease hand-off failures. We now study the soft hand-off effect in $CDMA$ systems that

may control the retrials. Even if a mobile customer who is getting into the tagged cell meets all circuits being busy, his service is not interrupted and he can continue to hold his circuit in the pre-existing cell until a circuit becomes free there. However, new inconvenience happens, that is, non-priority inactive customers cannot find any free circuits, whenever there are waiting active customers in the tagged cell. That period may be called Busy Period. An area being more congested, the busy period length becomes longer. The performance deterioration of non-priority customers cannot be avoided. We evaluate that performance deterioration.

New distributions of the discrete-time single-server retrial queueing system with general retrial times

P. Moreno – Pablo de Olavide University, Spain

A.G. Hernández-Díaz – Pablo de Olavide University, Spain

Abstract.– We consider a discrete-time $Geo/G/1$ retrial queue in which the retrial time has a general distribution. Following the techniques employed by Nobel and Moreno, we derive the distribution of the busy period, the distribution of the number of customers served in a busy period and the waiting time under an FCFS discipline in the orbit. Finally, some numerical examples are presented in order to illustrate the procedures used to obtain such distributions.

Retrial Queues with Service Dependent on the Number of Retrials

G. Mytalis – Athens University of Economics and Business, Greece

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Abstract.– We consider a queueing system with a single exponential serve, no buffer, and retrials. The service rate depends upon the number of retrials that a customer in orbit has already attempted. A markovian analysis based on matrix geometric techniques is given and numerical results are presented. We also give a heavy traffic analysis based on a corresponding fluid limit approximation.

Dynamic control of a retrial queueing system with abandonments

R.D. Nobel – Vrije University of Amsterdam, The Netherlands

Abstract.–A retrial queueing model is considered with Poisson input and

a controllable number of active servers. An active server is always idle or busy. Keeping an idle server active for upcoming services requires a standby cost. When upon arrival of a customer at least one of the active servers is idle, the newly arrived customer goes into service immediately. Otherwise the decision must be made to send the customer into orbit, or to activate a new server for immediate service of the arrived customer. Activating a new server goes with a set-up cost. A customer in orbit tries to re-enter the system some random time later, but he will abandon the orbit forever once his patience time has expired. Retrial times and patience times follow an exponential distribution. For customers in orbit linear holding costs are incurred and every customer who abandons the orbit incurs a penalty cost for the system. To reduce standby costs, the system has the option to deactivate an idle server at service completion epochs. The service times of the customers are independent and follow a *Coxian* – 2 distribution. The problem is to find the optimal policy for activating new servers at arrival epochs and deactivating idle servers at service completion epochs which guarantees a minimal long-run average cost per unit time.

Using Markov decision theory this policy can be calculated in principle, but due to the large state space showing up in the mathematical description of the system a straightforward application of well-known methods like the policy-iteration algorithm is hampered. By introducing so-called fictitious decision epochs it is shown how the problem of a large state space can be circumvented.

Mean value analysis of single server retrial queues

J. Resing – Eindhoven University of Technology, The Netherlands

J.R. Artalejo – Complutense University of Madrid, Spain

Abstract.– Mean value analysis is an elegant tool for determining mean performance measures in queueing models. We use the technique to analyze the $M/G/1$ retrial queue with exponential retrial times. We also show how the relations can be adapted to obtain mean performance measures in more advanced $M/G/1$ -type retrial queues.

Retrial queues with limited number of retrials

Y.W. Shin – Changwon National University, South Korea

Abstract.– Retrial queues are characterized by the phenomenon that arriving customers who find the service facility at the capacity leave the system immediately and retry for service after random amount of time. We con-

sider the retrial queues where the number of retrials of each customer is limited by a finite number, say m . That is, if a customer fails to enter the service facility at m^{th} retrial, then the customer leaves the system forever. The monotonicity results and approximations for the number of customers that have failed k ($k = 0, 1, \dots, m$) times to enter the service facility are presented with some numerical examples.

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