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Book review

Invitation to fixed-parameter algorithms

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1. Introduction

Classic complexity theory indicates that a large number of natural combinatorial problems are inherently hard to solve algorithmically (in technical terms, they are NP-hard and, thus, it is not expected that they admit a polynomial time algorithm). The parameterization idea tries to cope with this rather discouraging fact by asking whether it is possible to to design algorithms where the non-polynomial part of their running times depends exclusively on some parameter of the problem input. The idea in this approach is that the chosen parameterization reflects a good part of real instances of the problem.

Parameterized complexity and algorithms have developed rapidly during the last three decades. Since the fundamental work of Downey and Fellows in [1,3–5], parameterized complexity theory introduced numerous innovative ideas in algorithmic design and offered insightful results in almost all disciplines of theoretical computer science.

The first monograph in this field was the book: [R.G. Downey and M.R. Fellows. *Parameterized complexity*. Monographs in Computer Science. Springer-Verlag, New York, 1999.] The next two monographs in the field appeared during 2006. One was the book: [Rolf Niedermeier. *Invitation to fixed-parameter algorithms*, volume 31 of *Oxford Lecture Series in Mathematics and its Applications*. Oxford University Press, Oxford, 2006] and the other was the book: [J. Flum and M. Grohe. *Parameterized complexity theory*. Texts in Theoretical Computer Science. An EATCS Series. Springer-Verlag, Berlin, 2006]. Also, surveys on the topic appeared in [6–12]. In this review we will focus on the book of Rolf Niedermeier [14].

2. Contents and merits

The contents of [14] are divided into three parts. The first describes the foundations of the whole theory. It gives the main approach of the theory, using examples emerging from real-word problems. In my opinion this is the best way to start an "invitation" as the whole idea of parameterized complexity is justified by its impact on the solution of real problems. The same line of presentation is followed later, at the end of the first part, when the key issue of "how to parameterize" is discussed in a more formal way. The first part is devoted to the main arguments of the book. Also it introduces the formal definitions of the theory both from the complexity and the algorithmic point of view. This is done by describing the main complexity classes (FPT, W[1], W[2]). The author devotes a whole section to the key problem of VERTEX COVER as one of the most studied problems in this field. This helps to introduce the main algorithmic ideas presented in the second part of the book. I need to say that the first part of the book is already a self-contained introduction to the theory. One can have a first idea on parameterized complexity by just reading this part.

The second part of the book presents the main algorithmic methods on parameterized complexity. This is the core of the book and includes standard methods such as kernelization, search trees of bounded depth, dynamic programming, graph decompositions and further advanced techniques such as color-coding, integer linear programming, iterative compression, greedy localization and techniques emerging from the graph minor theory. The author tries to present each method using some concrete combinatorial problem where it applies. While the list of techniques presented is not (and cannot be) complete, the second part offers a very good survey of the techniques required for an introduction in this field. I believe that this part will be particularly useful for students. Also it will serve as a well-structured source of information for mature researchers who wish to get advice on the progress and merits of parameterized algorithm design.

The third part of the book contains three sections. The first concerns parameterized complexity theory. It gives the

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definitions of the main parameterized complexity classes W[1] and W[2] and presents a good collection of reductions for proving hardness for these classes. It seems to be a choice of the author not to present any further theory on the growing collection of parameterized complexity classes such as W[*P*], W[*SAT*], XP, para-NP, A[2], or AW[*P*] and others. Regarding the algorithmic focus of the book, I find this choice understandable: the book avoids advanced and specialised complexity issues and concentrates its material on the complexity results that are closer to the mainstream combinatorial problems. This option is also justified by the presentation of the class M[1], related to the non-existence of parameterized algorithms with subexponential contribution of the parameter in their running times. The second section of the last part is devoted to the relations between parameterized complexity and approximability and the third section spans a wide variety of the fields in computer science where parameterized algorithms have been designed. The collection of problems offered in this last section is adequate but by no means could ever be complete. In any case, it covers, a good part of the landscape and certainly offers a good topic-oriented complement to the techniques and approaches presented in the second part.

3. Comparison

Any comparison of this book with the two other monographs on Parameterized Complexity cannot avoid stressing its clear algorithmic focus. The earlier monograph of Downey and Fellows [2] and the contemporary one of Flum and Grohe [13] tend to be more extended in material, devoting a good part to the parameterized complexity theory. However, the algorithmic material of [2] has already been superseded by the rapid development of the field. Currently, [14] constitutes a rich and up-to-date source of algorithmic techniques. It also seems that the algorithmic content of [14] is more extended and more topic-oriented than the excellent book by Flum and Grohe [13]. However, I should stress that, in [13], there are advanced topics such as automata-oriented techniques, monadic second-order logic, subexponentiality and counting parameterized algorithms that are missing in [14]. On the other hand, the content and the structure of [14] certainly makes it more inviting for a student or a beginner in this field.

4. Conclusion

My overall opinion is that Rolf Niedermeir's book fills a void in the publishing landscape on algorithms and should become a standard point of reference on fixed-parameter algorithms. Taking into account the abundance of existing and emerging results, the book makes a good and well-structured choice of material. In fact, it really succeeds to be what it intended to be in its title: *An invitation to fixed-parameter algorithms*. I think that it is the correct book to read or to suggest for anybody who wants to have a solid and self-contained immersion in this rapidly growing field.

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REFERENCES

[1] Karl A. Abrahamson, Rodney G. Downey, Michael R. Fellows, Fixed-parameter tractability and completeness. IV. On completeness for W[P] and PSPACE analogues, Ann. Pure Appl. Logic 73 (3) (1995) 235–276.
[2] R.G. Downey, M.R. Fellows, Parameterized complexity, in: Monographs in Computer Science, Springer-Verlag, New York, 1999.
[3] Rod Downey, Michael Fellows, Fixed-parameter tractability and completeness. III. Some structural aspects of the W hierarchy, in: Complexity Theory, Cambridge Univ. Press, Cambridge, 1993, pp. 191–225.
[4] Rod G. Downey, Michael R. Fellows, Fixed-parameter tractability and completeness. I. Basic results, SIAM J. Comput. 24 (4) (1995) 873–921.
[5] Rod G. Downey, Michael R. Fellows, Fixed-parameter tractability and completeness II: On completeness for W[1], Theoret. Comput. Sci. 141 (1–2) (1995) 109–131.
[6] Rodney G. Downey, Michael R. Fellows, Parameterized computational feasibility, in: Feasible Mathematics, II, Ithaca, NY, 1992, in: Progr. Comput. Sci. Appl. Logic, vol. 13, Birkhäuser Boston, Boston, MA, 1995, pp. 219–244.
[7] Rodney G. Downey, Michael R. Fellows, Parameterized complexity after (almost) ten years: Review and open questions, in: Combinatorics, Computation & Logic '99, Auckland, in: Aust. Comput. Sci. Commun., vol. 21, Springer, Singapore, 1999, pp. 1–33.
[8] Rodney G. Downey, Michael R. Fellows, Ulrike Stege, Computational tractability: The view from Mars, Bull. Eur. Assoc. Theor. Comput. Sci. EATCS 69 (1999) 73–97.
[9] Rodney G. Downey, Michael R. Fellows, Ulrike Stege, Parameterized complexity: A framework for systematically confronting computational intractability, in: Contemporary Trends in Discrete Mathematics, Štiřín Castle, 1997, in: DIMACS Ser. Discrete Math. Theoret. Comput. Sci., vol. 49, Amer. Math. Soc., Providence, RI, 1999, pp. 49–99.
[10] Michael R. Fellows, Parameterized complexity: The main ideas and some research frontiers, in: Algorithms and Computation, Christchurch, 2001, in: Lecture Notes in Comput. Sci., vol. 2223, Springer, Berlin, 2001, pp. 291–307.
[11] Michael R. Fellows, Parameterized complexity: The main ideas and connections to practical computing, in: Experimental Algorithmics, in: Lecture Notes in Comput. Sci., vol. 2547, Springer, Berlin, 2002, pp. 51–77.
[12] Michael R. Fellows, New directions and new challenges in algorithm design and complexity, parameterized, in: Algorithms and Data Structures, in: Lecture Notes in Comput. Sci., vol. 2748, Springer, Berlin, 2003, pp. 505–519.
[13] J. Flum, M. Grohe, Parameterized Complexity Theory, in: Texts in Theoretical Computer Science. An EATCS Series, Springer-Verlag, Berlin, 2006.
[14] Rolf Niedermeier, Invitation to Fixed-parameter Algorithms, in: Oxford Lecture Series in Mathematics and its Applications, vol. 31, Oxford University Press, Oxford, 2006.